Application of LQR for Phase-Locked Loop Control Systems

Somyos Khumma*, Taworn Benjanarasuth*, Don Isarakorn*, Jongkol Ngamwiwit,*
Somsak Wanchana** and Noriyuki Komine***

*Faculty of Engineering and Research Center for Communication and Information Technology
King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand.
(Tel: +662-326-4221, Fax: +662-326-4225, Email: knjongko@kmitl.ac.th)
**Faculty of Electrical and Electronics, Chonburi Technical College, Chonburi, Thailand
(Tel: +663-844-3066, Fax: +663-844-3701, Email: s_wanchana@chontech.ac.th)
***School of Information Technology and Electronics, Tokai University
1117 Kitakaname, Hiratsuka-Shi, Kanagawa-Ken, 259-1292, Japan
(Tel: +81-463-58-1211, Fax: +81-463-50-2240, Email: komine@keyaki.cc.u-tokai.ac.jp)

Abstract: A phase-locked loop control system designed by using the linear quadratic regulator approach is presented in this paper. The system thus designed is optimal system when system is in locked state and the parameter value of loop filter which is an active PI filter can be obtained easily. By considering the structure of loop filter of phase-locked loop is included in the process to be controlled, a type 1 servo system can be constructed when voltage control oscillator is considered as an integrator. The integral gain of the proposed system obtained by linear quadratic regulator approach can be used as an optimal value to design the parameter of loop filter. The implemented result in controlling the second-order lag pressure process by using the proposed scheme show that the system response is fast with no overshoot and no steady-state error. Furthermore, the experimental results are also shown in term of output disturbance effect rejection, tracking and process parameter changed.

1. INTRODUCTION

In the industry, root-locus and frequency-response methods are mainly used to design the control system to meet the desired performances. They required the transfer function for design an acceptable performance system but cannot apply for designing the optimal control system. For linear optimal control system, the system is expressed in state-space representation. The most fundamental control system design approach is the linear quadratic regulator (LQR) approach where the process or plant is assumed linear and the controller is constrained to be linear [1].

On the other hand, the phase-locked loop (PLL) technique has been extensively applied in various fields. The standard PLL is composed of a phase frequency detector (PFD), a loop filter (LF) and voltage control oscillator (VCO) [2]. In control system applications, it is well known that the PLL techniques give the result of controlling the system accurately because it uses the more stable reference frequency. However, the PLL technique tends to be slow response of the process control system. The PLL incorporated with PI controller that gives fast response but slow rejection of the disturbance effect has been presented [3]. It has also been proposed that the adaptive phase-locked loop for process control system to give fast rejection of disturbance effect but the speed of the response at transient state is still slow [4]. The feedforward adaptive PLL for controlling the process that improves both transient state and disturbance effect has been presented recently [5]. However, the method for selecting the proper value of LF of PLL is still trial and error method [6]. This will lead to time consuming in PLL control system design.

Therefore, this paper presents the PLL control system designed by using LQR approach. The system thus designed is optimal system when system is in locked state and the parameter value of LF can be obtained easily. By putting the loop filter structure, which is a active PI filter with a time constant $\tau_{F1} = 10\tau_{F2}$, into the process to be controlled, a type 1 servo system can be constructed when the VCO is considered as an integrator.

The state feedback gains and integral gain obtained by LQR approach for the proposed type 1 servo system are optimal values. The parameter value of LF for the PLL control system can be assigned from the integral gain thus obtained. Hence, this proposal allows designer easy to assign or change the PLL parameters and also the integral gain in order to meet the desired specification.

The results of implementation of the designed PLL to control the process variables of the second order lag pressure process will also be shown.

2. TYPE 1 SERVO SYSTEM

The structure of PLL control system will be described in this section first. Then how to construct a type 1 servo system will be described later.

2.1 PLL control system

A basic block diagram of the process control system using PLL technique is illustrated in Fig. 1. It is known that the PLL is a feedback-controlled system maintaining a constant phase/frequency difference between a reference input signal and a feedback output signal [2]. It composes of two exactly matches VCO, a PFD, a LF and a process. One VCO is employed to provide a phase or frequency ω_r according to the reference voltage V_r and another one converts the output voltage V_o to feedback output phase or frequency ω_o. A PFD compares the phase or frequency ω_r with the phase or frequency ω_o. A phase detector output pulse of the PFD is generated in proportion to that phase difference. This output pulse is smoothed by passing it through a LF. The resulting dc component from output of LF is used as the input voltage for controlling the process variable. So the process output voltage V_o is related to the phase difference. The output frequency ω_o is fed back to the PFD input for comparison, which in turn controls the VCO oscillating frequency to minimize the phase difference. Therefore, both frequency and phase are regulated until the synchronization, i.e., $\theta_i = \theta_o$ and $\omega_i = \omega_o$, such that the phase and frequency of the VCO and the reference signal source are in a locked state.

In order to meet the desired performance, the parameters of PLL such as a filter time constant τ_F of the LF must be specified properly and easily. Consequently, the LQR approach will be employed.

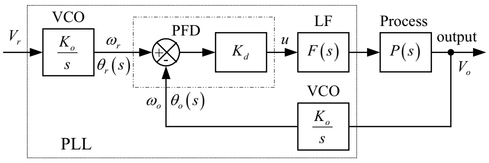

Fig. 1 PLL control system.

2. 2 Type 1 servo system arrangement

In this sub-section, a type 1 servo system obtained from the PLL control system shown in Fig. 1 is described first. The integral gain relating to parameter assigning of LF of PLL is then described later.

By merging the structure of LF into the process to be controlled, its state equation and output equation can be respectively given as

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x \in \mathrm{R}^{n\times 1}$ is the state of the process including LF, $u \in \mathrm{R}^1$ is the system control input and $y \in \mathrm{R}^1$ is the system output. The dimension of matrices A, B and C are $n \times n$, $n \times 1$ and $1 \times n$, respectively. In order to express the PLL control system into type 1 servo system, the summing point of the block PFD in Fig. 1 is moved to the front of the block VCO which will make the type 0 system of the process including LF be a type 1 system [7]. Therefore, a type 1 servo system arranged from the PLL control system with

$$\dot{e}(t) = r(t) - y(t) = r(t) - Cx(t), \tag{3}$$

can be expressed as

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u(t) + \begin{bmatrix}0\\ 1\end{bmatrix}r(t), \tag{4}$$

$$y(t) = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}, \tag{5}$$

where $e(t)$ is the error signal and $r(t)$ is the reference signal. The block diagram of the type 1 servo system arrangement can be illustrated as Fig. 2. K_d in the figure is the constant gain in V/rad for the PFD and the voltage control oscillator is represented by $\frac{K_o}{s}$, where K_o is the gain in rad/sec/V.

When the system (4) is completely controllable, the control law

$$u(t) = -\begin{bmatrix}K & -k_I\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} = -\frac{1}{R}\begin{bmatrix}Bp_{11} & -Bp_{12}\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}, \tag{6}$$

that minimizes the performance index

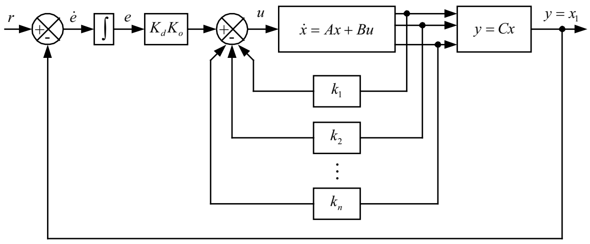

Fig. 2 Type 1 servo system arrangement.

$$J = \int_0^\infty \left(\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}^T \begin{bmatrix}Q & 0\\ 0 & q\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} + u^T R u \right) dt, \tag{7}$$

can be found, where $\begin{bmatrix}Q & 0\\ 0 & q\end{bmatrix} \geq 0$ and $R > 0$, and where the matrix $P = \begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix} > 0$ is the unique solution of the following Riccati equation

$$\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}^T P + P\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix} - P\begin{bmatrix}B\\ 0\end{bmatrix}\frac{1}{R}\begin{bmatrix}B\\ 0\end{bmatrix}^T P + \begin{bmatrix}Q & 0\\ 0 & q\end{bmatrix} = 0. \tag{8}$$

Then the following feedback system can be expressed as

$$\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}A - BK & Bk_I\\ -C & 0\end{bmatrix}\begin{bmatrix}x(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}r(t), \tag{9}$$

where K is the state feedback gain matrix and k_I is the integral gain, and this system is stable. The integral gain k_I will play an important rule to assign the parameter of LF.

3. CONTROLLER DESIGN

The procedure for LF parameter assigning and feedback gain matrix obtaining for the type 1 servo system which is rearranged from the PLL control system will be described in this section.

3.1 Loop filter design

The steps to find the parameter of LF that relates to integral gain k_I in Eq. (6) are as follows:

1. Find p_{12} from Eq. (8) which can be expressed as

   $$p_{12} = \frac{1}{B}\sqrt{Rq}.$$

2. Find integral gain k_I which is expressed as $k_I = \frac{1}{R}Bp_{12}$.
3. Substituting p_{12} into k_I which leads to $k_I = \sqrt{\frac{q}{R}}$.
4. From Fig. 2, $k_i = K_dK_o$ and the unit of K_dK_o is in frequency, therefore, it must be divided by natural frequency ω_n of the PLL which is expressed as [8] $\omega_n = \sqrt{\frac{K_dK_o}{\tau_{F1}}}$. Finally, the parameter of the active PI filter can be obtained as

$$\tau_{F1} = \frac{1}{K_d K_o}\frac{q}{R}. \tag{10}$$

3.2 Feedback gain matrix

The feedback gain matrix of the type 1 servo system shown in Fig.2 can be found as the followin steps:

1. The process is assumed to be a $(n-1)^{th}$ order and type zero system with no zero. The LF is active PI filter represented as $\frac{\tau_{F2}s+1}{\tau_{F1}s}$ for a time constant $\tau_{F1} = 10\tau_{F2}$. The matrix A, B and C of Eqs.(1)-(2) constituted from process and LF are expressed as

$$A = \begin{bmatrix} A_p & B_p \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} B_p \frac{\tau_{F2}}{\tau_{F1}} \\ \frac{1}{\tau_{F1}} \end{bmatrix} \text{ and } C = \begin{bmatrix} C_p & 0 \end{bmatrix}.$$

2. Construct the type 1 servo system of Eq. (4) and Eq.(5) by using the system constituted in step 1.
3. Find the control law as stated in section 2 so that the feedback matrix gain $\begin{bmatrix} K & -k_I \end{bmatrix}$ can be asigned.

4. EXPERIMENTAL RESULTS

In this section, the strucure of pressure process in laboratory is described first and the experimental results of the proposed control system will be investigated later.

4.1 Structure of pressure process

A second-order lag pressure to be controlled by the proposed controller is illustrated in Fig. 3. The corresponding state equation and output equation of the process are then expressed as

$$\begin{bmatrix} \dot{x}_{p1}(t) \\ \dot{x}_{p2}(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2C_2} & \frac{1}{R_2C_2} \\ \frac{1}{R_2C_1} & -\left(\frac{R_1+R_2}{R_1R_2C_1}\right) \end{bmatrix} \begin{bmatrix} x_{p1}(t) \\ x_{p2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_p}{R_1C_1} \end{bmatrix} u_p(t), \tag{11}$$

and

$$y_p(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{p1}(t) \\ x_{p2}(t) \end{bmatrix}, \tag{12}$$

where R_1 and R_2 are the gas flow resistance of valve V_1 and V_2, and C_1 and C_2 are the capacitance of pressure tank 1 and tank 2 respectively, and where u_p is the control input, y_p is the output pressure of tank 2, x_{p1} is the pressure at tank 2 and x_{p2} is the pressure at tank 1.

The unknown values of process gain K_p, R_1, R_2, C_1 and C_2 in Eq. (11) must be known so that the proposed desgin method can be employed. These values can be found from the experimental result as $K_p = 1$, the time constant of the first tank $R_1C = 10$ seconds and the second tank $R_2C = 5$ seconds, where the capacity of the two tanks are same ie., $C_1 = C_2 = C$. Furthermore, the values of $K_d = 1.4324$ V/rad and $K_o = 3.4548 \times 10^4$ rad/sec/V must also be known respectively.

By choosing the values of the weighting matrices $\begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix} = diag\begin{bmatrix} 0 & 0 & 0 & 0.05 \end{bmatrix}$ and $R = 0.1$, and adopt the LF design steps in sub-section 3.1, the value of τ_{F1} is obtained as 1×10^{-5} second. Consequently, the value of τ_{F2} is also found to be 1×10^{-6} second. The system (4) is then obtained as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} -0.2 & 0.2 & 0 & 0 \\ 0.2 & -0.3 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01 \\ 1\times10^5 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t).$$

From sub-section 3.2, the following optimal feedback gain matrix for the type 1 servo system can be found as

$$\begin{bmatrix} K & -k_I \end{bmatrix} = \begin{bmatrix} 0.2892 & 0.0121 & 0.0002 & -0.7071 \end{bmatrix}.$$

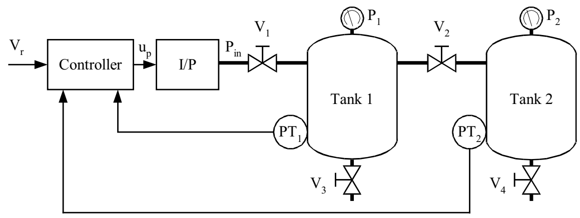

Fig. 3. Pressure process control system.

4.2 Experimental results

The controller obtained in sub-section is implemented to control the laboratory pressure process in order to demonstrate the practicability of the proposed scheme. The reference frequency of PLL is 225.73 kHz which corresponds to the pressure of 7.5 psi. The two pressure transmitters are calibrated to give a voltage of 2.5 volts at this pressure. The experimental results are recorded by the recorder model μR.180 (Yokogawa) at speed 7200 mm/hr, which can assure the accuracy of the experimental result. The IC chip No. CD4046BE is employed for PFD and VCO.

System responses

When apply the step reference signal at 7.5 psi to the type 1 servo system rearranged from PLL control system using LQR approach, its response is shown in Fig. 4. It is found that the response is fast with small overshoot, the rise time t_r is 13.03 seconds, the settling time t_s is 16.13 seconds and steady-state error is zero.

In order to show the effectiveness of the proposed control system in term of output disturbance rejection, the valve V_4 is opened at 38.5 seconds and closed when the pressur drops to 4.5 psi . The effect of the disturbance is also shown in Fig.4. It is seen that the effect of the output disturbance can also be rejected and converge to its reference signal again without steady-state error.

The response of the PLL control system (see Fig. 1) for the same pressure process and the same values of the LF is shown in Fig. 5. It can observe that the transient response resulting from both designed methods exhibit almost the same results. However, the small oscillation in the steady state is encountered due to the system is in unlocked state when the PLL control system is employed.

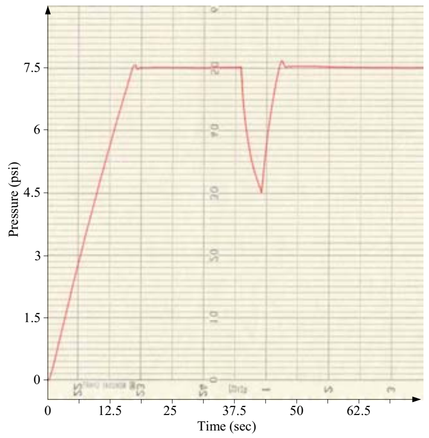

Fig. 4. Response of type 1 servo system.

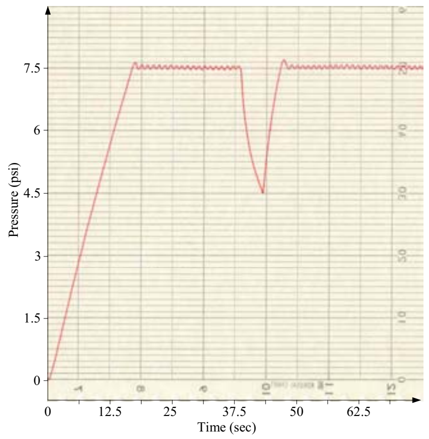

Fig. 5. Response of PLL control system.

Tracking capability

The capability of tracking of the proposed control system is investigated here. The step reference signal is considered to change from 0 psi to 4.5 psi, from 4.5 psi to 6.75 psi and from 6.75 psi to 8.25 psi of the interval 25 seconds respectively. The expeimental result of the proposed control system is shown in Fig. 6. It is seen that the system response can track the changed reference signal properly.

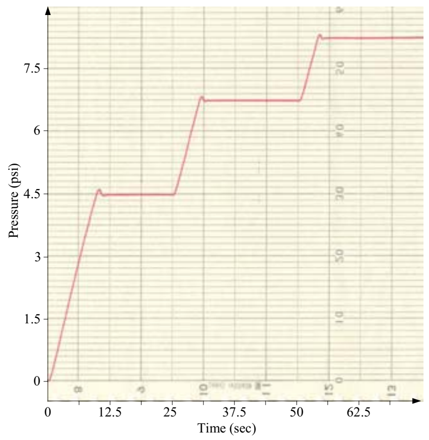

Fig. 6. Tracking capabilty.

Effect of process parameter changed

When the pressure process gain K_p is increased 25% from the nominal value without changing any values of the previous designed controller, it is seen that the system is still stable as shown in Fig. 7. It is also seen that the result is almost same as the result of Fig. 4.

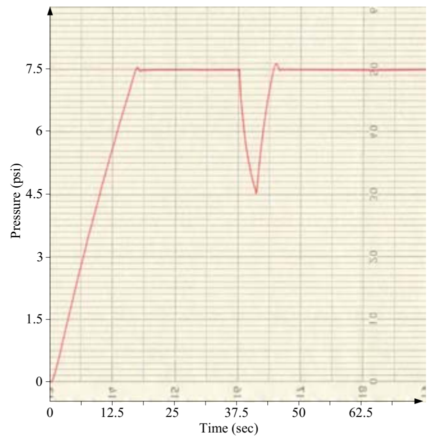

Fig. 7. Response when the pressure process gain is changed.

5. CONCLUSION

The type 1 servo system arranged from PLL control system designed by LQR approach has been proposed in this paper. The proposed technique allows the designer can assign the parameter of the LF from the weighting matrices. The better performances at steady state, the fast output disturbance effect rejection, good tracking capability and the stable system when process gain changed have also been shown in this paper.

REFERENCES

[1] Anderson B. D. O. and Moore J. B., *Optimal Control: Linear quadratic methods*, Englewood Cliffs, New Jersey, Prentice-Hall Inc., 1989.

[2] G. C. Hsieh and J. C. Hung, "Phase-Locked Loop Techniques—A Survey," *IEEE Trans. Ind. Electron.,*Vol. 43, No. 6, pp. 609-615, Dec. 1996.

[3] S. Wanchana, T. Benjanarasuth, N. Komine and J. Ngamwiwit, "PLL in cooperated with PI Controller for Flow Control System," *proceeding of 1999 IEEE International Conference on System, Man, and Cybernetics (SMC'99)*, Tokyo, Japan, pp.V-27-V-32, 1999.

[4] S. Wanchana, T. Benjanarasuth, N. Komine and J. Ngamwiwit, "Adaptive Phase-Locked Loop for Process Control System," *Proceeding of the International Conference on Control, Automation and Systems (ICCAS2001)*, Jeju, Korea, pp. 896-899, 2001.

[5] S. Wanchana, T. Benjanarasuth, N. Komine and J. Ngamwiwit, "Feedforward Adaptive Phase-Locked Loop for Process Control System," *Proceeding of SICE Annual Conference 2002 in Osaka (SICE2002)*, Osaka, Japan, pp. 2861-2864, Aug, 2002.

[6] A. W. Moore, "Phase-Locked Loops for Motor-Speed Control," *IEEE Spectrum*, pp. 61-67, Apr, 1973.

[7] Katsuhiko Ogata., *Modern Control Engineering*, 2nd Ed. New Jersey, Prentice-Hall Inc., 1990.

[8]. R. E. Best, *Phase-Lock Looped*, 4th Ed. McGraw-Hill Inc., 1997.