Input Time-Delay Compensation for a Nonlinear Control System

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Abstract: In most physical processes, the transfer function includes time-delay, and in the general distributed control system using computer network, there exists inherent time-delay caused by the spatial separation between controllers and actuators. This work deals with the synthesis of a discrete-time controller for a nonlinear system and proposes a new effective method to compensate the influence of input time-delay. The controller is synthesized by using input/output linearization. Under the circumstance that input time-delay exists, the system response has more overshoot and tends to diverge. For these reasons, the controller has to produce future input value that will be needed for the system. In order to calculate the future input value, some predictors are adopted. Using the discretization via Euler's method, numerical simulations about the Van der Pol system are performed to evaluate the performance of the proposed method.

Keywords: Nonlinear System, Input/Output Linearization, Predictor, Input Time-Delay, Compensation

1. INTRODUCTION

Time-delay occurs in control systems when there is a delay between the commanded response and the start of the output response [1]. It is caused by distance of time and space existing among the components of the control system. Time-delay in a control system decreases gain margin and phase margin; in conclusion, it deteriorates performance of the control system and makes the system unstable. Time-delay has primarily been studied in the field of process control, but the importance of research about time-delay is rising now because of the development of remote control systems through networks [2-4].

Much research has been done about linear systems in the continuous time domain limitedly, while practical physical systems have nonlinearity inherently. Recently, most of control systems are designed by using digital computer. Therefore, it is important to analyze nonlinear system with time-delay and design a controller in discrete time domain.

In order to remove the time-delay effect, Pade's approximation method was proposed. It is a controller synthesis scheme to analyze system and design controller including approximated dynamic model of time-delay element [5].

A stabilizing method was proposed for a force-reflection remote robot system. It provides inertia moment constraint and gain range of reflected torque to make the reflected torque of the master/slave system converge to zero. This method offers an easy way to design controller for the force-reflection system and focuses on improvement of relative stability [6].

Predictive control is the representative approach for the time-delay. In the field of process control, methods using Smith predictor have been proposed. It eliminates the affect of time-delay algebraically in the closed loop transfer function by modeling of system and time-delay. The Smith predictor can make us treat a system as if it has no time-delay and design controller for the system. This method is a structural time-delay compensation scheme, but there are two constraints. One is that the method can be applied to a linear system only, and the last is that it requires exact model equation of system and time-delay [7-8].

An estimator was adopted to compensate the time-delay between sensor and controller. This method uses the time-domain solution of state equation to estimate the change of states during the delay time. It is a comparatively exact way to compensate the sensor-to-controller delay, but it cannot afford the input time-delay between controller and actuator [4].

In this article, stability region analysis of a system is presented according to the sampling period and time-delay at discretization. Then, the influence of time-delay in control systems is shown and input/output linearization is described, which is a controller design scheme for nonlinear systems. Then, some prediction models are proposed to compensate the input time-delay in the discrete-time nonlinear control system. Finally, some simulations are performed to evaluate the proposed method.

2. TIME-DELAY EFFECT

2.1 Stability region

Generally, fast sampling is required to approximate the system more accurately, but fast sampling in distributed control system increases the load of network and causes the time-delay. Therefore, sampling rate is important for the robustness against the time-delay and the desired control performance.

By plotting the stability region about sampling period h and time-delay τ, the relation between stability and the two parameters can be proved. Consider a linear system described by a continuous time state-space model [9]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u), \\ y(t) &= Cx(t), \\ u(t) &= -Kx(t-\tau), \end{aligned} \tag{1}$$

where τ (less than sampling period h) denotes the time-delay. The discrete time state-space model of Eq. (1) is bellow:

$$\begin{aligned} \dot{x}((k+1)h) &= \Phi x(kh) + \Gamma_0 u(kh) + \Gamma_1 u((k-1)h), \\ y(kh) &= Cx(kh), \end{aligned} \tag{2}$$

$$\Phi = e^{Ah}, \quad \Gamma_1 = \int_{h-\tau}^{h} e^{As} B ds, \quad \Gamma_0 = \int_0^{h-\tau} e^{As} B ds. \tag{3}$$

Defining $w(kh) = \left[x^T(kh), \quad u^T((k-1)h) \right]^T$ as the augmented state vector, the augmented closed-loop system is

$$\begin{aligned} &w((k+1)h) = \tilde{\Phi}(k) w(kh), \\ &\tilde{\Phi}(k) = \begin{bmatrix} \Phi - \Gamma_0 K & \Gamma_1 \\ -K & 0 \end{bmatrix}. \end{aligned} \tag{4}$$

The eigenvalues of the above matrix can define the stability region of the system.

Example 1. Consider the integrator system [4]:

$$\begin{aligned} &\dot{x}(t) = u(t), \\ &u(t) = -Kx(t-\tau). \end{aligned} \tag{5}$$

The characteristic equation of the discrete time closed-loop system can be represented as bellow:

$$z^2 - (1 - K(h-\tau))z + K\tau = 0\,. \tag{6}$$

Fig. 1 is the stability region of the system Eq. (5), and step responses for some particular points (Table 1) on the stability region are shown in Fig. 2.

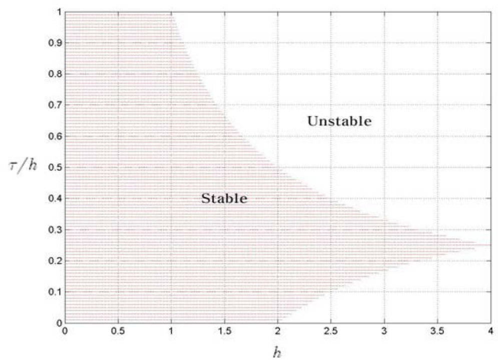

Fig. 1 Stability region of the integrator system.

The stability region presents the relation between sampling period and time-delay in stability aspect.

Table 1. Particular points on the stability region.

	Time-delay[sec]	τ/h	Stability
$h=1$	0.0 0.5 1.0	0.0 1.5 2.1	Stable Stable Marginal stable
$h=2$	0.4 0.8 1.0	0.2 0.4 0.5	Stable Stable Marginal Stable
$h=3$	0.3 0.9 1.2	0.1 0.3 0.4	Unstable Stable Unstable

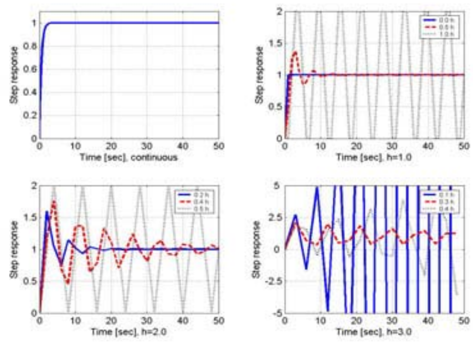

Fig.2 Step responses according to h and τ.

2.2 Time-delay effect of a linear control system

If there exists time-delay in the closed-loop system $G(z)$, the aggregated transfer function becomes Eq. 7:

$$G_d(z) = \frac{z^{-N} G(z)}{1 + z^{-N} G(z)} \tag{7}$$

N is the time-delay of sampling period size. As the time-delay increases, the order of increase, and poles move toward the outside of unit circle on the z-plane. Therefore, output has much more oscillation and becomes unstable.

Example 2. Consider a simplified model [10]:

$$G(z) = \frac{1}{2} \frac{1}{(z-1)^2}\,. \tag{8}$$

Let us suppose that position controller of Eq. (8) was designed as bellow:

$$D(z) = 0.389 \frac{z - 0.8201}{z - 0.135}\,. \tag{9}$$

As the time-delay increases, the order of aggregated transfer function including time-delay increases and location of system poles change according to the time-delay.

$$G_d(z) = \frac{1}{2} \frac{0.389(z-0.8201)}{z^N (z-1)^2 (z-0.135) + 0.389(z-0.8201)} \tag{10}$$

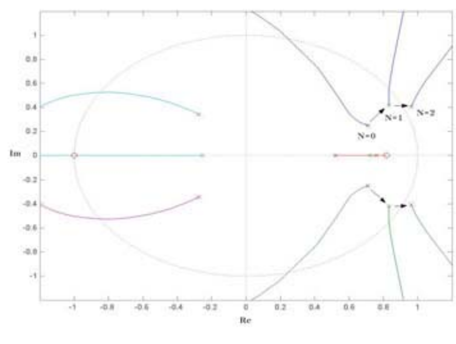

Fig. 3 Pole movement according to the time-delay

It is shown in Fig. 3, and Fig. 4 shows the step responses of each delay condition.

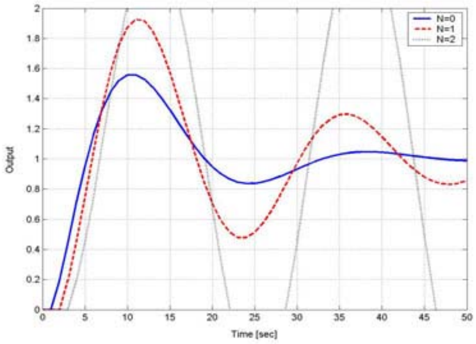

Fig. 4 Output properties according to the time-delay

3. CONTROLLER SYNTHESIS

3.1 Continuous-time input/output linearization

Input/output lineariztion is a controller synthesis scheme for a nonlinear system. It uses exact state transformation and feedback to linearize the system algerbrically and then applies linear control scheme.

Consider a nonlinear system described in general form:

$$\dot{x} = \Phi\left[x, \quad u\right], \qquad y = h\left[x\right]. \tag{11}$$

If a system is controllable, then it has finite relative order between output y and input u. The relative order r can be calculated by differential operations as bellow:

$$\frac{d^r}{dt^r} y = h^{(r-1)}\left[\Phi(x,u)\right] = f_2(x)u + f_1(x) . \tag{12}$$

The direct relation of output and input is revealed in Eq. (12). If the control input is set up like Eq. (13), the nonlinearity of the system will be eliminated from the nonlinear system and a simple linear dynamics of Eq. (14) will hold:

$$u = \frac{1}{f_2}(\nu - f_1) \tag{13}$$

$$y^{(r)} = \nu . \tag{14}$$

The tracking problem of the system can be solved by using linear control scheme. Eq. (15) is tracking error. In Eq. (16), if the coefficients k of error dynamics are set up to make the error converge to 0, then the internal input ν can be calculated as Eq. (17).

$$e = y - y_d . \tag{15}$$

$$e^{(r)} + k_r e^{(r-1)} + k_{r-1} e^{(r-2)} + \cdots + k_1 e = 0 . \tag{16}$$

$$\nu = y_d - k_r e^{(r-1)} - k_{r-1} e^{(r-2)} - \cdots - k_1 e . \tag{17}$$

3.2 Discrete-time input/output linearization

Controller synthesis for a discrete-time nonlinear system by using Input/output linearization is same with the case of continuous-time system. The relative order of Eq. (18) can be induced by shift operations and the control input of Eq. (20) eliminates the nonlinearity in the Eq. (19).

$$x(k+1) = \Phi\left[x(k), u(k)\right], \qquad y(k) = h\left[x(k)\right] \tag{18}$$

$$y(k+r) = f_1(x(k)) + f_2(x(k))u(k) \tag{19}$$

$$u(k) = \frac{1}{f_2 x(k))}\left\{\nu(k) - f_1(x(k))\right\} \tag{20}$$

The system can be derived as a linear system of Eq. (21). The track problem of this system can be solved by discrete-time linear control method described in the previous section.

$$y(k+r) = \nu(k) . \tag{21}$$

4. COMPENSATION FOR THE INPUT TIME-DELAY

If there exists input time-delay in the discrete-time nonlinear control system synthesized by using input/output linearization, the control input to the plant at time k is bellow:

$$u(k-N) = \frac{1}{f_2(x(k-N))}\left\{\nu(k-N) - f_1(x(k-N))\right\} . \tag{22}$$

In order to compensate the input time-delay, the controller should provide future input value to the plant at current time k:

$$w(k) \simeq \hat{u}(k+N \mid k) = \frac{1}{\hat{f}_2(x(k+N))}\left\{\hat{\nu}(k+N) - \hat{f}_1(x(k+N))\right\} . \tag{23}$$

Therefore, predictors about the internal input ν and the combined state function f are adopted to calculate future control input $w(k)$. The proposed prediction models use the time-series data and the predicted values are available for the controller to calculate the control input that will be used for the system. The states of the system for the next prediction are renewed continuously by the control input.

Each predictor has the following forms:

$$\text{FIR:} \quad \hat{f}(k+1) = \sum_{n=0}^{M} b_n f(k-n) , \tag{24-a}$$

$$\text{ARX:} \quad \hat{f}(k+1) = \sum_{n=0}^{M} a_n f(k-n) + \sum_{n=0}^{N} b_n w(k-n) , \tag{24-b}$$

$$\text{ARMAX:} \quad \hat{f}(k+1) = \sum_{n=0}^{M} a_n f(k-n) + \sum_{n=0}^{N} b_n w(k-n) + \sum_{n=0}^{O} c_n e_{pre}(k-n) . \tag{24-c}$$

Even though the control input had time-delay, the plant can be handled by approximated current input of Eq. (25). It can compensate the influence of the input time-delay.

$$w(k-N) \simeq \hat{u}(k) \tag{25}$$

The structure of this proposed method is shown in Fig. 5.

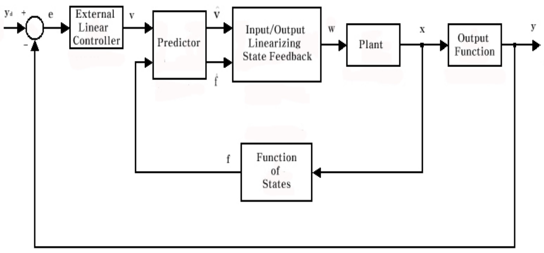

Fig. 5 Block diagram for the input time-delay compensation.

5. SIMULATIONS

In this paper, some simulations for the Van der Pol system that has input time-delay are performed. The Van der Pol equation is a representative nonlinear system. It can be interpreted as a 2nd order mass-spring-damper system that has position-dependency damping coefficient or a RLC electronic circuit that has nonlinear resistance.

If it has any initial value except equilibrium point, then this system sustains oscillation. There exists a closed curve in the portraits. This closed curve corresponds to a limit cycle. Fig. 6 shows the phase portraits of the Van der Pol system used in simulations.

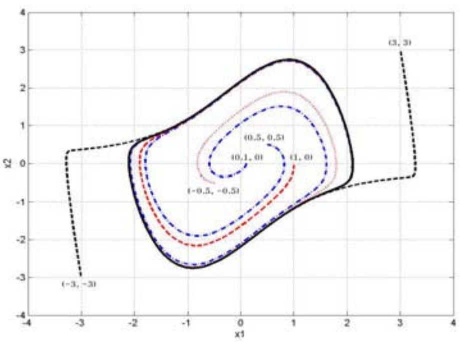

Fig. 6 Phase portraits of the Van der Pol system

The dynamic equation of the system is Eq. (26):

$$\ddot{x} = \dot{x}(1-x^2) - x + u\,, \tag{26}$$

and the state space representation of that is Eq. (27):

$$\begin{aligned} \dot{x}_1 &= x_2\,, \\ \dot{x}_2 &= x_2(1-x_1^2) - x_1 + u\,. \end{aligned} \tag{27}$$

Euler's method is used for discretization of the system as bellow:

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k)\cdot T\,, \\ x_2(k+1) &= x_2(k) \\ &\quad + \left\{x_2(k)\cdot\left(1-x_1^2(k)\right) - x_1(k)\right\} x_2(k)\cdot T \\ &\quad + u(k)\cdot T\,, \end{aligned} \tag{28}$$

$$y(k) = x_1(k)\,. \tag{29}$$

In the case that the sampling period is $T = 0.05$ [sec], the initial state is $x_{(0,0)} = \begin{bmatrix}0.1 & 0\end{bmatrix}^T$, and the input is $u(k) = 0$, the behavior of system output is like Fig 7.

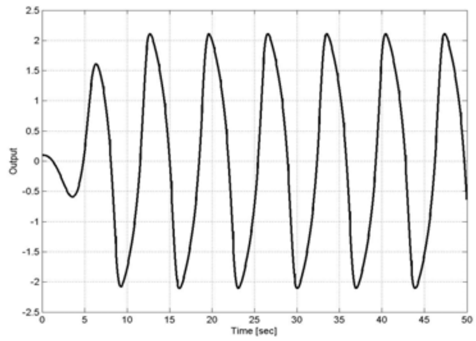

Fig. 7 Zero-input response of the system

The relative order of the system is $r = 2$. The controller synthesized by using input/output linearization is bellow:

$$u(k) = \frac{1}{T^2}\left\{v(k) - f(k)\right\}, \tag{30}$$

in this notation, the internal input v and function of states are Eq. (30-a) and Eq. (30-b).

$$\begin{aligned} v(k) &= y_d \\ &\quad + k_2\left\{y_d - \left(x_1(k) + x_2(k)\cdot T\right)\right\} \\ &\quad + k_1\left\{y_d - x_1(k)\right\} \end{aligned} \tag{30-a}$$

$$\begin{aligned} f(k) &= x_1(k) + x_2(k)\cdot T \\ &\quad + \left[x_2(k) + \left\{x_2(k)\left(1-x_1^2(k)\right) - x_1(k)\right\}\cdot T\right]\cdot T \end{aligned} \tag{30-b}$$

y_d is the desired output, T is the sampling period. k_1 and k_2 are the coefficients of the error dynamics in Eq. (31).

$$e(k+2) + k_2 e(k+1) + k_1 e(k) = 0 \tag{31}$$

In Eq. (31), the coefficients are set as $(k_1, k_2) = (0.53, -1.4)$ to the roots be located at $0.7 \pm j0.2$ in the z-plane.

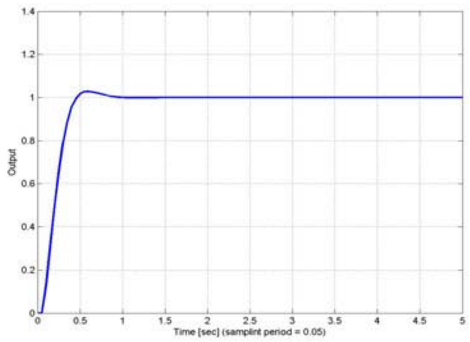

Fig. 8 System output by the controller without delay.

The output property follows the error dynamics of Eq. (31). The output of the system controlled by Eq. (30) is shown in Fig. 8.

If there exists input time-delay of $N = 1$, the output has the typical effect of time-delay like as Fig. 9. It shows that increased percent overshoot and the settling time.

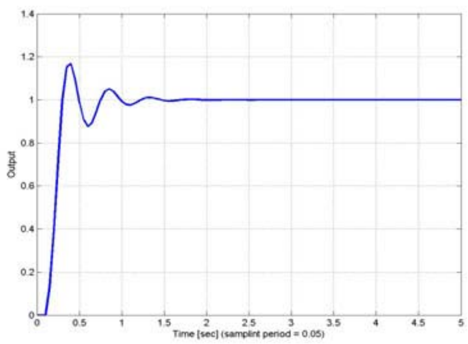

Fig. 9 Output of the system with input time-delay $N = 1$.

It is verified that the influence of input time-delay can be compensated by the proposed method in Fig. 10. The percent overshoot and settling time are decreased effectively.

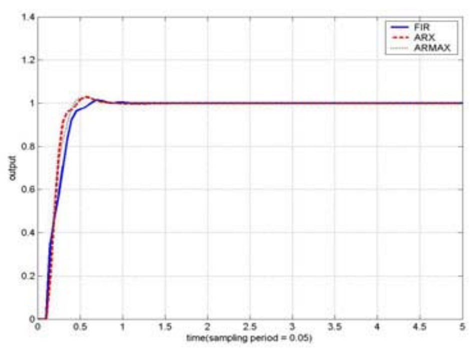

Fig. 10 Compensation for the input time-delay $N = 1$.

In the case that the input time-delay is $N = 2$ in the control system, it becomes unstable and the output diverges. That is shown in Fig. 11.

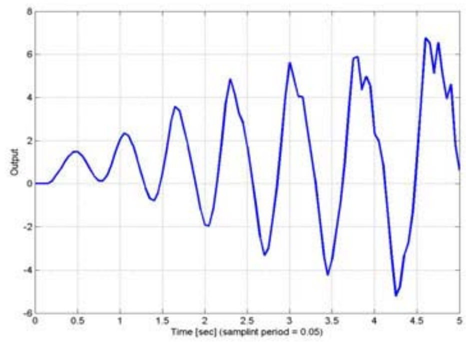

Fig. 11 Output of the system with input time-delay $N = 2$.

The proposed method makes the control system stable and satisfies the tracking performance. In the case that the time-delay is longer than sampling period, it shows the effective compensation property for the input time-delay as shown in Fig. 12.

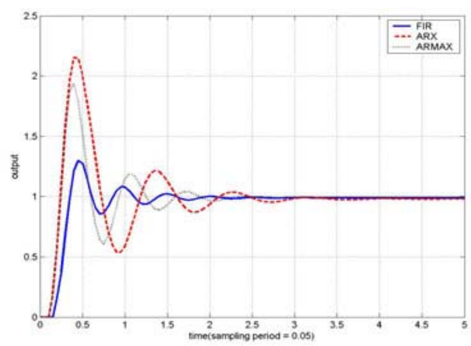

Fig. 12 Compensation for the input time-delay $N = 2$.

6. CONCLUSION

In this paper, the stability region is analyzed about sampling period and time-delay in discretization of a control system. A controller of discrete-time nonlinear system is designed by using input/output linearization. It is shown that the performance of the controller is deteriorated and the system becomes unstable under the circumstance of input time-delay existence.

A new approach is proposed for the input time-delay compensation. It is based on the predictor. It can obtain the future value of states combination and internal input that will be needed to calculate the control input. It is adopted to a nonlinear control system that is synthesized without considering the input time-delay.

Through the simulations, it is validated that the proposed method can compensate for the input time-delay effectively. Another analysis of the system for the time-delay or a controller change is not required in the proposed method.

There are needs to develop an exact and simple discretization method, study on variable time-delay and improve property of the prediction model for longer time-delay.

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