System Identification of Aerodynamic Coefficients of F-16XL (ICCAS 2004)

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Abstract: This paper presents the aerodynamic coefficient modeling with a new model structure explored by Least Squares using Modulating Function Technique (LS/MFT) for an F-16XL airplane using wind tunnel data supplied by NASA/LRC.
A new model structure for aerodynamic coefficient was proposed, one that considered all possible combination terms of angle of attack $\alpha(t)$ and $\dot{\alpha}(t)$ given number of harmonics *K*, and was compared with Pearson's model, which has the same number of parameters as the new model. Our new model harmonic results show better agreement with the physical data than Pearson's model. The number of harmonics in the model was extended to 6 and its parameters were estimated by LS/MFT. The model output of lift coefficient with *K*=6 correspond reasonably well with the physical data. In particular, the estimation performances of four aerodynamic coefficients were greatly improved at high frequency by considering all harmonics included in the input $\alpha(t)$, and by using the new model. In addition, the importance of each parameter in the model was analyzed by parameter reduction errors. Moreover, the estimation of three parameters, i.e., amplitude, phase and frequency, for a pure sinusoid and a finite sum of sinusoids- using LS/MFT is investigated.

Keywords: LS/MFT, System identification, Aerodynamic coefficient, F-16XL

1. INTRODUCTION

This paper concerns parameter estimation for aerodynamic modeling under unsteady flow effects motivated by early work of Tobak and coworkers [1,2], and continued by Klein and Murphy [3]; therein are described integro-differential equation models (including the following) which link the angle of attack and pitch rate variables, $\alpha(t)$ and $q(t)$ respectively, to a generic aerodynamic coefficient $C_a(t)$:

$$C_a(t) = C_a(\infty;\alpha(t),q(t)) - \int_0^t F_{a_\alpha}(t-\tau;\alpha(\tau),q(\tau))\dot{\alpha}(\tau)d\tau - \frac{l}{V}\int_0^t F_{a_q}(t-\tau;\alpha(\tau),q(\tau))\dot{q}(\tau)d\tau \quad (1)$$

where $C_a(\infty;\alpha(t),q(t))$ is the total aerodynamic coefficient that would correspond to steady flow with α and q fixed at the instantaneous values $\alpha(t)$ and $q(t)$, and the kernel functions (F_{a_α}, F_{a_q}), which are called deficiency functions, account for the dynamic effects of transient variables (α, q).
In wind-tunnel testing, the q effect cannot be separated from that of $\dot{\alpha}(t)$. Since we are analyzing wind-tunnel data, $\dot{\alpha}(t)$ will be used instead of q in the following investigation. Thus,

$$q(t) = \dot{\alpha}(t) \quad (2)$$

corresponding to which the model (1) reverts to

$$C_a(t) = C_a(\infty;\alpha(t)) - \int_0^t F_{a_\alpha}(t-\tau;\alpha(\tau),\dot{\alpha}(\tau))\dot{\alpha}(\tau))d\tau - \frac{l}{V}\int_0^l F_{a_q}(t-\tau;\alpha(\tau),\dot{\alpha}(\tau))\ddot{\alpha}(\tau)d\tau \quad (3)$$

Equation (3) will be investigated in this report with respect to parameterizations of the kernel functions (F_{a_α}, F_{a_q}) and the static term $C_a(\infty;\alpha(t))$ that facilitate a linear least squares estimation of the parameters given oscillatory data for $\alpha(t)$ and $C_a(t)$. Physical data for the modeling was taken in NASA LaRC's 12-Foot Low-Speed Tunnel for the F-16XL 10 percent scale model. Large amplitude (35°) forced oscillatory data were sampled at 100 and 40 Hz for forcing periods *T*={1.0, 1.33, 1.72, 2.38, 4.0} sec and *T*=12.0 sec respectively. Characteristic length to velocity is l/V =0.0213 sec, where *V* is airspeed in $m/\sec$ and characteristic length l is the half wing mean aerodynamic chord $\left(l = \bar{c}/2\right)$. The basic set of oscillatory data was obtained at 6 frequencies, one mean value of the angle of attack α_0, and an amplitude, $|a| = 35^\circ$. Following a frequency analysis of a portion of the data in Section 2, estimation of two different kinds of sinusoidal signals using LS/MFT is explored in Section 3. One specific parameterization is presented in Section 4. Algorithms for parameter estimation are given in Section 5, comparison of the resulting model outputs based on the analyzed data are given in Section 6, prior to the concluding section, Section 7.

2. DATA ANALYSIS

The physical data consists of wind tunnel measurements of various aerodynamic coefficients corresponding to sinusoidal motions in the angle of attack $\alpha(t)$ at a specified set of frequencies as given by the inverses to six periods: $T = \{1.0, 1.33, 1.72, 2.38, 4.0, 12.0\}$ seconds which corresponds to nondimensionalized reduced frequency *k* = {0.1338, 0.1006, 0.0778, 0.0562, 0.0335, 0.0112} respectively. Each sinusoid possesses a nominal value (DC) of 35° and an amplitude of 35°: $\alpha(t) = 35 + 35\cos(\omega t)$, (in degrees)

Time histories of $\alpha(t)$ and one aerodynamic coefficient, lift coefficient $C_L(t)$, are shown in Fig. 1 which indicates that several periods of the oscillatory data are given at one input frequency T = 1.0 sec. Although the data appears to be nearly perfectly periodic, close examination reveals some anomalies. In particular, there is a short transient interval in the output lasting several seconds for each of the input frequencies, and the time history for $\alpha(t)$ shows a slightly variable amplitude and frequency or phase in the *T*=1.0 case.

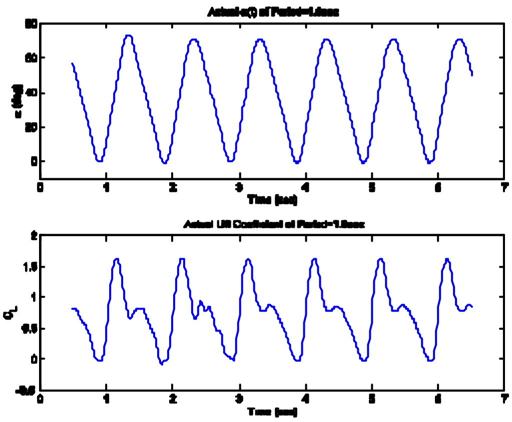

Fig. 1 $\alpha(t)$ and C_L for T=1.0 sec

Since the data is oscillatory, it is of interest to learn the extent to which it can be represented by a finite Fourier series, using as a fundamental frequency the inverse of the specified period T in each case. As is well known, if $f(t)$ is a T-periodic function that is representable by a finite Fourier series of K harmonics, then the following equality holds for all t:

$$f(t) = \sum_{k=-K}^{K} F[k]e^{ik\omega t}, \quad \omega = \frac{2\pi}{T} \quad and \quad i = \sqrt{-1} \tag{4}$$

where the Fourier coefficients $F[k]$ are specified by

$$F[k] = \frac{1}{T}\int_0^T f(t)e^{-ik\omega t}dt, \quad k = 0, \pm 1, \ldots \pm K. \tag{5}$$

Assuming the sampling rate exceeds the Nyquist rate for the signal $f(t)$, the coefficients $F[k]$ can be computed using the discrete Fourier transform which can be calculated by a standard FFT algorithm applied to the sampled data. Thus, if f_j denotes samples of the continuous-time signal $f(t)$ at $t = 0 + jT/N, \quad j = 0,1,\ldots,N$, then the Euler approximation for the right side of (5) yields

$$F[k] \approx \frac{1}{N}\sum_{j=0}^{N-1} f_j e^{-i2\pi jk/N} \tag{6}$$

By computing the Fourier coefficients from (6), and reconstructing the data with (4) for the given oscillatory data, the answer can be approached as to the question of what value to assign the integer K.

The angle of attack motion is assumed to be a pure sinusoid, and $\alpha(t)$ and $\dot{\alpha}(t)$ can be reconstructed with Fourier coefficients as follows:

$$\alpha(t) = \alpha_0 + 2|F[1]|\cos(\omega t + \angle F[1]), \tag{7}$$

$$\dot{\alpha}(t) = -2|F[1]|\omega \sin(\omega t + \angle F[1]) \tag{8}$$

If $\alpha(t)$ includes K harmonics like for T=1.00 sec case, $\alpha(t)$ and $\dot{\alpha}(t)$ can be obtained as follows using Fourier series;

$$\alpha(t) = \sum_{k=1}^{K}\left\{F[k]e^{ik\omega t} + F[k]^* e^{-ik\omega t}\right\} + F[0] \tag{9}$$

$$\dot{\alpha}(t) = \sum_{k=1}^{K}\left\{F[k]ik\omega e^{ik\omega t} - F[k]^* ik\omega e^{-ik\omega t}\right\} \tag{10}$$

3. ESTIMATION OF INPUT $\alpha(t)$

3.1 Estimation of pure sinusoids.

Let' s assume we don' t know the DC value, the amplitude, and the frequency (period) (α_0, a, ω) of input $\alpha(t)$, which has only a fundamental frequency, and want to identify the parameters comprising $\alpha(t)$:

$$\alpha(t) = \alpha_0 + ae^{i\omega t} + a^* e^{-i\omega t} \tag{11}$$

The following identity holds for $\alpha(t)$:

$$p(p^2 + \omega^2)\alpha(t) = 0 \tag{12}$$

$$p^3\alpha(t) = -\omega^2 p\alpha(t) \quad \text{and let} \quad y(t) = \alpha(t)$$

$$p^3 y(t) = \theta p y(t) \quad where \quad \theta = -\omega^2$$

Convert this equation to the frequency domain using order 3 modulation functions and estimate $\omega^2 = -\theta$.

Following this estimation, let $\alpha(t) = \alpha_0 + 2|a|\cos(\hat{\omega}t + \angle a)$.

After we expand the cosine term, we get

$$\alpha(t) = \alpha_0 + 2H\cos(\hat{\omega}t + \varphi)$$

$$= \alpha_0 + \hat{A}\cos\hat{\omega}t - \hat{B}\sin\hat{\omega}t$$

where $H = |a|, \quad \varphi = \angle a, \quad \hat{A} = 2H\cos\varphi, \quad \hat{B} = 2H\sin\varphi$

$$\alpha(t) = [1, \ \cos\hat{\omega}t, \ -\sin\hat{\omega}t]\begin{bmatrix}\hat{\alpha}_0 \\ \hat{A} \\ \hat{B}\end{bmatrix} \tag{13}$$

Now, we can estimate $(\hat{\alpha}_0, \hat{A}, \hat{B})$ using least squares estimation and from this, we can compute

$$\hat{H} = \frac{\hat{A}}{2\cos\hat{\varphi}} \qquad \hat{\varphi} = \tan^{-1}\frac{\hat{B}}{\hat{A}} \tag{14}$$

Finally,

$$\hat{\alpha}(t) = \hat{\alpha}_0 + (\hat{H}\cos\hat{\varphi} + j\hat{H}\sin\hat{\varphi})e^{j\hat{\omega}t} + (\hat{H}\cos\hat{\varphi} - j\hat{H}\sin\hat{\varphi})e^{-j\hat{\omega}t} \tag{15}$$

3.2 Estimation of Finite Sum of Sinusoids using LS /MFT

Periodic $y(t)$ with finite sum of sinusoids can be expressed as follows: Explicitly,

$$y(t) = \alpha_0 + \sum_{k=1}^{N} H_k \cos(\omega_k t + \varphi_k) \tag{16}$$

where $\omega_k = 2\pi / T_k$, or implicitly,

$$p(p^2 + \omega_1^{\ 2})(p^2 + \omega_2^{\ 2})\cdots(p^2 + \omega_N^{\ 2})y(t) = 0 \tag{17}$$

Again we take p^2 out from all parentheses, then

$$p^{2N+1}(1 + \omega_1^{\ 2}/p^2)(1 + \omega_2^{\ 2}/p^2)\cdots(1 + \omega_N^{\ 2}/p^2)y(t) = 0$$

Now use Bruzzone and Kaveh' s rule [6] to expand the above equation

$$p^{2N+1}\sum_{i=0}^{N} a_i p^{-2i} y(t) = 0 \tag{18}$$

where

$$\begin{aligned}
a_0 &= 1, \\
a_1 &= \sum_{i=1}^{N} \omega_i^{\ 2} \\
a_2 &= \sum_{i=2}^{N}\sum_{j=1}^{i-1} \omega_i^{\ 2}\omega_j^{\ 2}, \\
a_3 &= \sum_{i=3}^{N}\sum_{j=2}^{i-1}\sum_{k=1}^{j-1} \omega_i^{\ 2}\omega_j^{\ 2}\omega_k^{\ 2}, \\
a_4 &= \sum_{i=4}^{N}\sum_{j=3}^{i-1}\sum_{k=2}^{j-1}\sum_{l=1}^{k-1} \omega_i^{\ 2}\omega_j^{\ 2}\omega_k^{\ 2}\omega_l^{\ 2}, \\
&\ \vdots \\
a_N &= \prod_{i=1}^{N} \omega_i^{\ 2}.
\end{aligned} \tag{19}$$

Again, arrange (18) as following

$$p^{2N+1}y(t) = \left[-p^{2N-1}y(t), -p^{2N-3}y(t), \cdots, -py(t)\right]\cdot\begin{bmatrix}\theta_1 \\ \theta_2 \\ \vdots \\ \theta_N\end{bmatrix} \tag{20}$$

where $\theta_1 = a_1, \theta_2 = a_2, \cdots, \theta_N = a_N$

Apply $2N+1$ order MFT and transform it to the Frequency domain, and estimate θ_1 through θ_N. We have N unknowns and N equations for ω so we can calculate $\omega_1, \omega_2, \cdots, \omega_N$. Substitute these ω's into equation (16) and expand *cos* terms again.

$$\alpha(t) = \alpha_0 + \sum_{k=1}^{N} (A_k \cos\hat{\omega}_k t - B_k \sin\hat{\omega}_k t) \tag{21}$$

where $A_k = H_k \cos\varphi_k$, *and* $B_k = H_k \sin\varphi_k$

$$\alpha(t) = u_F \theta'_F \tag{22}$$

where $\theta_F = [\hat{\alpha}_0, \hat{A}_1, \hat{B}_1, \hat{A}_2, \hat{B}_2, \cdots, \hat{A}_N, \hat{B}_N]$ and

$u_F = [1, \cos\omega_1 t, -\sin\omega_1 t, \cos\omega_2 t, -\sin\omega_2 t, \cdots, \cos\omega_N t, -\sin\omega_N t]$

Using the LS/MFT, we can estimate $\hat{\alpha}_0, \hat{A}_1, \hat{B}_1, \hat{A}_2, \hat{B}_2, \cdots, \hat{A}_N$ and $\hat{B}_N$. From these estimated parameters, we can calculate θ''s and H's.

$$\hat{\theta}_k = \tan^{-1}\frac{\hat{B}_k}{\hat{A}_k} \qquad \hat{H}_k = \frac{\hat{A}_k}{\cos\hat{\theta}_k} \quad or \quad \hat{H}_k = \frac{\hat{B}_k}{\sin\hat{\theta}_k} \tag{23}$$

Reconstructed input $\alpha(t)$ with estimated parameters is

$$\hat{\alpha}(t) = \hat{\alpha}_0 + \sum_{k=1}^{N} \hat{H}_k \cos(\hat{\omega}_k t + \hat{\varphi}_k) \tag{24}$$

4. MODEL PARAMETERIZATIONS

Since K products of steady state linear operations on a sinusoid can produce K harmonics of the fundamental frequency, it is natural to seek explanations for the data of Section 2 using such products. Although many structures can be envisaged, the following specification reflects this notion and is consistent with the modeling discussed in Klein and Murphy as well as with the general Tobak model: The kernel functions (F_{a_α}, F_{a_q}) are assumed to satisfy separability-type conditions;

$$F_{a_\alpha}(t;\alpha,q) = h_\alpha(t) P_\alpha(\alpha,q) \quad and \quad h_q(t) P_q(\alpha,q) \tag{25}$$

where $P_\alpha(\alpha,q), P_q(\alpha,q)$ are polynomial functions of their arguments, and $(h_\alpha(t), h_q(t))$ are functions with rational Laplace transfer functions. Combining this with the single degree of freedom constraint (2) facilitates the following input/output operator representation for (3).

$$y(t) = r_0' u(\alpha(t)) + H(p) z(t) \tag{26}$$

where $p = d/dt$, $z(t) = g(\alpha(t), \dot{\alpha}(t))$

Here prime denotes vector transpose, $y(t) = C_a(t)$ represents the scalar-valued output, the $r_0' u(\alpha(t))$ term is for the compensation of DC value in the output, $z(t) = g(\alpha(t), \dot{\alpha}(t))$ is a column vector-valued forcing function to be structured in a way that contributes to the K harmonics observed in the output during oscillatory operation. $r_0' u(\alpha(t))$ represents parameterized effects of the static term $C_a(\infty; \alpha(t))$ in (3) with r_0' comprising a $K+1$ row vector of parameters, and the K+1 dimensional column vector function $u(\alpha(t))$ is defined by

$$u(\alpha) = [1, \alpha, \alpha^2, \cdots, \alpha^K]' \tag{27}$$

$H(s)$ is a row vector-valued rational transfer function in the Laplace variable s which will be parameterized to facilitate a linear least squares for the model.

4.2 Parameterizing *H(s)*

From the previous work [7], we found that the noncausal model provides a better fit to the C_L data than a causal model. Representing a noncausal system from input to output, an improper rational transfer function of user-specified order n, endowed with appropriate parameterization for a linear least squares estimation, is defined as follows [7]

$$H_{NC}(s) = \sum_{j=1}^{n+1} b_j' s^{j-1} \tag{28}$$

The model order n was chosen to be zero and it showed good performance. The parameter vector b_1 is separated into two subvectors, r_1 and r_2, Thus, $H_{NC}(s) = b_1' = (r_1', r_2')$

In general, the proposed model with K harmonics can be expressed as follows;

$$y = r_0' \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \\ \vdots \\ \alpha^K \end{bmatrix} + r_1' \begin{bmatrix} \dot{\alpha} \\ \dot{\alpha}^2 \\ \dot{\alpha}^3 \\ \dot{\alpha}^4 \\ \vdots \\ \dot{\alpha}^K \end{bmatrix} + r_2' \begin{bmatrix} 1 \\ \alpha \\ \dot{\alpha} \\ \alpha^2 \\ \alpha\dot{\alpha} \\ \dot{\alpha}^2 \\ \vdots \\ \alpha^{K-2} \\ \alpha^{K-3}\dot{\alpha} \\ \vdots \\ \alpha\dot{\alpha}^{K-3} \\ \dot{\alpha}^{K-2} \end{bmatrix} \alpha\dot{\alpha}, \tag{29}$$

Hence, a total of $1 + 2K + \sum_{k=1}^{K} \{(k-2)+1\}$ parameters are needed to define the input/output model (29) for this structure, using the causal transfer function of order zero. This model is for a special case of a causal system when $n = 0$.

5. LEAST SQUARES FORMULATIONS

Using the parameterized transfer functions of the previous section, the input/output model (26) can be equivalently expressed as an equation error in differential operator format with the unknown parameters appearing linearly as coefficients. But, in the case of the special noncausal transfer function (29):

$$y(t) = r_0' u(\alpha(t)) + r_1' v(\dot{\alpha}(t)) + r_2' w(\alpha(t), \dot{\alpha}(t))$$

$$= \begin{bmatrix} u'(\alpha(t)), & v'(\dot{\alpha}(t)), & w'(\alpha(t), \dot{\alpha}(t)) \end{bmatrix} \cdot \begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix} \tag{30}$$

where $u'(\alpha) = [1, \alpha, \alpha^2, \cdots, \alpha^K]$, $v'(\dot{\alpha}) = [\dot{\alpha}, \dot{\alpha}^2, \cdots, \dot{\alpha}^K]$,

$w'(\alpha, \dot{\alpha}) = [\alpha\dot{\alpha}, \alpha^2\dot{\alpha}, \alpha\dot{\alpha}^2, \alpha^3\dot{\alpha}, \cdots, \alpha^{K-1}\dot{\alpha}, \alpha^{K-2}\dot{\alpha}^2, \cdots, \alpha\dot{\alpha}^{K-1}]$

Equation (30) is now in a form for direct application of the Fourier-based modulating function technique (MFT) developed in [5]. Order $n = 0$ modulating functions are used to convert (30) to the frequency domain, given the functions $[u(\alpha(t)), v(\dot{\alpha}(t)), w(\alpha(t), \dot{\alpha}(t)), y(t)]$ on some interval $[t_0, t_1]$. But in the general $n \neq 0$ case, whether that data is transient or in steady state makes no difference since the MFT method obviates dealing with unknown or unspecified initial/boundary conditions for time-limited data. Although the output function $y(t)$ is given directly from the oscillatory data, the vector functions $(u(\alpha(t)), v(\dot{\alpha}(t)), w(\alpha(t), \dot{\alpha}(t)))$ have to be computed separately given the angle of attack variable $\alpha(t)$. This can be done either directly from the defining equations (27) or indirectly using a Fourier representation for $\alpha(t)$.

After conversion of (30) to the frequency domain via the MFT technique and transferring the output to the left side, a complex valued equation results which is of the form

$$c'Y_m = c'\Gamma_m\theta, \quad m = 0,1,\cdots,M. \tag{31}$$

Here θ denotes the composite column vector of parameters comprised of the vectors r_j, and c' is a row vector of binomial coefficients, each defined as follows:

$$\theta = (r_0', r_1', r_2')', \quad c' = (c_0, c_1, \cdots, c_n) \quad where \quad c_k = \binom{n}{k} \tag{32}$$

The (Y_m, Γ_m) denote a vector/matrix pair constructed from the Fourier coefficients of the data in the following way:

$$Y_m = \begin{bmatrix} Y[m] \\ Y[m+1] \\ \vdots \\ Y[m+n] \end{bmatrix} \quad and \quad \Gamma_m = \begin{bmatrix} \Gamma[m] \\ \Gamma[m+1] \\ \vdots \\ \Gamma[m+n] \end{bmatrix} \tag{33}$$

where $Y[k]$ denotes k^{th} harmonic Fourier coefficient on the data variable $y(t)$ relative to an arbitrary data-length time interval $[0, T_0]$, i.e.,

$$Y[k] = \frac{1}{T_0}\int_0^{T_0} y(t)e^{-ik\omega_0 t}dt, \quad \omega_0 = 2\pi / T_0, \quad k = 0, \pm 1, \cdots, \pm(M+n) \tag{34}$$

and $\Gamma[k]$ is the k^{th} harmonic Fourier coefficient of a row vector function $\Gamma(t)$ stemming from the data pair $\left(u(\alpha(t)), v(\dot{\alpha}(t)), w(\alpha(t), \dot{\alpha}(t)\right)$ which is defined as follows:

$$\Gamma(t) = \left[u'(\alpha(t)), v'(\dot{\alpha}(t)), w'(\alpha(t), \dot{\alpha}(t))\right], \quad t \in [0, T_0] \tag{35}$$

and $\Gamma[k]$ is a row vector of frequency-modulated Fourier coefficients of the data variables $[u(\alpha(t)), v(\dot{\alpha}(t)), w(\alpha(t), \dot{\alpha}(t))]$ relative to $[0, T_0]$ which is constructed as follows:

$$\Gamma[k] = \frac{1}{T_0}\int_0^{T_0} \Gamma(t)e^{-ik\omega_0 t}dt, \quad k = 0, \pm 1, \cdots, \pm(M+n) \tag{36}$$

A real-valued "measurement equation" for least squares estimation can then be formed from (31), viz.

$$Z = \Phi\theta \tag{37}$$

where

$$Z = \begin{bmatrix} c'Y_0 \\ \mathrm{Re}\, c'Y_1 \\ \vdots \\ \mathrm{Re}\, c'Y_M \\ \mathrm{Im}\, c'Y_1 \\ \vdots \\ \mathrm{Im}\, c'Y_M \end{bmatrix}, \quad \Phi = \begin{bmatrix} c'\Gamma_0 \\ \mathrm{Re}\, c'\Gamma_1 \\ \vdots \\ \mathrm{Re}\, c'\Gamma_M \\ \mathrm{Im}\, c'\Gamma_1 \\ \vdots \\ \mathrm{Im}\, c'\Gamma_M \end{bmatrix}, \tag{38}$$

Equation (37) pertains to a single data pair $(\alpha(t), C_a(t))$, which to reiterate need not be in the steady state insofar as the least squares estimation via the MFT technique is concerned. In order to perform the least squares estimation of θ over several such data pairs, let a generic $\alpha(t)$ be represented in complex format

$$\alpha(t) = \alpha_0 + ae^{i\omega t} + a^* e^{-i\omega t}$$

and let the triplet

$$\xi = (\alpha_0, a, \omega) \tag{39}$$

be used to characterize a particular sinusoid $\alpha(t)$. Then all such data pairs can be enumerated by the L triplets ξ_l, $l = 1,2,\cdots,L.$ In the present study only frequency distinguishes the various data pairs, for L=6 pairs corresponding to the six T- values: T={1.0, 1.33, 1.72, 2.38, 4.0, 12.0} seconds. However, the complex valued amplitude a as well as the DC value α_0 could in other situations be the quantities that initiate various data pairs. Thus, for each such pair it is obtained from (37) that

$$Z_l = \Phi_l'\theta, \quad l = 1,2,\cdots,L \tag{40}$$

which essentially define the *regressand* and the *regressors* for this problem. Hence, the normal equation for estimating θ is

$$\sum_{l=1}^{L} \Phi_l' Z_l = \sum_{l=1}^{L} \Phi_l' \Phi_l \theta \tag{41}$$

and the LSE, assuming linearly independent regressors, is given by

$$\hat{\theta} = \left[\sum_{l=1}^{L} \Phi_l' \Phi_l\right]^{-1} \sum_{l=1}^{L} \Phi_l' Z_l \tag{42}$$

6. MODELING RESULTS

The entire six data sets for lift coefficient were not used for the estimation of the coefficients. By using only a part of the data sets for each coefficient, for instance L=3, we found a combined model using equation (42), and then utilized the remaining data sets for model verification. In addition, the entire data points for each frequency were not used for the parameter estimation of the aerodynamic coefficients model. By using only a part of the data points for each frequency, we were able to get better performance. In other words, the numbers of the data points used for the identification of an individual model before we combine several frequencies, are 256 I/O data for T={1.0, 1.33, 1.72} sec, 512 I/O data for T={2.38} sec, 1024 I/O data for T={4.0, 12.0} sec, and the corresponding resolving frequencies are 0.392, 0.1957, 0.098, and 0.039 Hz, respectively. The highest modulating indices M that are used are {40, 25, 50, 41, 51, 42} for T={1.0, 1.33, 1.72, 2.38, 4.0, 12.0} sec, respectively, which give the best estimation performance for each individual model. Simulation procedures for the real data model estimation are as follows;

1. take one observation interval (T_0) after the transient state.
2. reconstruct $\alpha(t)$ with equation (7) and (9) using Fourier coefficients.
3. construct $\dot{\alpha}(t)$ using $\alpha(t)$ obtained in step 2.
4. construct $u(\alpha(t))$, $v(\dot{\alpha}(t))$, and $w(\alpha(t), \dot{\alpha}(t))$ using $\alpha(t)$ and $\dot{\alpha}(t)$.
5. estimate parameters r_0 , r_1 , and r_2 using LS/MFT in (42)
6. Reconstruct model output using estimated parameters (r_0 , r_1 , r_2) and input, $[u(\alpha(t))\,, v(\dot{\alpha}(t)), w(\alpha(t), \dot{\alpha}(t))]$.

Since the actual sample interval is not even, the true $\dot{\alpha}(t)$ cannot be generated by the numerical method. Thus, model error is inevitable. Only for T=1.0 sec, three harmonics, K=3, were used for $\alpha(t)$ and $\dot{\alpha}(t)$ in equation (9) and (10). For other data sets of angle of attack, only the fundamental frequency was used.

First of all, we will compare our model results with Pearson's model in [7]. Model I represents Pearson's model, and Model II in Table 1, our model proposed in this study. Model I has 15 parameters with the following form;

$$y = r_0'\begin{bmatrix}1\\ \alpha\\ \alpha^2\\ \alpha^3\end{bmatrix} + r_1'\begin{bmatrix}\dot\alpha\\ \ddot\alpha\\ \alpha\dot\alpha\\ \alpha\ddot\alpha\\ \alpha^2\dot\alpha\\ \alpha^2\ddot\alpha\end{bmatrix} + r_2' p\begin{bmatrix}\ddot\alpha\\ \alpha\dot\alpha\\ \alpha\ddot\alpha\\ \alpha^2\dot\alpha\\ \alpha^2\ddot\alpha\end{bmatrix}, \qquad p = d/dt \;. \tag{43}$$

Model II is a new structure with same number of parameters, 15 parameters, but includes 4th harmonics with the following form;

$$y = r_0'\begin{bmatrix}1\\ \alpha\\ \alpha^2\\ \alpha^3\\ \alpha^4\end{bmatrix} + r_1'\begin{bmatrix}\dot\alpha\\ \dot\alpha^2\\ \dot\alpha^3\\ \dot\alpha^4\end{bmatrix} + r_2'\begin{bmatrix}1\\ \alpha\\ \dot\alpha\\ \alpha^2\\ \alpha\dot\alpha\\ \dot\alpha^2\end{bmatrix}\alpha\dot\alpha, \tag{44}$$

Note that both models have the same number of parameters. The combined models for the lift coefficient of Model I and Model II in Table 1 were built based on three input/output data pairs corresponding to: $T \in \{1.00, 1.33, 4.0\}$ sec. This combination was found to produce the highest average output SER, among all other combinations of three data pairs, within the entire data set available. Adapting Model II, which has the same number of parameters as Model I, we can obtain a higher average model output SER of 4 aerodynamic coefficients than in Model I. Moreover, there is an advantage in that we can expand K, so that we can get better results.

Model	AVG SER (dB)	Model Output SER (dB) for different frequency (T sec)					
		T= 1.0	T= 1.33	T= 1.72	T= 2.38	T= 4.0	T= 12.0
I	19.3	19.8	21.2	21.2	19.6	17.8	16.3
II	21.9	20.3	23.0	21.8	24.3	23.7	18.2

Table 1 Comparison of C_L model output SER(dB)

Fig. 2 shows the mean output SER and standard deviation (STD) of combined Model II for several K's, K=4,5 and 6, of the lift coefficients. In Fig. 2 C_L, C_D, C_m, C_N represent lift coefficient, drag coefficient, pitching moment coefficient and normal force coefficient, respectively. As K increases in model II, the model produces better results but it needs more parameters. Whenever K is increased by 1, K+1 more parameters are needed.

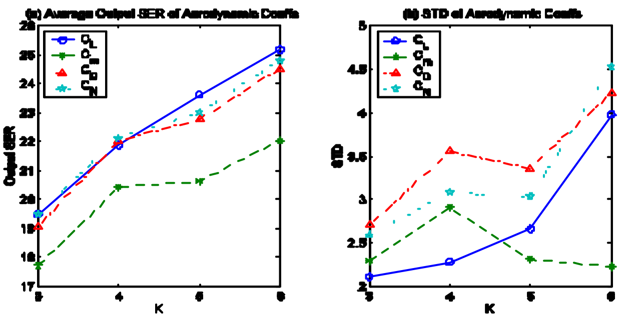

Fig. 2 Output SER and STD versus K

When K=6, the Model II has 28 parameters, and they are shown in the following;

$$\begin{aligned} y(t) &= r_0' u_6(\alpha(t)) + r_1' v_6(\dot\alpha(t)) + r_2' w_6(\alpha(t), \dot\alpha(t)) \\ &= \left[u_6'(\alpha(t)),\; v_6'(\dot\alpha(t)),\; w_6'(\alpha(t), \dot\alpha(t))\right]\cdot\begin{bmatrix}r_0\\ r_1\\ r_2\end{bmatrix} \end{aligned} \tag{45}$$

where $u_6'(\alpha) = [1, \alpha, \alpha^2, \cdots, \alpha^6]$, $v_6'(\dot\alpha) = [\dot\alpha, \dot\alpha^2, \cdots, \dot\alpha^6]$,
$w_6'(\alpha, \dot\alpha) = [\alpha\dot\alpha, \alpha^2\dot\alpha, \alpha\dot\alpha^2, \alpha^3\dot\alpha, \cdots, \alpha^5\dot\alpha, \alpha^4\dot\alpha^2, \cdots, \alpha\dot\alpha^5]$

Although the model combined with five frequencies produces the best performance, only one data set is available for model verification, since six sets of data corresponding to 6 different frequencies are available (see Seo's Ph.D thesis [8]). Thus, we used three frequencies as input data to estimate the parameters for the combined model with 6 harmonics, and verified the model with the other three frequencies data sets.

Table 2 shows the estimated parameters for the lift coefficient with a model in (45) of K=6. Three frequencies were combined: $T \in$ {1.0, 1.72, 4.0} for C_L because these combinations produce the best performances.

The lift coefficient obtained from the aerodynamic model in (45) is compared with the original test data in Fig. 3 and is found to agree well with the physical data. To verify the aerodynamic model, the data sets not involved in the training set were applied to the model. Compared with the model in [7], our models show greatly improved performances, especially at low and high frequency case, T=1.0 and T=12.0 sec.

DC	α	α^2	α^3	α^4	α^5	α^6
0.0226	2.6058	2.1746	-12.152	16.271	-11.530	3.4392
$\dot\alpha$	$\dot\alpha^2$	$\dot\alpha^3$	$\dot\alpha^4$	$\dot\alpha^5$	$\dot\alpha^6$	$\alpha\dot\alpha$
-0.0228	-0.0365	0.0096	0.0070	-0.0003	-0.0004	-0.0665
$\alpha^2\dot\alpha$	$\alpha\dot\alpha^2$	$\alpha^3\dot\alpha$	$\alpha^2\dot\alpha^2$	$\alpha\dot\alpha^3$	$\alpha^4\dot\alpha$	$\alpha^3\dot\alpha^2$
2.6957	-0.2818	-1.3004	0.3997	-0.0664	-2.5240	0.1900
$\alpha^2\dot\alpha^3$	$\alpha\dot\alpha^4$	$\alpha^5\dot\alpha$	$\alpha^4\dot\alpha^2$	$\alpha^3\dot\alpha^3$	$\alpha^2\dot\alpha^4$	$\alpha\dot\alpha^5$
-0.1773	0.0157	1.5303	-0.1889	0.1705	-0.0251	0.0050

Table 2 Estimated parameters for the lift coefficient

Some estimated parameters in Table 2, say the ones for $\dot\alpha^5$ and $\dot\alpha^6$ terms, are very small. Can we zero these small parameters out? To answer this question, we tested the importance of each parameter using the Parameter Reduction Error (PRE). We investigated the effect of each term in the model by computing output SER of lift coefficient after we deleted term after term from the 3 frequencies combined model with K=6 in (45). PRE is defined as follows:

$$PRE = \frac{SER_f - SER_r}{SER_f} \times 100\% \tag{46}$$

where SER_f is an output SER with full parameters and SER_r is the output SER with 1 less parameter which is shown in the last column of Table 3. Table 3 shows the PRE for lift coefficient in percentages, (PRE for other parameters are not shown because of considerations of length, see [8]).

Z.T in Table 3 stands for zeroed term.

Some observations on the PRE in Table 3 are discussed below;

1. Negative (-) numbers in the table means that the specific term in the rightmost column affects the specific frequency of the aerodynamic coefficient negatively, i.e., if this term were not included in the model, then the output SNR of the frequency would increase.

	PRE (%) for Lift Coeff.						
	T=1.0 sec	1.33 sec	1.72 sec	2.38 sec	4.00 sec	12.0 sec	Z.T
r_1	37.67	-10.08	-1.15	1.28	0.01	-0.00	$\dot{\alpha}^5$
	93.99	27.91	9.44	0.82	-0.02	-0.00	$\dot{\alpha}^6$

Table 3 Parameter Reduction Error (%) for C_L

2. Zeros in the table mean that the specific term in the rightmost column does not affect the specific frequency of the aerodynamic coefficient at all, i.e., even though this term is not included in the model, the output SNR of the frequency will not change.
3. The bigger the number the greater the importance of the term for the coefficient.
4. The r_1 parameters for the column $(\dot{\alpha}^5, \dot{\alpha}^6)$ in the table are unnecessary in the low frequency data for lift coefficient, but they are important in the high frequency data. Thus, these parameters get more important with increased frequency.
5. Although some estimated parameters are very small on the order of 10^{-3}, they can not be zeroed out without significantly decreasing the output SER in some frequency data.
6. To eliminate 1 parameter from the full model in equation (45), the PER for all frequencies should be around zero.
7. Some parameters are unnecessary for the low frequency data but become necessary in the high frequency data. This fact explains why we can't get a good combined model which fits well over all frequencies.

7. CONCLUDING REMARKS

A new model structure for aerodynamic coefficient was proposed, one that considered all possible combination terms of $\alpha(t)$ and $\dot{\alpha}(t)$ given K, and was compared with Pearson's model [7], which has the same number of parameters as the new model. Our new model harmonic results show better agreement with the physical data than Pearson's model. The number of harmonics in the model was extended to 6 and its parameters were estimated by LS/MFT. The model output of lift coefficient with K=6 correspond reasonably well with the physical data. In particular, the estimation performances of lift aerodynamic coefficient was greatly improved at high frequency by considering all harmonics included in the input $\alpha(t)$, and by using the new model. In addition, the importance of each parameter in the model was analyzed by parameter reduction errors. Moreover, the estimation of three parameters, i.e., amplitude, phase and frequency, for a pure sinusoid and a finite sum of sinusoids using LS/MFT is investigated, which could result in more improved performance by the AWLS.

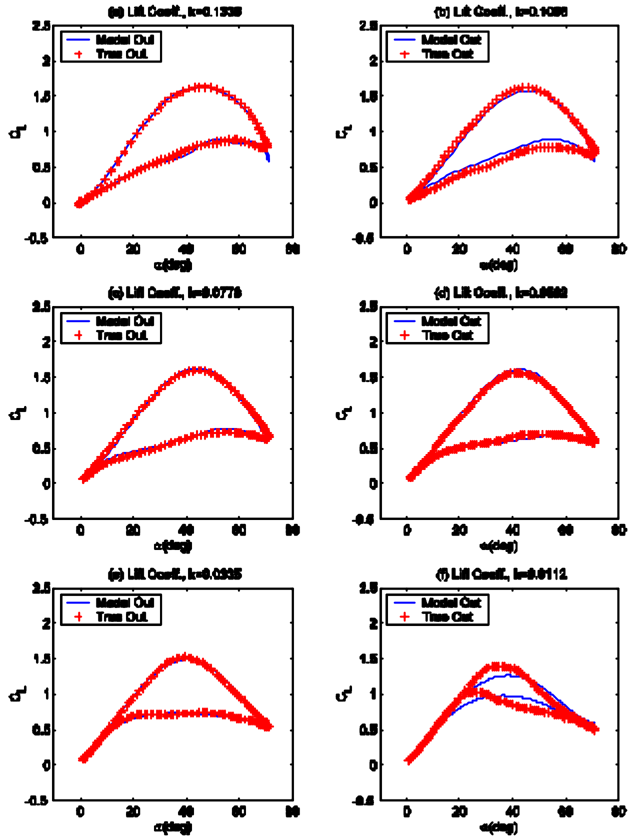

Fig. 3 Model output of lift coefficient

REFERENCES

[1] M. Tobak and W.E. Pearson. A study of nonlinear longitudinal dynamic stability. NASA TR R-209, 1964.

[2] M. Tobak and L.W. Schiff. On the formulation of the aerodynamic characteristics in aircraft dynamics. *Von Karman Inst.* For Fluid Dynamics Lecture Series 80.

[3] Klein, Vladislav and Patrick C. Murphy. Estimation of aircraft nonlinear unsteady parameters from wind tunnel data. NASA TM-1998-208969, 1998.

[4] A. E. Pearson. Interim Report One a NASA Grant: Regression Models and algorithms for Aerodynamics Applications. February 26, 1999.

[5] A. E. Pearson. Aerodynamic parameter estimation via Fourier modulating function technique. NASA CR 4654.

[6] S.P. Bruzzone and M. Kaveh. Information Tradeoffs in Using the Sample Autocorrelation Function in ARMA parameter estimation. *IEEE Transactions on Acoustics, Speech, and Signal Processing* ASSP-32, pp 701-715, Aug 1984.

[7] A. E. Pearson. Final Technical Report for a NASA Grant NAG-1-2201: Regression Models and Algorithms for Aerodynamics Applications. May 31, 2001.

[8] In-Yong Seo. *Frequency Domain Estimation and Parameter Decoupling Using Modulating Function Techniques*. Ph.D. thesis, Division of Engineering, Brown University, Providence, Rhode Island, 2003