Adjustable Phase, Discrete Time Sinewave Generator

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Abstract: The following paper proposes the new design of digital sine wave generator which allows users to define the phase shift of the out put sinewave according to user's demands. This new sinewave generator will have 2 outputs, $\cos(\omega_0 n)$ and $\cos(\omega_0 n + \phi)$ The design of the new system starts from the construction of discrete time system with impulse response as $\cos(\omega_0 n)$ in a pair of conjugate complex poles and a pair of zeros at the origin and the real axis. If users want to make a phase shift of sign wave, users can change the position of zero at the real axis. The results of the experiment have shown that the new design of sign wave generator has generated sine wave with the correct phase shift according to the theory.

Keywords: direct form, measurement system, phase shift, sinewave

1. INTRODUCTION

Digital sine oscillator is constructed as the impulse response of the discrete time system with a pair of conjugate complex pole on the circumference of unit circle. There are 2 kinds of structure of digital sign wave oscillator that can be described as follows;

1. Direct Form Structure with single output and the impulse response as $\sin(\omega_0 n)$ or $\cos(\omega_0 n)$ [1]
2. Coupled-form Structure with double outputs with the impulse response as $\sin(\omega_0 n)$ and $\cos(\omega_0 n)$ [1,2]

After making an assessment on the output from both sets of digital sine generator structure, the researchers found that both sets of structure cannot generate digital sine wave with phase shift according to the users' demands. Direct form structure will generate only single output while couple form structure will generate double output with phase difference of 90° since one output is $\sin(\omega_0 n)$ while the other output is $\cos(\omega_0 n)$

Therefore, the new digital sin oscillator that can change the phase shift according to the user's demands will be introduced. The discrete time system with the result as $\cos(\omega_0 n)$ will be used for the design and then the system will be cascaded with another discrete time system to change the zero position on the real axis. This will make output response of cascaded discrete time system will have a phase shift after comparing with $\cos(\omega_0 n)$

2. THEORY AND PRINCIPLES

2.1 Previous Design

Direct form structure of digital sine oscillator is shown in Fig. 1 and the output response or impulse response of the system after using $\delta(n)$ as an input will be shown in Eq. (1)

$$y(n) = \sin(n+1)\omega_0 \qquad (1)$$

The coupled-form structure will be displayed in Fig. 2 and 2 output equations of the system will be shown in Eqs. (2) and (3).

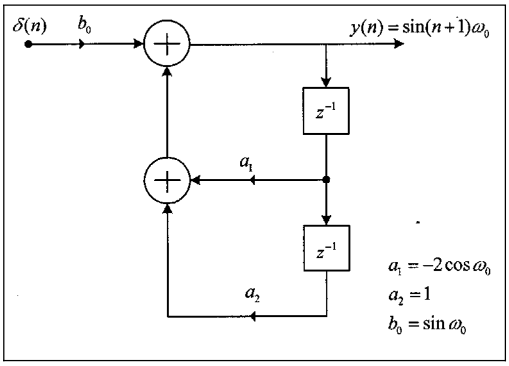


Fig. 1 Structure of Direct Form Digital Sine Oscillator

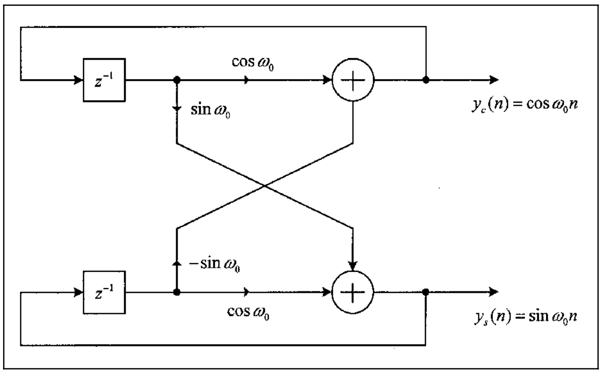


Fig. 2 Structure of Coupled-Form Digital Sine Oscillator

$$y(n)_C = (\cos\omega_0)y_C(n-1) - (\sin\omega_0)y_S(n-1) \qquad (2)$$

$$y(n)_S = (\sin\omega_0)y_C(n-1) + (\cos\omega_0)y_S(n-1) \qquad (3)$$

The output generated from the oscillator with direct form structure will give the output as a sine wave which cannot be compared with phase-shift sine waves. The output generated from the oscillator with coupled form as shown in Eq. (2) will give cosine signal while the other output will be shown as sine wave which states that both outputs have fixed 90° or $\left(\frac{\pi}{2}\right)$ and phase shift which cannot shift the phase to fit the users' demands.

2.2 Proposed Design

Data sequence is defined as $x_1(n) = \cos(\omega_0 n)$ as shown in Fig. 3 and z-transform [3] of $x(n)$ will be defined as (4)

$$X_1(z) \equiv Z\{x_1(n)\}$$

$$X_1(z) = \frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}} \tag{4}$$

If the discrete time system has the transfer function as shown in Eq. (4), it can be rewritten into Eq. (5) with pole-zero placements as shown in Fig. 4. If defining the input for discrete time system mentioned in Eq. (5) as $\delta(n)$, the output response or impulse response $h_1(n)$ will be shown in Eq. (6)

$$H_1(z) = \frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}} \tag{5}$$

$$h_1(n) = \cos(\omega_0 n) \tag{6}$$

Fig. 4 shows that the discrete time system will have 2 zeros with one zero at the origin while the other zero will be on the real axis. If defining the sequence shown in Fig. 5 as $x_2(n) = \cos(\omega_0 n + \phi)$, z-transform of $x_2(n)$ will be shown in (7) as follows

$$X_2(z) = \frac{\cos(\phi)-\cos(\omega_0-\phi)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}} \tag{7}$$

Define the transfer function of discrete time system as shown in (7), users can be rewrite the equation to become (8) and the pole-zero position of the new system will be shown in Fig. 6. If define the input of discrete time system as $\delta(n)$ on (8), the output response or impulse response $h_1(n)$ will be shown in (9)

$$H_2(z) = \frac{\cos(\phi)-\cos(\omega_0-\phi)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}} \tag{8}$$

$$h_2(n) = \cos(\omega_0 n + \phi) \tag{9}$$

Pole-zero plot in Fig. 4 and Fig. 6 show that the zero position changes in the real axis result in the phase shift of $\cos(\omega_0 n)$. The implementation of sinusoidal signal generator system with phase shift will be started from rewriting transfer function of discrete time system in Eq. (5) into the direct form as shown in Fig. 7, and then changing the position of zeros on the real axis by constructing another first order discrete time system with transfer function according to Eq. (10) with the structure as shown in Fig. 8. After cascading both discrete time systems, the new system will be shown in Fig. 9.

$$H(z) = \frac{\cos(\phi)-\cos(\omega_0-\phi)z^{-1}}{1-\cos(\omega_0)z^{-1}} \tag{10}$$

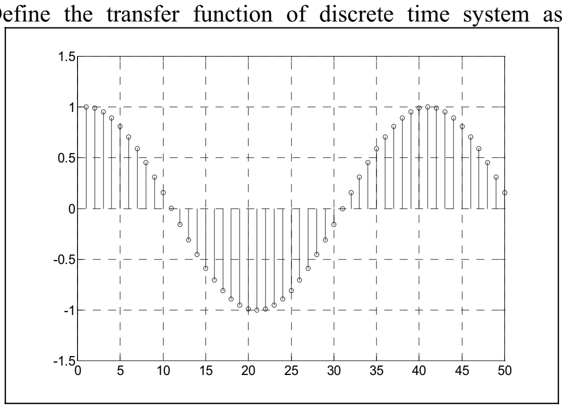


Fig. 3 Sequence of $x_1(n) = \cos(\omega_0 n)$

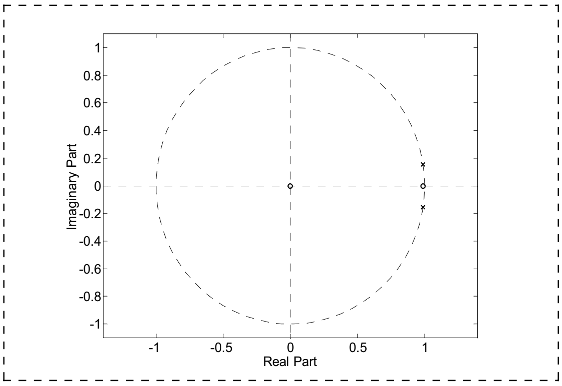


Fig. 4 Pole-Zero Plot of $H_1(z)$

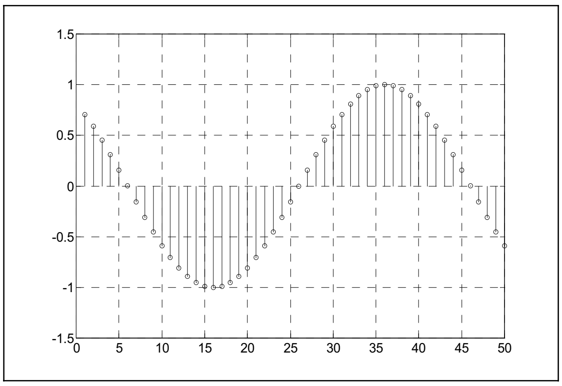


Fig. 5 Sequence of $x_2(n) = \cos(\omega_0 n + \phi)$

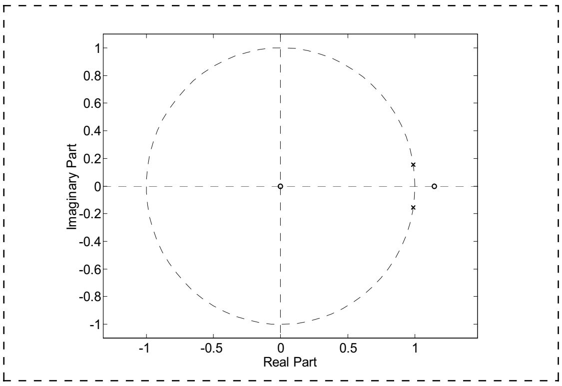


Fig. 6 Pole-Zero Plot of $H_2(z)$

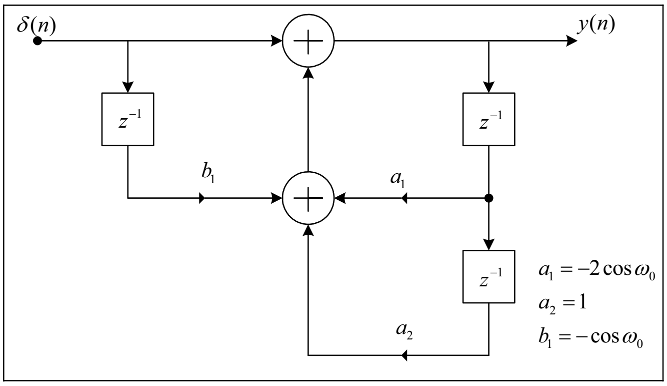


Fig. 7 Structure of Direct Form Digital Sine Oscillator

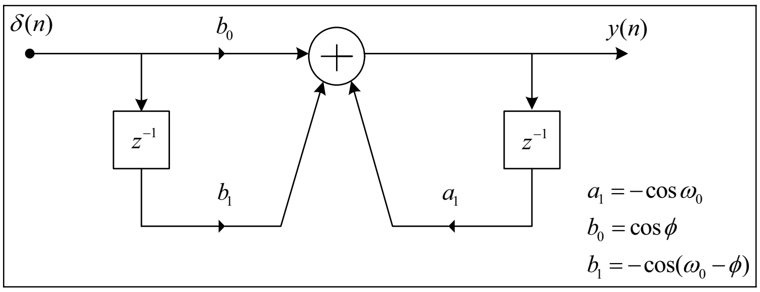


Fig. 8 First Order Discrete Time System

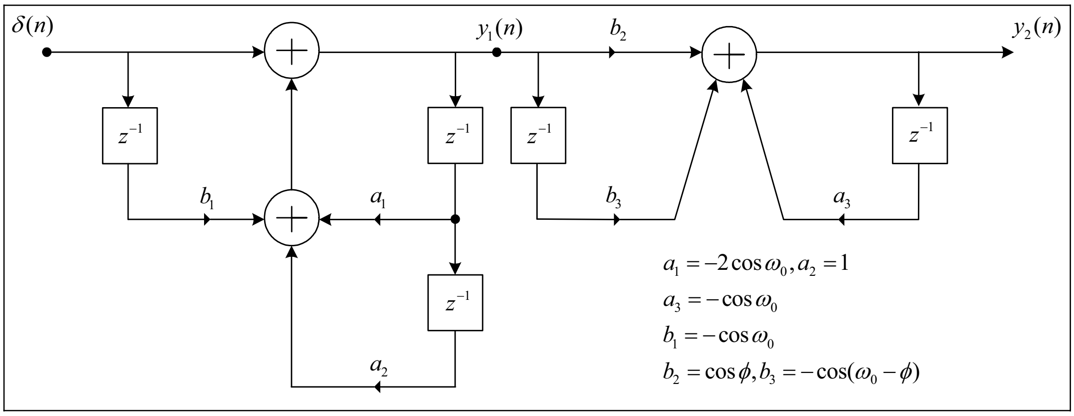


Fig. 9 Structure of Two-phase Digital Sine Oscillator

Fig. 9 has shown that after feeding input into the discrete system as impulse, the output response or impulse response of the system $y_1(n)$ will be shown in (12). Where $x_1(n) = \delta(n)$.

$$\frac{Y_1(z)}{X_1(z)} = \frac{1 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{1 - \cos(\omega_0) z^{-1}}{1 - 2\cos(\omega_0) z^{-1} + z^{-2}} \tag{11}$$

$$y_1(n) = h_1(n) = \cos(\omega_0 n) \tag{12}$$

and $y_2(n)$ will be shown in (14)

$$\frac{Y_2(z)}{X_1(z)} = \left[\frac{1 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}\right]\left[\frac{b_2 + b_3 z^{-1}}{1 + a_3 z^{-1}}\right] \tag{13}$$

$$\frac{Y_2(z)}{X_1(z)} = \left[\frac{1 - \cos(\omega_0) z^{-1}}{1 - 2\cos(\omega_0) z^{-1} + z^{-2}}\right]\left[\frac{\cos\phi - \cos(\omega_0 - \phi) z^{-1}}{1 - \cos(\omega_0) z^{-1}}\right]$$

$$\frac{Y_2(z)}{X_1(z)} = \frac{\cos\phi - \cos(\omega_0 - \phi) z^{-1}}{1 - 2\cos(\omega_0) z^{-1} + z^{-2}}$$

$$y_2(n) = h_2(n) = \cos(\omega_0 n + \phi) \tag{14}$$

3. DESIGN EXAMPLES

The design examples have specified the frequency at $\omega_0 = 0.02\pi$, $\phi = \pi, \frac{\pi}{3}$ and $\frac{2}{3}\pi$ in the proposed design by applying the factors into Eqs. (11) and (13), b_1, a_1, a_2, b_2, b_3 and a_3 as shown in Table1.and difference equations shown in Eqs (15)-(20).

Table 1 The result from proposed design.

	$\omega_0 = 0.02\pi$ $\phi = \pi$	$\omega_0 = 0.02\pi$ $\phi = \frac{\pi}{3}$	$\omega_0 = 0.02\pi$ $\phi = \frac{2}{3}\pi$
b_1	-0.9980	-0.9980	-0.9980
a_1	-1.9961	-1.9961	-1.9961
b_2	-1	0.5000	-0.5000
a_2	1	1	1
b_3	0.9980	-0.5534	0.4446
a_3	-0.9980	-0.9980	-0.9980

The 1st Design will define $\omega_0 = 0.02\pi$ and $\phi = \pi$

$$y_1(n) = \delta(n) - \cos(0.02\pi)\delta(n-1) + 2\cos(0.02\pi)y_1(n-1)$$
$$-y_1(n-1) \qquad (15)$$
$$y_2(n) = \cos(\pi)y_1(n) - \cos(0.02\pi - \pi)y_1(n-1)$$
$$+2\cos(0.02\pi)y_2(n-1) - y_2(n-2) \qquad (16)$$

The 2nd Design defines $\omega_0 = 0.02\pi$ and $\phi = \dfrac{\pi}{3}$

$$y_1(n) = \delta(n) - \cos(0.02\pi)\delta(n-1)$$
$$+2\cos(0.02\pi)y_1(n-1) - y_1(n-1) \qquad (17)$$
$$y_2(n) = \cos(\frac{\pi}{3})y_1(n) - \cos(0.02\pi - \frac{\pi}{3})y_1(n-1)$$
$$+2\cos(0.02\pi)y_2(n-1) - y_2(n-2) \qquad (18)$$

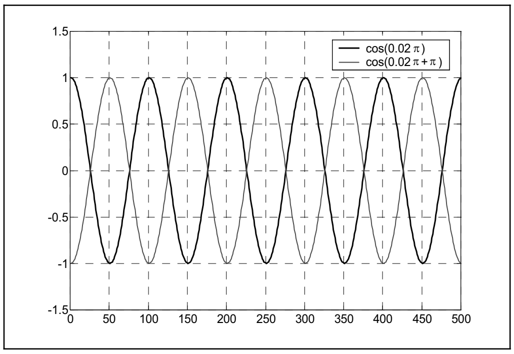
The 3rd Design will define $\omega_0 = 0.02\pi$ and $\phi = \dfrac{2}{3}\pi$

$$y_1(n) = \delta(n) - \cos(0.02\pi)\delta(n-1)$$
$$+2\cos(0.02\pi)y_1(n-1) - y_1(n-1) \qquad (19)$$
$$y_2(n) = \cos(\frac{2}{3}\pi)y_1(n) - \cos(0.02\pi - \frac{2}{3}\pi)y_1(n-1)$$
$$+2\cos(0.02\pi)y_2(n-1) - y_2(n-2) \qquad (20)$$

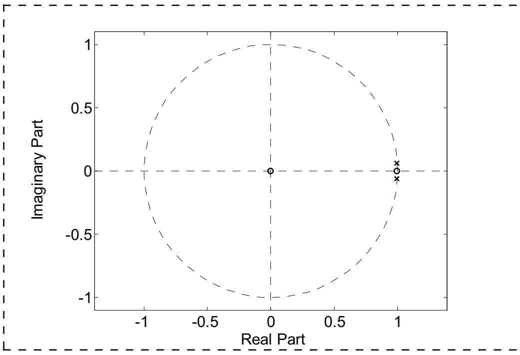
After that, rewrite expression (15) to (20) into the simulation program in Matlab.

4. RESULTS OF EXPERIMENT

The results from Matlab simulation for the 1st, 2nd, and 3rd design will be shown on Fig. 10, 11 and 12 respectively.

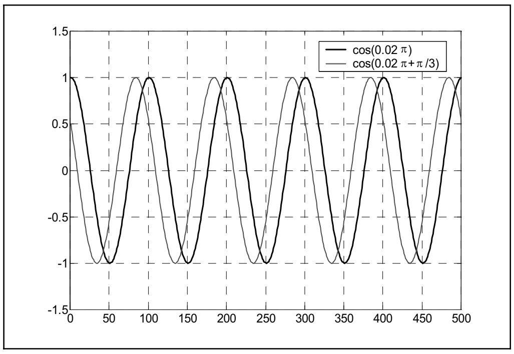


a) Output Response

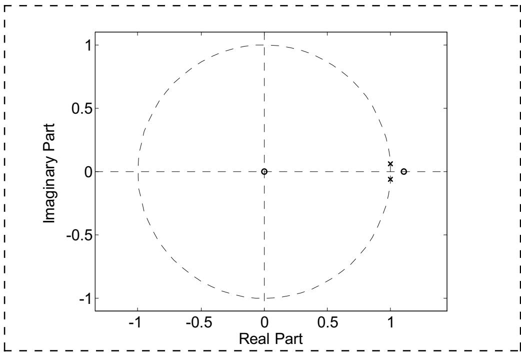


b) Pole-Zero plot

Fig. 10 $\omega_0 = 0.02\pi$ and $\phi = \pi$

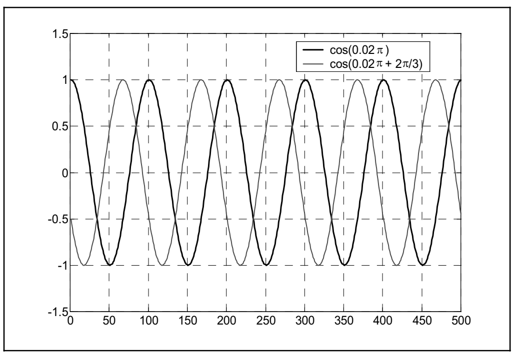


a) Output Response

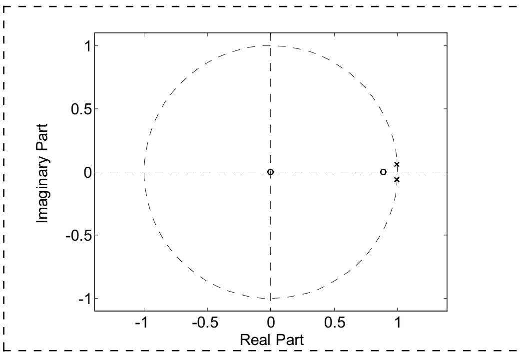


b) Pole-Zero plot

Fig. 11 $\omega_0 = 0.02\pi$ and $\phi = \dfrac{\pi}{3}$



a) Output Response



b) Pole-Zero plot

Fig. 12 $\omega_0 = 0.3\pi$ and $\phi = \dfrac{2}{3}\pi$

5. CONCLUSION

The researchers have found that the new designs of sine wave generator according to the proposed principles will give the output as sign wave with correct phase shift according to users demands. This digital sine wave generator with adjustable phase shift will have the applications on digital sign oscillators to measurement system [4,5], control power electronic instruments and machines such as motors, heaters. Further development will create multiple phase digital sine oscillators by cascading at least 2 discrete systems and then applying the numerical methods to find out the optimal coefficients for discrete time system.

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