Two-Degree-of-Freedom Controller Designed by Coefficient Diagram Method

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Abstract: This paper is concerned with a design method of two-degree-of-freedom controller by Coefficient Diagram Method (CDM). The two-degree-of-freedom control system satisfies both of tracking and disturbance rejection performances, but requires tuning of many parameters. By the proposed design method, these parameters can be obtained properly without any adjustment. Furthermore, the tracking and disturbance rejection performances can be simultaneously designed. The effectiveness of the proposed method is demonstrated by the simulation.

Keywords: Two-degree-of-freedom control system, coefficient diagram method, transient response, disturbance rejection

1. INTRODUCTION

The number of closed-loop transfer function that can be adjusted independently is called the degree of freedom of a control system [1]. It is known that the conventional one-degree-of-freedom (abbreviated as 1DOF) controller such as Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers have been widely used in various industrial applications. These controllers with well-tuned parameters can give a good system performance such as fast rise time, small overshoot and short settling time [2]. For conventional 1DOF control systems, the parameters of the controllers are usually tuned so that tracking performances are satisfactory. However, this tuning may frequently lead to poor disturbance rejection. This is a drawback of 1DOF control system. In order to design the control system to meet both tracking and regulation performances which are the most important requirements for every control systems [3], a two-degree-of-freedom (abbreviated as 2DOF) control system must be adopted. However, it is quite complicated to design and tune the controller parameters.

Hence, this paper proposes the application of Coefficient Diagram Method [4] to design 2DOF control system. This method is an efficient tool to design the parameters of 2DOF controller based on the stability and the speed of the controlled system so that the desired performance criteria can be met. Stability is designed from the stability index γ_i, and speed is designed from either the equivalent time constant τ or the tuning factor α. In this work, the transfer function of the 2DOF control system in term of stability index γ_i, equivalent time constant τ and tuning factor α has been developed in general form. The coefficients of the numerator and denominator of the transfer function are related to the 2DOF controller parameters algebraically in an explicit form. Consequently, the 2DOF controller parameters can be obtained appropriately from the values of stability index γ_i, equivalent time constant τ and tuning factor α. It is known that the effect of disturbance can be reduced by assigning the appropriate location of the closed-loop poles and the tracking response can be further optimized by adding the zeros to the system via feedforward compensator. By CDM concept, the good disturbance response can be achieved by selecting the appropriate values of the equivalent time constant τ and the stability index γ_i, while the speed of the response is directly effected by the tuning factor α. Hence, the effects of variation in the stability index γ_i, the equivalent time constant τ and the tuning factor α are investigated. The proposed control design methodology is tested on a numerical example via simulation.

2. OVERVIEW OF 2DOF CONTROL SYSTEM

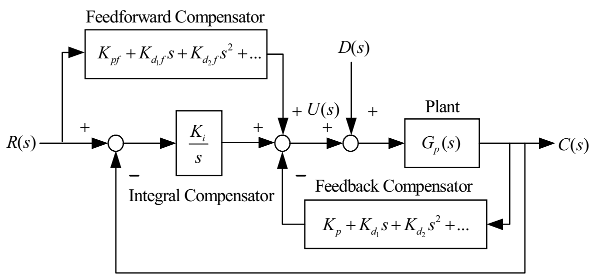


Fig. 1 Structure of 2DOF control system.

The proposed structure of 2DOF control system shown in Fig. 1 consists of a plant $G_p(s)$, an integral compensator K_i/s, a feedforward compensator $\left(K_{pf} + K_{d_1f}s + K_{d_2f}s^2 + ...\right)$ and a feedback compensator $\left(K_p + K_{d_1}s + K_{d_2}s^2 + ...\right)$. The transfer function from $R(s)$ to $C(s)$ and the transfer function from $D(s)$ to $C(s)$ are respectively given as

$$\frac{C(s)}{R(s)} = \frac{G_p(s)\left[K_{pf} + K_i/s + K_{d_1f}s + K_{d_2f}s^2 + ...\right]}{1 + G_p(s)\left[K_p + K_i/s + K_{d_1}s + K_{d_2}s^2 + ...\right]} \tag{1}$$

and

$$\frac{C(s)}{D(s)} = \frac{G_p(s)}{1 + G_p(s)\left[K_p + K_i/s + K_{d_1}s + K_{d_2}s^2 + ...\right]}. \tag{2}$$

It is seen from Eq. (2) that the disturbance response is irrelevant to the feedforward compensator. The tuning may be done without the parameters of feedforward compensator for disturbance rejection first, and then they are selected to obtain the satisfactory tracking capability. The 2DOF control system can be designed to satisfy both the regulation and tracking performances via selection of poles and zeros, but requires tuning of the additional parameters of feedforward compensator. Practically, it may be too complex for users to design and tune all of parameters.

3. COEFFICIENT DIAGRAM METHOD

The CDM is an algebraic control design approach. This method uses polynomials for system representation. The denominator and the numerator of the transfer function are considered independently from each other. In this section, the CDM standard block diagram and the basic mathematical relations concerning the CDM will be described.

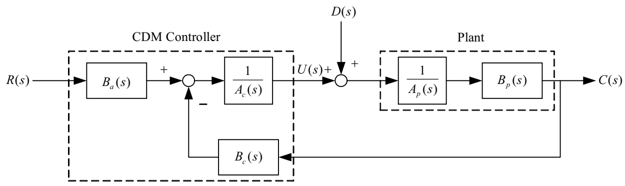


Fig. 2 CDM standard block diagram of SISO system.

The standard block diagram of the CDM design for a single-input-single-output system is shown in Fig. 2. $A_p(s)$ and $B_p(s)$ are the polynomials of the plant, $A_c(s)$, $B_c(s)$ and $B_a(s)$ are the polynomials of the standard CDM controller. $D(s)$ is the disturbance entering to the controlled system. The transfer function of the plant in polynomial form can be expressed as

$$A_p(s) = p_k s^k + p_{k-1}s^{k-1} + \ldots + p_0 \,, \tag{3a}$$

$$B_p(s) = q_m s^m + q_{m-1}s^{m-1} + \ldots + q_0 \,, \tag{3b}$$

and the controller in the polynomial forms are given by

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1}s^{\lambda-1} + \ldots + l_0 \,, \tag{4a}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1}s^{\lambda-1} + \ldots + k_0 \,, \tag{4b}$$

$$B_a(s) = k_0 \,, \tag{4c}$$

where $\lambda < k$ and $m < k$. $B_a(s)$ is a pre-filter and is set to be k_0 in order to obtain the step response with zero steady-state error. Hence, the characteristic polynomial of the closed-loop system is

$$P(s) = A_c(s)A_p(s) + B_c(s)B_p(s)$$

$$= \sum_{i=0}^{n} a_i s^i \,, \tag{5}$$

where $a_0, a_1, \ldots, a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i, the equivalent time constant τ and stability limit γ_i^* are defined as follows:

$$\gamma_i = a_i^2 \big/ a_{i+1}a_{i-1} \,, \tag{6}$$

$$\tau = a_1 \big/ a_0 \,, \tag{7}$$

$$\gamma_i^* = 1\big/\gamma_{i+1} + 1\big/\gamma_{i-1} \;; \quad \gamma_0, \gamma_n = \infty \tag{8}$$

where $i = 1, \ldots, n-1$. In general, the equivalent time constant τ and the standard stability index γ_i are chosen as:

$$\tau = \frac{t_s}{2.5} \sim \frac{t_s}{3} \,, \tag{9}$$

$$\gamma_i = [2.5, 2, 2, \ldots, 2] \;; \quad i = 1, \ldots, n-1 \tag{10}$$

where t_s is the user specified settling time. In the actual design, the stability index $\gamma_1 = 2.5$, $\gamma_2 = \gamma_3 = 2$ are strongly recommended. However, for $\gamma_{n-1} \sim \gamma_4$, the condition can be relaxed as

$$\gamma_i > 1.5\gamma_i^* \;; \quad n-1 \geq i \geq 4 \,. \tag{11}$$

Sometimes the larger value of stability index γ_i is selected in order to improve robustness related parameter change. The standard stability index values stated in Eq. (10) can be used to design the controller if the following condition is satisfied

$$p_k / p_{k-1} > \tau / (\gamma_{n-1}\gamma_{n-2} \cdots \gamma_1) \,, \tag{12}$$

where p_k and p_{k-1} are the coefficients of the plant at k^{th} and $(k-1)^{th}$. If the above condition is not met, γ_{n-1} is first increased then γ_{n-2} and so on, until Eq. (12) is satisfied. From Eq. (6) to Eq. (8), the coefficient a_i can be written by

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_2^{i-2}\gamma_1^{i-1}} \,. \tag{13}$$

Then the characteristic polynomial can be expressed in term of a_0, τ and γ_i as

$$P(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] . \tag{14}$$

4. DESIGN OF 2DOF CONTROL SYSTEM

A designing method of the 2DOF control system by CDM to meet desired performance criteria is discussed in this section. The parameters of 2DOF controller are designed based on the stability and the speed of the controlled system. Stability is designed from the standard stability index γ_i, and speed is designed from the equivalent time constant τ and the tuning factor α. The stability index γ_i, the equivalent time constant τ, and the tuning factor α are defined based on the closed-loop transfer function. These coefficients are related to the controller parameters algebraically in an explicit form.

In order to employ CDM to design the controller parameters properly, the controlled system consisting of the CDM standard block diagram of SISO system with the feedforward and feedback compensators is proposed (see Fig. 3). $B_{ff}(s)$ and $B_{fb}(s)$ are the polynomials of the feedforward compensator and the feedback compensator respectively. After rearranging the plant and the feedback compensator shown in Fig. 3, $A_p^*(s)$ and $B_p^*(s)$ which are the polynomials of the modified plant $G_p^*(s)$ can be obtained and is shown in Fig. 4.

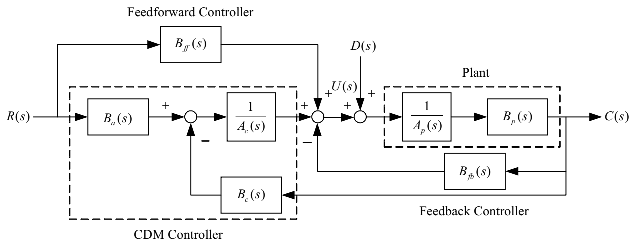


Fig. 3 CDM standard block diagram of SISO system with the feedforward and feedback compensators.

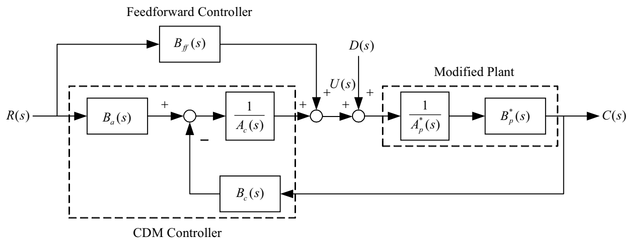


Fig. 4 Rearranged CDM standard block diagram.

From the block diagram of Fig. 4 it follows:

$$\frac{C(s)}{R(s)} = \frac{B_p^*(s)[B_a(s) + B_{ff}(s)A_c(s)]}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)} \tag{15}$$

and

$$\frac{C(s)}{D(s)} = \frac{A_c(s)B_p^*(s)}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)}. \tag{16}$$

It is seen from Eq. (15) and Eq. (16) that the feedforward compensator $B_{ff}(s)$ has an influence on the transfer function from $R(s)$ to $C(s)$ and can be used to increase the speed of the transient response of the controlled system, while the transfer function from $D(s)$ to $C(s)$ is not affected.

In order to design the 2DOF control system in term of γ_i, τ and α, Eq. (15) may assume to be

$$\frac{C(s)}{R(s)} = \frac{B(s)}{A(s)}$$

$$= \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}, \tag{17}$$

where $m \le n$, and a's and b's are constants. The denominator polynomial $A(s)$ is the characteristic polynomial of the 2DOF control system, and its coefficients can be found from

$$A(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right]. \tag{18}$$

This equation is the same form of Eq. (14).

The CDM is mainly used to design the controller of the closed-loop system. However, this method can be extended to design the coefficients of the numerator polynomial $B(s)$ as well [5-6]. Thus, the relationship among the coefficients of the numerator polynomial $B(s)$ can be defined as

$$b_i = b_0(\alpha\tau)^i \frac{1}{\gamma_{i-1}\ldots\gamma_2^{i-2}\gamma_1^{i-1}}$$

$$= b_0(\alpha\tau)^i \prod_{j=1}^{i-1}\frac{1}{(\gamma_{i-j})^j}, \tag{19}$$

where α is the tuning factor. The equivalent time constant τ is scaled by tuning factor α so that the response speed can be adjusted. The value of tuning factor α is defined as $0 < \alpha \le 1$. Substituting each coefficient b_i into the numerator polynomial $B(s)$, yields

$$B(s) = b_0\left[\left\{\sum_{i=2}^{m}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\alpha\tau s)^i\right\} + \alpha\tau s + 1\right]. \tag{20}$$

Hence, the transfer function of 2DOF control system in term of γ_i, τ and α can be obtained by

$$\frac{C(s)}{R(s)} = \frac{b_0\left[\left\{\sum_{i=2}^{m}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\alpha\tau s)^i\right\} + \alpha\tau s + 1\right]}{a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}}\right)(\tau s)^i\right\} + \tau s + 1\right]}. \tag{21}$$

This transfer function is a general form for designing the proposed 2DOF control system. Then, the parameters of 2DOF controller can be designed by following procedures:

1) Derive the transfer function (15) which contains the unknown parameters of 2DOF controller. In this step, a designer must select the number of the feedforward and the feedback parameters. Usually, the number of feedback parameters should equal to the order of the plant $G_p(s)$.

2) Define the settling time t_s in order to find the equivalent time constant τ from Eq. (9), and determine the proper values of stability index γ_i and tuning factor α. Then, substituting these parameters to the transfer function (21). Noting that the settling time t_s cannot be specified in case that the number of the feedback compensator parameters is less than the order of the plant $G_p(s)$.

3) The 2DOF controller parameters are simultaneously obtained by equating the transfer function (15) to the transfer function (21).

5. NUMERICAL EXAMPLES

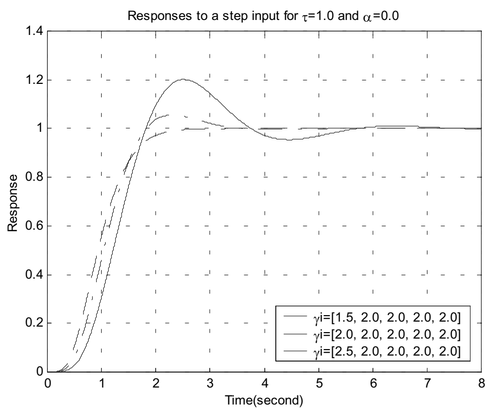
The effectiveness of the proposed 2DOF control system will be evaluated in this section. A series of simulations are performed. The 2DOF controller parameters will be designed with various stability index γ_i values in order to study the tracking capability and disturbance rejection property. Then, the responses to the step input with different values of the equivalent time constant τ are investigated. Finally, the effect of tuning factor will be regarded. In this study, the plant is assumed to be a fifth-order system and expressed as

$$G_p(s) = \frac{800}{s^5 + 12s^4 + 88s^3 + 407s^2 + 2000s + 400} \ . \tag{22}$$

Thus, the transfer function (15) is given by Eq. (23). Next, the transfer function of the 2DOF control system in term of γ_i, τ, and α is derived. The transfer function of a fifth-order plant with 2DOF controller is then shown in Eq. (24). To obtain the desired system performance, the values of γ_i, τ and α must be selected properly and substituting into the Eq. (24). Finally, equating the Eq. (23) to Eq. (24), the parameters of 2DOF controller will be obtained.

5.1 Responses with the variation of stability index

The 2DOF controller parameters are designed with parameter variations in γ_1, γ_2, γ_3 and γ_4 to study the system responses to the unit-step input and the step disturbance. The equivalent time constant τ is set to be 1.0 second, for the settling time $t_s = 2.5$ seconds, and the tuning factor α is zero. The unit-step responses are shown in Fig. 5. The responses that the unit-step disturbance is applied to the input terminal of the plant $G_p(s)$ are also shown in Fig. 6.



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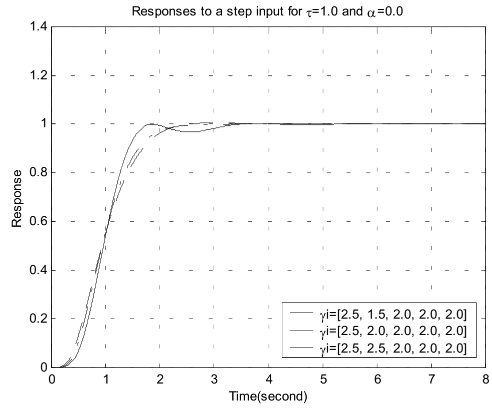

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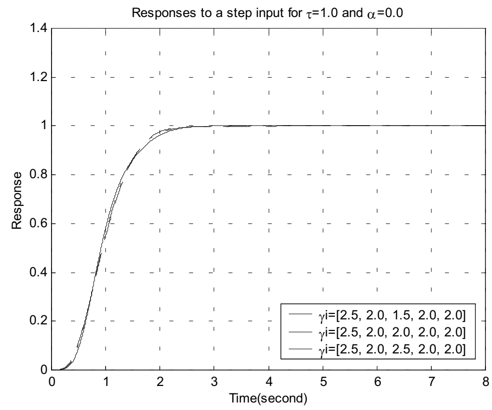

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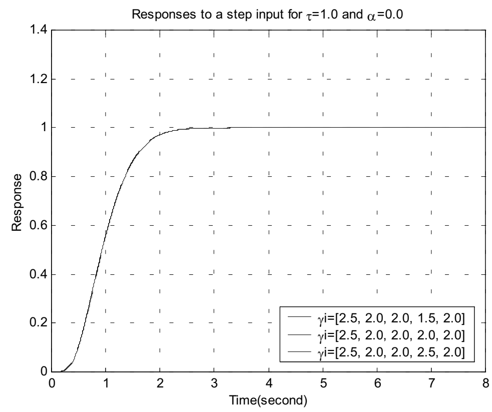
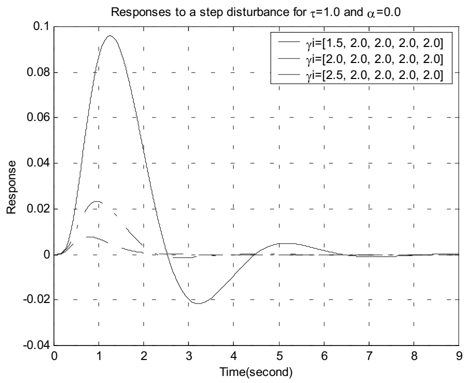


(d)

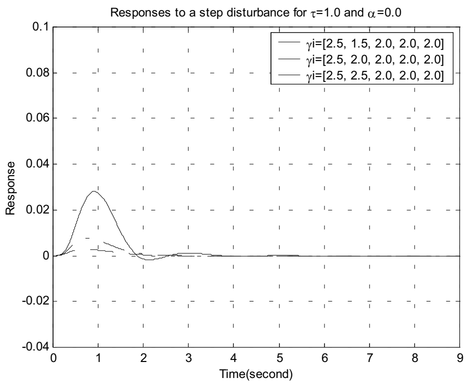
Fig. 5 Unit-step responses with the variation of stability index; (a) γ_1, (b) γ_2, (c) γ_3 and (d) γ_4.

$$\frac{C(s)}{R(s)} = \frac{800\left[K_{d_4f}s^5 + K_{d_3f}s^4 + K_{d_2f}s^3 + K_{d_1f}s^2 + K_{pf}s + K_i\right]}{s^6 + \left(12 + 800K_{d_4}\right)s^5 + \left(88 + 800K_{d_3}\right)s^4 + \left(407 + 800K_{d_2}\right)s^3 + \left(2000 + 800K_{d_1}\right)s^2 + \left(400 + 800K_p\right)s + 800K_i} \tag{23}$$

$$\frac{C(s)}{R(s)} = \frac{b_0\left(\frac{(\alpha\tau)^5}{\gamma_1^4\gamma_2^3\gamma_3^2\gamma_4}s^5 + \frac{(\alpha\tau)^4}{\gamma_1^3\gamma_2^2\gamma_3}s^4 + \frac{(\alpha\tau)^3}{\gamma_1^2\gamma_2}s^3 + \frac{(\alpha\tau)^2}{\gamma_1}s^2 + (\alpha\tau)s + 1\right)}{a_0\left(\frac{\tau^6}{\gamma_1^5\gamma_2^4\gamma_3^3\gamma_4^2\gamma_5}s^6 + \frac{\tau^5}{\gamma_1^4\gamma_2^3\gamma_3^2\gamma_4}s^5 + \frac{\tau^4}{\gamma_1^3\gamma_2^2\gamma_3}s^4 + \frac{\tau^3}{\gamma_1^2\gamma_2}s^3 + \frac{\tau^2}{\gamma_1}s^2 + \tau s + 1\right)} \tag{24}$$



(a)



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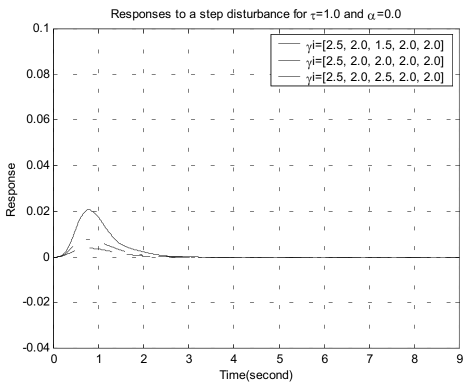

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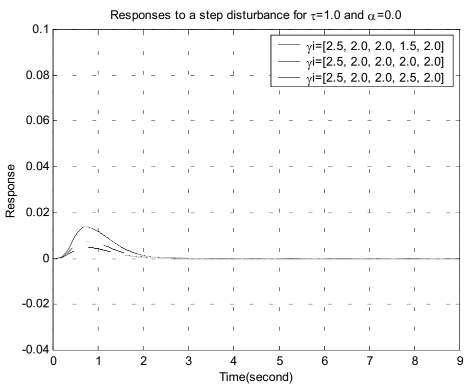

(d)

Fig. 6 Disturbance responses with the variation of stability index; (a) γ_1, (b) γ_2, (c) γ_3 and (d) γ_4.

It is observed from Fig. 5 that the transient responses to a step input are effected by changing the values of γ_1 and γ_2, which are the most dominant factors dictating the shape of the response than the rest. The overshoot of the transient response can be reduced by increasing the values of γ_1 and γ_2. Fig. 6 shows the fact that the disturbance effect can be decreased when the 2DOF controller is designed by larger value of stability index. In this case, the stability indices γ_1 and γ_2 are still the dominant factor. However, the rest of stability index also have effect on the disturbance rejection capability. To illustrate these results, the three systems for which the stability indices given in Table 1 are considered.

Table 1 Higher stability index variation.

Systems	α	τ	γ_1	γ_2	γ_3	γ_4	γ_5
1	0	1.0	2.5	2.0	2.0	2.0	2.0
2	0	1.0	2.5	2.5	2.5	2.0	2.0
3	0	1.0	2.5	2.5	2.5	2.5	2.5

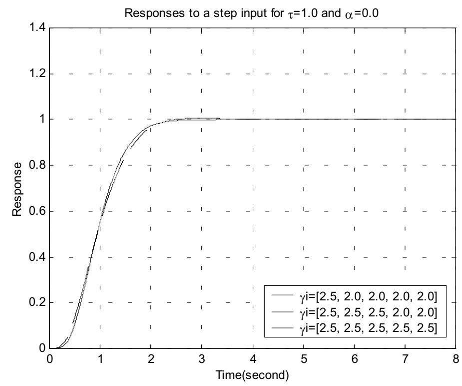

Fig. 7 Unit-step responses.

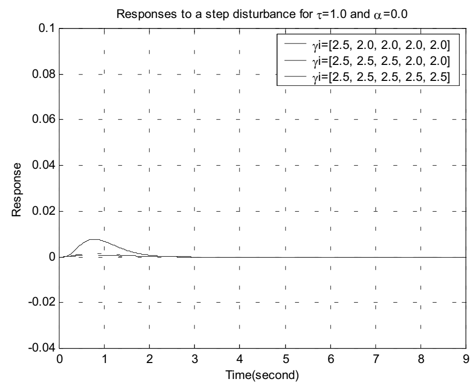

Fig. 8 Disturbance responses.

All of three systems have the same value of γ_1. The γ_2 and γ_3 of the system 1 are smaller than those of system 2 and 3. The γ_4 and γ_5 are largest for system 3. The unit-step responses and disturbance responses are respectively shown in Fig. 7 and Fig. 8. Fig. 7 shows that the unit-step responses of all three systems are almost same due to the same value of γ_1. However, the system 3 shows the best disturbance rejection capability because increasing of γ_i for $i \geq 3$ greatly effects to the disturbance responses. Thus, the properties of the disturbance can be defined by CDM directly.

5.2 Responses with the variation of equivalent time constant

In this sub-section, 2DOF controller parameters are redesigned with the variation of equivalent time constant τ. The standard stability indices $\gamma_i = [2.5, 2.0, 2.0, 2.0, 2.0]$ are selected and the tuning factor α is zero. The unit step responses are shown in Fig. 9.

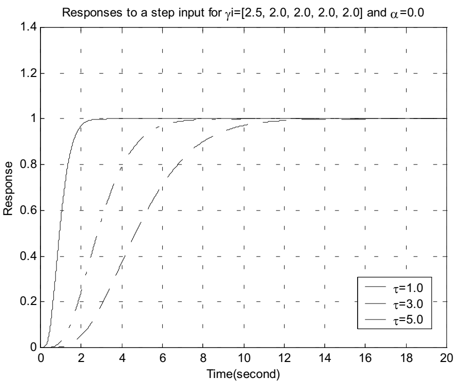

Fig. 9 Speeding up responses.

The simulation results in Fig. 9 show that the speedy up response can be achieved by decreasing the value of the equivalent time constant. Moreover, the settling time can be defined by a designer as in Eq. (9), which corresponds to the concept of CDM.

5.3 Effect of tuning factor

The responses from two previous sub-sections are satisfying both transient response and disturbance rejection property without considering the feedforward compensator ($\alpha = 0$). However, the transient response generally has long rise time, and also the equivalent time constant cannot be chosen freely in case that the number of the feedback compensator parameters is less than the order of the plant $G_p(s)$ [5]. Therefore, the response speed cannot be specified by a designer.

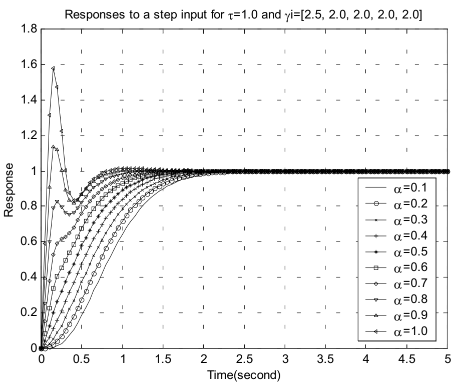

Fig. 10 Unit-step responses with the variation in tuning factor.

It is known that the response speed can be further improved by adding zeros to the system via feedforward compensator. Thus, the feedforward compensator is added to 2DOF control system in order to improve the response speed by varying the values of tuning factor α. Normally, a faster response can be obtained from a larger value of tuning factor α. However, the fast response may lead to high overshoot. Hence, the effect of tuning factor α to response speed should be studied in order to find its satisfied value that gives a fast response without overshoot. As the tuning factor α directly affects the response speed but not the disturbance rejection capability, only the responses to the step input with various tuning factor values are considered.

Fig. 10 shows that a faster response can be obtained from a larger value of tuning factor, but the response has high overshoot. The satisfied value of tuning factor that gives a fastest response without overshoot should be around 0.6.

6. CONCLUSION

In this paper, the two-degree-of-freedom controller designed by CDM to satisfy the tracking and regulation performances simultaneously has been studied. The transfer function of the proposed 2DOF control system has been developed in general form by means of CDM concept. This form can be applied not only to the low-order plant, but also to a higher-order plant as well. As the result, the 2DOF controller parameters can be obtained easily and properly by assigning the values of stability index, equivalent time constant and tuning factor. The explanations of the effects of CDM parameters to the responses of 2DOF control system were also done.

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