Simple Two-Degree of Freedom PID Controllers Tuning Table Based on CDM

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Abstract: This paper presents a simple two-degree of freedom PID tuning table based on the CDM design method. The structure of the control system will be composed of plant, P or PI or PID controller and a pre-filter. The finalized formula can be used based on the experimental test of the plant in the same manner as the Ziegler-Nichols' second method. That is; users just need to find the critical gain and critical period experimentally and the parameters of the P, PI or PID controller with the pre-filter can be obtained by substituting the values of critical gain and critical period in the tuning table. The simulation results of the control systems utilizing the proposed controllers compared with those using the Ziegler-Nichols' second method will also be demonstrated.

Keywords: PID controller, Coefficient Diagram Method, CDM, PID tuning

1. INTRODUCTION

In the Industry, PID-family controllers are still widely used even though there are many other controllers available owing to their simplicity and their sufficiency in controlling most industrial processes. It is recently reported that more than 90% of the industrial controllers used nowadays belong to PID-family type. As a result, many researchers have devoted their efforts to the development of the tuning formula for PID parameters. One of the most well known tuning rules is Ziegler-Nichols' tuning formula [1] due to its simplicity and experimentally obtainable parameters. The closed-loop control system design based on such method is expected to have quarterly decayed overshoot characteristic in the step response. Thus, fine-tuning is generally needed for the practical use to reduce the overshoot in the closed-loop response. However, it gives very good initial parameters for fine-tuning later.

In this paper, another PID tuning formula will be designed based on coefficient diagram method (CDM) [2]. CDM is an algebraic design algorithm using polynomial form structure. In CDM, the closed-loop characteristic polynomial is designed based on stability index and equivalent time constant that are used to determine stability and speed of the closed-loop response respectively. In addition, CDM algorithm also requires a known plant transfer function and a controller structure. There are many cases that use CDM in designing PI, PID controllers as well as other controllers for known specific plants [3]-[4]. The concept of CDM will also be explained briefly in section 2 of this paper.

However in the industrial practices, the plant transfer function might not be known and it is inconvenient for practitioners to determine the plant transfer function and solve the algebraic equations. In many cases, only the initial points for controller parameters are needed for fine-tuning later because, even for the exact design, fine-tuning is still needed when applying to the real systems. Consequently, the controller's parameters should rather be determined directly form actual experiments without knowing the exact plant model and solving tedious mathematics. In section 3, the model of the plant will be approximated in a simple form to fit the plant characteristics near its phase crossover frequency based on the critical gain and critical period obtained from the experiment. In section 4, the PID controller design based on CDM will be derived and summarized in a convenient tuning table. The simulation results of the control systems utilizing the proposed tuning rules compared with those using the Ziegler-Nichols' second method will be demonstrated in section 5. Finally, the conclusion will be drawn in section 6.

2. CONCEPT OF CDM

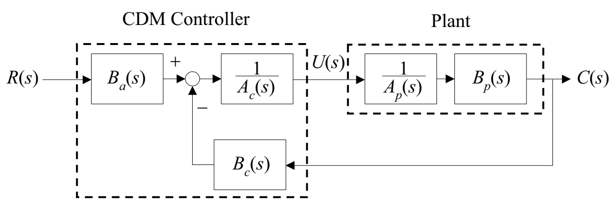


Fig. 1 Standard block diagram of CDM control systems.

The concept of CDM in designing the parameters of a controller so that the step response of the control system satisfies stability, fast response and robustness requirements [2] is described. Fig. 1 shows the standard block diagram of control system designed by CDM. It is composed of plant and CDM controller. Generally, the order of the controller is less than the order of the plant. The transfer function of the plant in the polynomial form in each block is

$$A_p(s) = p_k s^k + p_{k-1} s^{k-1} + ... + p_0 , \tag{1a}$$

$$B_p(s) = q_m s^m + q_{m-1} s^{m-1} + ... + q_0 , \tag{1b}$$

and the controller polynomials are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1} s^{\lambda-1} + ... + l_0 , \tag{2a}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1} s^{\lambda-1} + ... + k_0 , \tag{2b}$$

$$B_a(s) = k_0 , \tag{2c}$$

where $\lambda < k$ and $m < k$. The characteristic polynomial of the closed-loop system shown in Fig. 1 is given in the following forms

$$\begin{aligned} P(s) &= A_c(s)A_p(s) + B_c(s)B_p(s) \\ &= a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \\ &= \sum_{i=0}^{n} a_i s^i \end{aligned} \tag{3}$$

where $a_0, a_1, \ldots, a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i, the equivalent time constant τ and stability limit γ_i^* are defined as

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}}, \tag{4}$$

$$\tau = \frac{a_1}{a_0}, \tag{5}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0, \gamma_n = \infty, \tag{6}$$

where $i = 1, \ldots, n-1$. In general, the equivalent time constant τ is selected according to the specified settling time as $t_s = 2.5\tau$ and the standard stability index is recommended to be

$$\gamma_{n-1} = \ldots = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5. \tag{7}$$

The standard values stated in eq. (7) can be used to design the controller if the following condition is satisfied.

$$p_k / p_{k-1} > \tau / (\gamma_{n-1}\gamma_{n-2} \cdots \gamma_1), \tag{8}$$

where p_k and p_{k-1} are the coefficients of the plant at kth and (k–1)th. If the above condition is not satisfied, we can first increase γ_{n-1} then γ_{n-2} and so on, until eq. (8) is satisfied. From eq. (4) to eq. (6), the coefficient a_i can be written by

$$a_i = a_0\tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2}\gamma_1^{i-1}} = a_0\tau^i \prod_{j=1}^{i-1} \frac{1}{(\gamma_{i-j})^j}. \tag{9}$$

Then the characteristic polynomial to be used to design the parameters of a controller is

$$P(s) = a_0\left[\left\{\sum_{i=2}^{n}\left(\prod_{j=1}^{i-1} 1/\gamma_{i-j}^{j}\right)(\tau s)^i\right\} + \tau s + 1\right]. \tag{10}$$

Hence, the parameters of a controller can be obtained by equating the characteristic polynomial in eq. (3) to the characteristic polynomial in eq. (10) resulting from the known equivalent time constant τ and stability index γ_i when the mathematical model of the plant is known.

From the CDM standard block diagram, it can be rearranged as shown in Fig. 2 where the controller $G_c(s)$ and the pre-filter $G_{pf}(s)$ are

$$G_c(s) = \frac{B_c(s)}{A_c(s)}, \tag{11}$$

$$G_{pf}(s) = \frac{B_a(s)}{B_c(s)}. \tag{12}$$

In this paper, the controller $G_c(s)$ will belong to PID controller type and the pre-filter will be identified from the designing procedure.

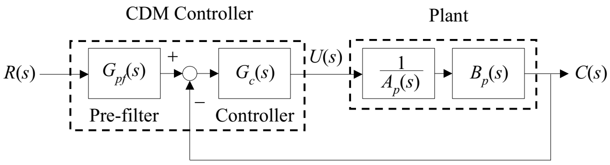


Fig. 2 Rearranged block diagram of CDM control systems.

3. PLANT MODEL APPROXIMATION

Assume that the plant to be controlled is tested by the similar ultimate sensitivity test in the Ziegler-Nichols' second method [1]. The resulting critical gain and critical period are determined experimentally and denoted by K_{cr} and P_{cr} respectively. This implies that the gain margin of the tested plant is K_{cr} and the phase crossover frequency ω_{cp} is

$$\omega_{cp} = \frac{2\pi}{P_{cr}} \text{ rad/sec.} \tag{13}$$

For the model approximation of the tested plant, the following transfer function will be used for its simplicity and sufficiency in the approximation near the phase crossover frequency.

$$\tilde{G}_p(s) = \frac{K}{s}e^{-sL}. \tag{14}$$

With simple mathematics, it can be shown that if the gain K and the delay time L are as in eq. (15) and eq. (16)

$$K = \frac{2\pi}{K_{cr}P_{cr}}, \tag{15}$$

$$L = \frac{P_{cr}}{4}, \tag{16}$$

the approximated plant model $\tilde{G}_p(s)$ will have the same critical gain and critical period as the actual plant. That is; $\tilde{G}_p(s)$ can be used as an approximated model of the actual plant near the phase crossover frequency.

However, this approximated model contains the delay time factor; thus, the CDM still cannot be applied. The delay time factor will then be approximated again by the Padé approximation as shown in eq. (17) and eq. (18) for the first order and the second order approximation respectively.

$$e^{-sL} \simeq \frac{1-(sL)/2}{1+(sL)/2} \tag{17}$$

$$e^{-sL} \simeq \frac{1-(sL)/2+(sL)^2/12}{1+(sL)/2+(sL)^2/12} \tag{18}$$

The selection of approximation order depends on the number of tuning parameters of the controller and will be discussed again in section 4.

4. PID CONTROLLER DESIGN

In this section, the tuning rules for P, PI and PID controller based on the approximated model and CDM concept will be derived. In general, the designing procedure will be done in 4 steps.

1) The closed-loop characteristic polynomial of the closed-loop system with the known approximated model and known controller structure is derived as in eq. (3). It will have controller's parameters as variables.
2) The stability index to be used must be defined. In this case, the standard stability index will be selected. However, the equivalent time constant will not be specified and considered as another variable to be solved. That is, the appropriate speed of the closed-loop systems will be implicitly determined by the design algorithm. Then the desired CDM closed-loop characteristic polynomial can be specified as in eq. (10).
3) Equating these 2 closed-loop characteristic polynomials will solve for the controller's parameters and equivalent time constant.
4) The pre-filter can be found by eq. (12).

4.1 P controller design

In this case, the controller in eq. (11) will be selected as

$$G_c(s) = K_p, \tag{19}$$

where K_p is the proportional gain. The first order Padé approximation as shown in eq. (17) is used for approximating the delay time. The rational approximated model becomes

$$\tilde{G}_p(s) \simeq \frac{K}{s} \cdot \frac{1-(sL)/2}{1+(sL)/2} = \frac{K(-(L/2)s+1)}{(L/2)s^2+s} \tag{20}$$

The closed-loop characteristic polynomial of this system indicated in eq. (3) is

$$P(s) = (L/2)s^2 + (1-(K_pKL)/2)s + K_pK \cdot \tag{21}$$

Using standard stability index $\gamma_1 = 2.5$, the desired CDM closed-loop characteristic polynomial in eq. (10) with the same order as of eq. (21) is

$$\begin{aligned} P(s) &= a_0\left[\frac{\tau^2}{\gamma_1}s^2 + \tau s + 1\right] \\ &= a_0\left[\frac{\tau^2}{2.5}s^2 + \tau s + 1\right]. \end{aligned} \tag{22}$$

Then symbolically equating eq. (21) and eq. (22) results in the proportional gain and the equivalent time constant as

$$K_p = \frac{0.4689}{KL} = \frac{K_{cr}}{3.35}, \tag{23}$$

$$\tau = 1.6328L = 0.41P_{cr} \cdot \tag{24}$$

The pre-filter can be solved accordingly to be $G_{pf}(s) = 1 \cdot$

4.2 PI controller design

In this case, the controller in eq. (11) will be

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right), \tag{25}$$

where T_i is the integral time. The controller design can be done by following the similar steps in the P controller design sub-section. Note that due to algebraic constraints, the first order Padé approximation is still used for approximating the delay time but the set of the standard stability index with $\gamma_1 = 2.5$ and $\gamma_2 = 2$ is employed. These will yield

$$K_p = \frac{K_{cr}}{2.72}, \tag{26}$$

$$T_i = P_{cr}, \tag{27}$$

$$\tau = 0.88P_{cr} \cdot \tag{28}$$

The pre-filter can be derived as

$$G_{pf}(s) = \frac{1}{T_i s + 1} \cdot \tag{29}$$

Form the equivalent time constant, it suggests that the speed of the response will be slowed down to compensate for the better steady-state response by introducing the integral action in the controller.

4.3 PID controller design

In this case, the controller in eq. (11) will be selected as

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{30}$$

where T_d is the derivative time. Following the similar steps in the P and PI controller design sub-section, the appropriate PID controller tuning rule can be designed. Note that due to algebraic constraints, the second order Padé approximation as shown in eq. (18) is utilized for approximating the delay time and the set of the standard stability index with $\gamma_1 = 2.5$, $\gamma_2 = 2$ and $\gamma_3 = 2$ is employed. These will give

$$K_p = \frac{K_{cr}}{1.59}, \tag{31}$$

$$T_i = 0.76P_{cr}, \tag{32}$$

$$T_d = 0.078P_{cr}, \tag{33}$$

$$\tau = 0.64P_{cr} \cdot \tag{34}$$

Similarly, the pre-filter can be solved accordingly to be

$$G_{pf}(s)=\frac{1}{T_dT_is^2+T_is+1}. \quad (35)$$

It can also be observed from the equivalent time constant that the speed of the response can be faster than using PI controller.

From the sub-section 4.1 to 4.3, the main result of this research can be summarized in Table 1. Without knowing these derivation backgrounds of these tuning rules, the users can easily employ this table by substituting corresponding values of K_{cr} and P_{cr} in the controller type to be used in the control systems.

5. SIMULATION DEMONSTRATION

From the tuning rules in Table 1, two examples of simple plants commonly found in the industry will be used to demonstrate the effectiveness of the proposed tuning rules. The unit step responses of the control systems using P, PI and PID controllers tuned by the proposed tuning rules will also be compared with those tuned by the Ziegler-Nichols' second method.

Example 1:

Consider the plant transfer function $G_{p1}(s)$ in eq. (36).

$$G_{p1}(s)=\frac{5}{(s+1)^3} \quad (36)$$

Testing the system by the ultimate sensitivity method, the critical gain K_{cr} and critical period P_{cr} can be determined and they are equal to $K_{cr}=1.6$ and $P_{cr}=4.53$ sec. By substituting these values in Table 1, the parameters of the controllers as well as the expected equivalent time constants τ can be calculated as shown in Table 2. With the corresponding controllers, the unit step responses of the closed-loop control systems are simulated by MATLAB/ Simulink and illustrated in Fig. 3.

Form the responses, it can be observed that the closed-loop control systems using the controllers tuned by the proposed tuning rules have less overshoot but slower rise time compared to those using Ziegler-Nichols' tuning rules. Noting that the time which the responses reach 63.2% of the steady-state values are 1.53, 6.17 and 3.68 seconds for the closed-loop systems with P, PI and PID controllers respectively. These values have the same trends with the pre-calculated equivalent time constant τ indicated in Table 2. Conversely, at the pre-calculated τ the responses reach 47.31%, 51.94% and 49.23% of their final values for P, PI and PID control systems respectively. Therefore, an addition advantage of the proposed tuning rules is that the speed of the responses can be roughly predicted in advance.

Table 2 Corresponding controllers' parameters in example 1.

	K_p	T_i (sec)	T_d (sec)	τ (sec)
P	0.4776	-	-	1.8572
PI	0.5882	4.5298	-	3.9862
PID	1.0063	3.4426	0.3533	2.8990

Example 2:

Consider the plant transfer function $G_{p2}(s)$ in eq. (37).

$$G_{p2}(s)=\frac{10}{s(s+1)(s+2)(s+3)} \quad (37)$$

Similarly, the critical gain K_{cr} and critical period P_{cr} can be found as $K_{cr}=1$ and $P_{cr}=6.2832$ sec. Thus, the parameters of the controllers as well as the expected equivalent time constants τ can be computed as in Table 3 and the simulated unit step responses of the closed-loop control systems are depicted in Fig. 4.

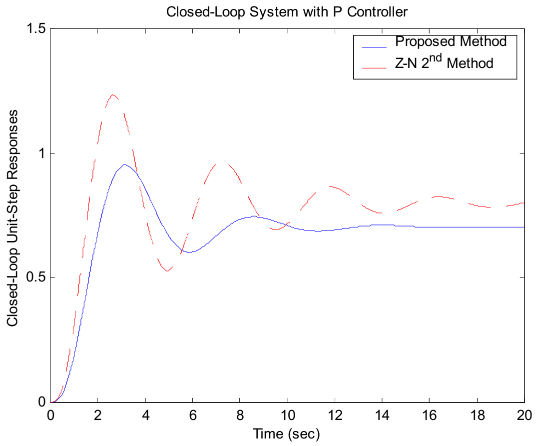
Table 3 Corresponding controllers' parameters in example 2.

	K_p	T_i (sec)	T_d (sec)	τ (sec)
P	0.2985	-	-	2.5761
PI	0.3676	6.2832	-	5.5292
PID	0.6289	4.7752	0.4901	4.0212

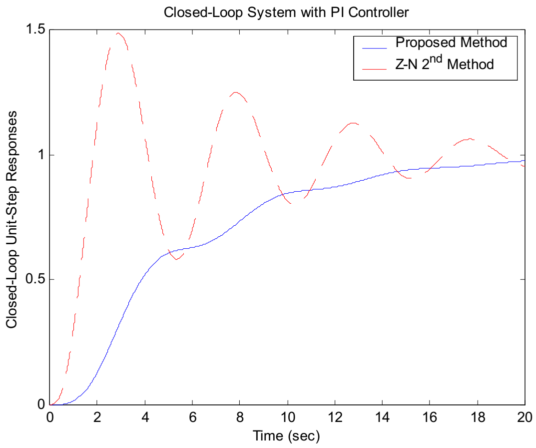
Similarly, it can be seen from Fig. 4 that the closed-loop control systems using the controllers tuned by the proposed tuning rules have less overshoot but slower rise time compared to those using Ziegler-Nichols' tuning rules. The time the responses reach 63.2% of the steady-state values for the closed-loop systems with P, PI and PID controllers are

Table 1 PID tuning rules and pre-filter structures based on CDM concept with predictive equivalent time constant.

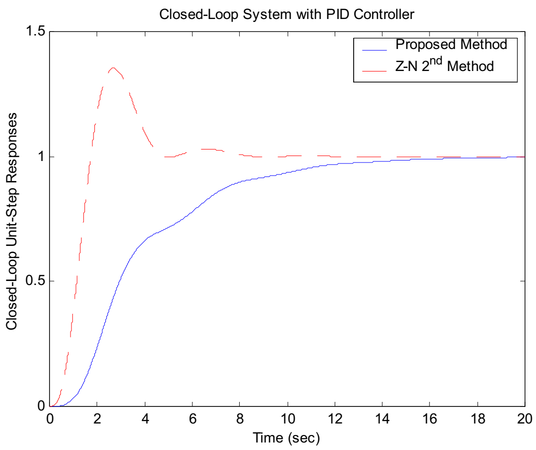
Type of Controller	K_p	T_i	T_d	Pre-filter	τ
P	$\frac{K_{cr}}{3.35}$	-	-	1	$0.41P_{cr}$
PI	$\frac{K_{cr}}{2.72}$	P_{cr}	-	$\frac{1}{T_is+1}$	$0.88P_{cr}$
PID	$\frac{K_{cr}}{1.59}$	$0.76P_{cr}$	$0.078P_{cr}$	$\frac{1}{T_dT_is^2+T_is+1}$	$0.64P_{cr}$



(a) Using P controller

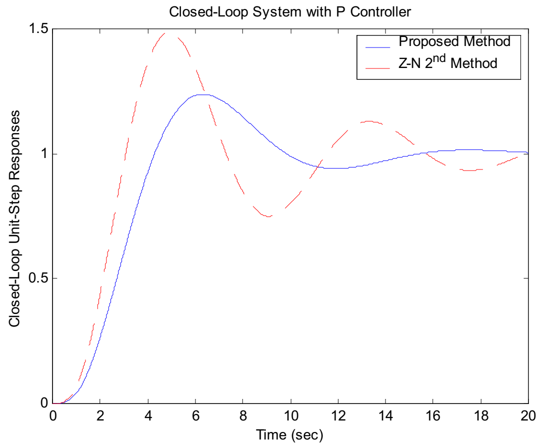


(b) Using PI controller

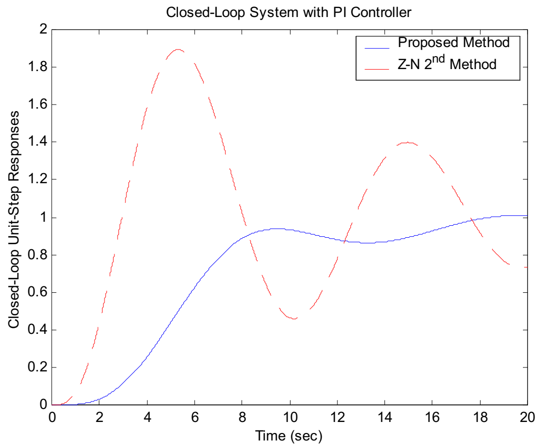


(c) Using PID controller

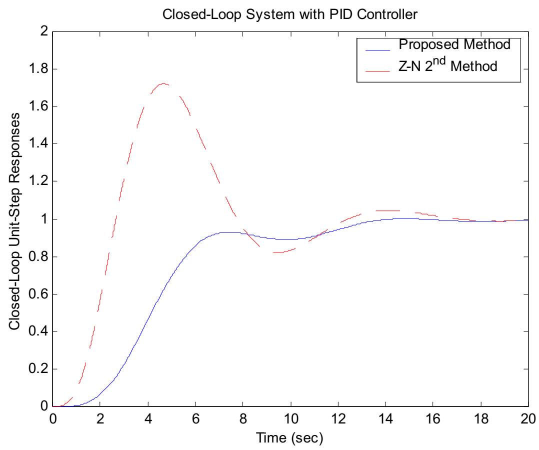
Fig. 3 Unit step responses of the closed-loop control systems with the controllers tuning by the proposed tuning rules (solid) and Ziegler-Nichols' tuning rules (dashed) for the example 1.



(a) Using P controller



(b) Using PI controller



(c) Using PID controller

Fig. 4 Unit step responses of the closed-loop control systems with the controllers tuning by the proposed tuning rules (solid) and Ziegler-Nichols' tuning rules (dashed) for the example 2.

respectively 3.07, 6.03 and 4.70 seconds. At the pre-calculated τ, the responses reach 45.80%, 54.16% and 46.93% of their final values for P, PI and PID control systems respectively. These agree with results in the example 1 stated previously and verify an addition advantage of the proposed tuning formula in forecasting the response speed in advance based on the pre-calculated equivalent time constant τ.

6. CONCLUSION

This paper presents the derivation and applications of the PID tuning rules inherited CDM design based on the experimental ultimate sensitivity test. The usage of the tuning rules is simple and straightforward. The numerical examples and corresponding simulation results show that the proposed tuning rules yield the closed-loop control systems in which the responses achieve smaller overshoot and less oscillation but with longer rise time compared to the well-known Ziegler-Nichols' tuning rules. Thus, the proposed tuning rules are suitable if the smaller overshoot is primarily desired. Furthermore, the speed of the closed-loop systems can be roughly predicted beforehand.

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