Design of Robust DC-DC Converter by High-Order Approximate 2-Degree-of-Freedom Digital Controller

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Abstract: In many applications of DC-DC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. Generally, design conditions are changed for each load and then each controller is re-designed. Then, a so-called robust DC-DC converter which can cover such extensive load changes and also input voltage changes with one controller is needed. Analog control IC is used usually for the controller of DC-DC converter. Simple integral control etc. are performed with the analog control IC. However it is difficult to retain sufficient robustness of DC-DC converter by these techniques. The authors proposed the method of designing an approximate 2-degree-of-freedom (2DOF) controller of DC-AC converter. This controller has an ability to attain sufficient robustness against extensive load and DC power supply changes. For applying this approximate 2DOF controller to DC-DC converter, it is necessary to improve the degree of approximation for better robustness. In this paper, we propose a method of designing good approximate 2DOF digital controller which makes the control bandwidth wider, and at the same time makes a variation of the output voltage very small at a sudden change of resistive load. The proposed good approximate 2DOF digital controller is actually implemented on a DSP and is connected to a DC-DC converter. Experimental studies demonstrate that this type digital controller can satisfy given specifications.

Keywords: DC-DC converter, 2-Degree-of Freedom system, digital control, robust control

1. Introduction

In many applications of DC-DC converters, loads cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. Generally, design conditions are changed for each load and then each controller is re-designed. Then, a so-called robust DC-DC converter which can cover such extensive load changes and also input voltage changes with one controller is needed. Analog control IC is used usually for the controller of DC-DC converter. Simple integral control etc. are performed with the analog control IC. Moreover, the application of the digital controller to a DC-DC converter designed by the PID or root locus method etc. has been recently considered[1], [2]. However it is difficult to retain sufficient robustness of DC-DC converters by these techniques.

The authors proposed the method of designing an approximate 2-degree-of-freedom (2DOF) controller of DC-AC converter[3], [4], [5]. Unlike other methods[6], [7],this controller has an ability to attain sufficient robustness against extensive load and DC power supply changes. For applying this approximate 2DOF controller to a DC-DC converter, it is necessary to improve the degree of approximation for better robustness. In this paper, we propose a method of designing good approximate 2DOF digital controller which makes the control bandwidth wider and at the same time makes a variation of the output voltage very small at a sudden change of resistive load. The good approximate 2DOF controller is constituted as follows : First, a model matching system with a specified control bandwidth is constituted by using the voltage and the current feedbacks. The current sensor is generally expensive and noisy. In order to avoid use of the current sensor, the current feedback is changed by using a dynamic compensator into the output feedback and control signal feedback equivalently. Secondly, an approximate model of this model matching system is derived by setting the high order by 1 than the former approximate model. The degree of approximation is improved by increasing the order of the approximate model. An inverse system of this good approximate model and a filter for realizing the inverse system are combined with the model matching system. Finally, an equivalent conversion of the portion of the controller is carried out, and a realizable good approximate 2DOF controller is obtained. This digital controller is actually realized by using a DSP. Some simulations and experiments show that the proposed good approximate 2DOF digital controller can satisfy given specifications.

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2. DC-DC converter

The power amplifier as shown in **Fig.1** has been manufactured. In order to realize the good approximate 2DOF digital controller which satisfies given specifications, we use the DSP(TI TMS320LF2401). This DSP has a built-in AD converter and a PWM switching signal generating part. The triangular wave carrier is adopted for the PWM switching signal. The switching frequency is set at 300[KHz] and the peak-to-peak amplitude C_m is 66[V]. The LC circuit is a filter for removing carrier and switching noises. C_0 is 308[μF] and L_0 is 1.4[μH]. If the frequency of control signal u is smaller enough than that of the carrier, the state equation of the DC-DC converter at a resistive load in Fig.1 except for the controller in DSP can be expressed from the state

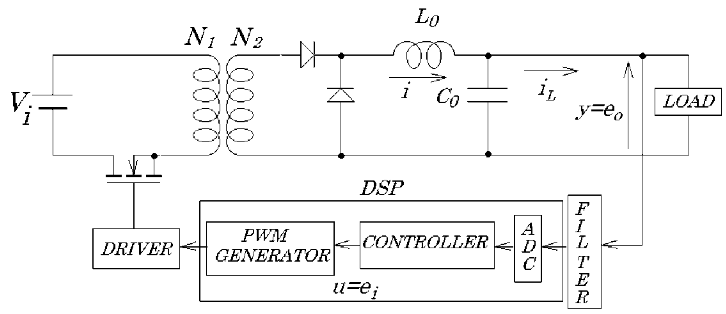

Fig. 1. DC-DC converter

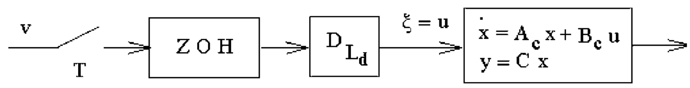

Fig. 2. Controlled object with input dead time $L_d(\leq T)$

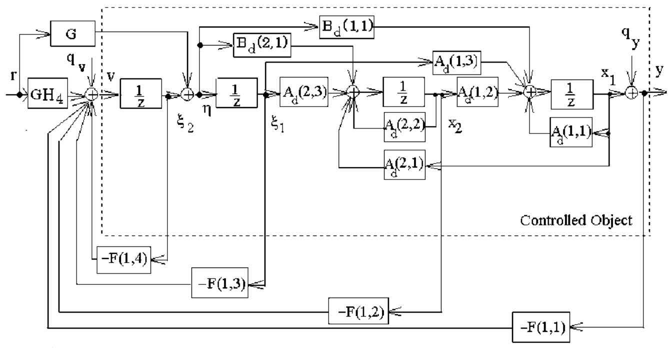

Fig. 3. Equivalent disturbances due to load variations (parameter variations) and model matching system with state feedback

equalizing method[8] as follows :

$$\begin{cases} \dot{x} = A_c x + B_c u \\ y = Cx \end{cases} \quad (1)$$

where

$$x = \begin{bmatrix} e_o \\ i \end{bmatrix} \quad A_c = \begin{bmatrix} -\frac{1}{C_0 R_L} & \frac{1}{C_0} \\ -\frac{1}{L_0} & -\frac{R_0}{L_0} \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ \frac{K_p}{L_0} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad u = e_i \quad y = e_0 \quad K_p = -\frac{V_i N_2}{C_m N_1}$$

and R_0 is the total resistance of coil, ON resistance of FET, etc and the value is 0.015[Ω].

When realizing a digital controller by a DSP, a delay time exists between the starting time of sampling operation and the outputting time of control signal due to the calculation and AD/DA conversion times. This delay time is considered to be equivalent to the input dead time which exists in the controlled object as shown in **Fig.2**. Then the state equation of the system in Fig.2 is expressed as follows :

$$\begin{cases} x_{dw}(k+1) = A_{dw} x_{dw}(k) + B_{dw} v(k) \\ y(k) = C_{dw} x_{dw}(k) \end{cases} \quad (2)$$

where

$$x_{dw}(k) = \begin{bmatrix} x_d(k) \\ \xi_2(k) \end{bmatrix} \quad x_d(k) = \begin{bmatrix} x(k) \\ \xi_1(k) \end{bmatrix}$$

$$A_{dw} = \begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix} \quad B_{dw}(k) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A_d = \begin{bmatrix} e^{A_c T} & e^{A_c(T-L_d)} \int_0^{L_d} e^{A_c \tau} B_c d\tau \\ 0 & 0 \end{bmatrix}$$

$$B_d = \begin{bmatrix} \int_0^{T-L_d} e^{A_c \tau} B_c d\tau \\ 1 \end{bmatrix}$$

$$C_{dw} = \begin{bmatrix} C_d & 0 \end{bmatrix} \quad C_d = \begin{bmatrix} C & 0 \end{bmatrix} \quad \xi_1(k) = u(k)$$

In practical use of a DC-DC converter, the characteristics of a startup transient response and a dynamic load response are important. The DC-DC converter with the following specifications 1-6 is designed and manufactured by constituting digital control systems to a DC-DC switching part :

1. Input voltage V_i is 48[V] and output voltage e_o is 3.3[V].
2. Startup transient reponses are almost the same at resistive load and parallel load of resistance and capacity, where $0.165 \leq R_L < \infty$[Ω] and $0 \leq C_L \leq 200$[μF].
3. The rising time of the startup transient reponse is smaller than 100[μs].
4. Against all the loads of spec.1, an over-shoot is not allowable in the startup transient response.
5. The dynamic load response is smaller than 50[mV] against 10[A] change of load current.
6. The specs. 2, 3, 4 and 5 are satisfied also to change of input voltage of ±20%.

The load changes for the controlled object and the input voltage change are considered as parameter changes in eq.(1). Such parameter changes can be transformed to equivalent disturbances q_v and q_y as shown in **Fig.3** even in discrete-time systems. Moreover, if the saturation in the input arises or the input frequency is not so small in comparison with the carrier frequency, the controlled object will be regarded as a class of nonlinear systems. Such changes in characteristics can be also transformed to equivalent disturbances as shown in Fig.3. Therefore, what is necessary is just to constitute the control systems whose pulse transfer functions from equivalent disturbances q_v and q_y to the output y become as small as possible in their amplitudes, in order to robustize or suppress the influence of these parameter changes, i.e., load changes and input voltage change. In the next section, an easy designing method which makes it possible to suppress the influence of such disturbances with the target characteristics held will be presented.

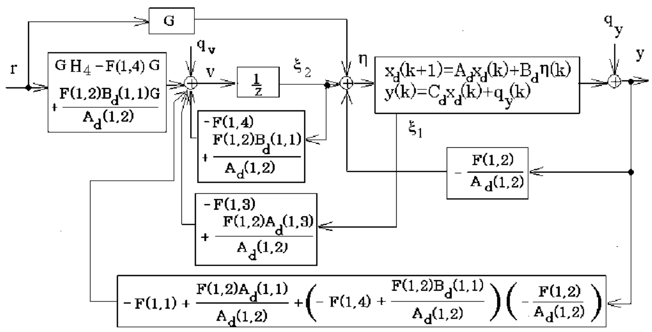

Fig. 4. Model matching system using only voltage (output) feedback

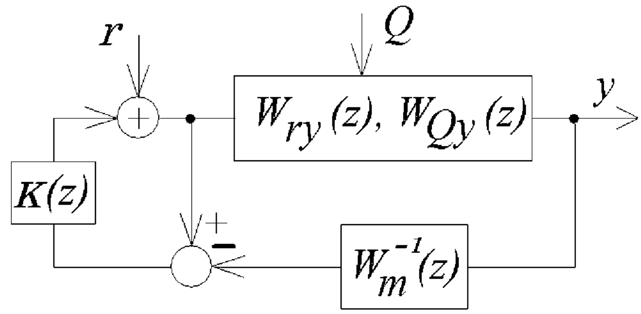

Fig. 5. System reconstituted with inverse system and filter

3. Design method of good approximate 2DOF digital integral-type control system

First, the transfer function between the reference input r and the output y is specified as follows :

$$\begin{aligned} W_{ry}(z) &= \frac{(1+H_1)(1+H_2)(1+H_3)}{(1-n_1)(1-n_2)(z+H_1)} \\ &\times \frac{(z-n_1)(z-n_2)(z+H_4)}{(z+H_2)(z+H_3)(z+H_4)} \end{aligned} \tag{3}$$

where, n_1 and n_2 are the zeros for the discrete-time control object (2). It shall be specified that the relation of H_1 and H_2, H_3 becomes $|H_1|$ and $|H_2| \gg |H_3|$. Then $W_{ry}(z)$ can be approximated by the following:

$$W_{ry}(z) \approx W_m(z) \approx \frac{(1+H_1)(1+H_2)(z-n_0)}{(z+H_1)(z+H_2)(1-n_0)} \tag{4}$$

This target characteristic $W_{ry}(z) \approx W_m(z)$ is specified so that it satisfies the specs.3 and 4.
Applying a state feedback

$$\begin{aligned} v &= -Fx^* + GH_4 r \\ x^* &= [y \;\; x_2 \;\; \xi_1 \;\; \xi_2]^T \end{aligned} \tag{5}$$

and a feedforward

$$\xi_1(k+1) = Gr \tag{6}$$

to the discrete-time controlled object as shown in Fig. 3, we determine $F = [F(1,1) \;\; F(1,2) \;\; F(1,3) \;\; F(1,4]$ and G so that $W_{ry}(z)$ becomes Eq.(3). Current feedback is used in Fig.3. This is transformed into voltage and control signal feedbacks, without changing the pulse transfer function between $r-y$ by an equivalent conversion. The following relation is obtained from Fig. 3 :

$$\begin{aligned} -F(1,2)x_2(k) &= -\frac{F(1,2)}{A_d(1,2)}(x_1(k+1) - A_d(1,1)x_1(k) \\ &- A_d(1,3)\xi_1 - B_d(1,1)\eta) \end{aligned} \tag{7}$$

If the current feedback is transformed equivalently using the right-hand side of this equation, the control system with only voltage feedback as shown in **Fig.4** will be obtained.

The transfer function $W_{Qy}(z)$ between this equivalent disturbance $Q = [q_v \;\; q_y]^T$ and the output y of the system in Fig.4 desfined as

$$W_{Qy}(z) = \left[\; W_{q_v y}(z) \quad W_{q_y y}(z) \; \right] \tag{8}$$

The system added the inverse system and the filter to the system in Fig.4 is constituted as shown in **Fig.5**. In Fig.5, the transfer function $F(z)$ becomes

$$K(z) = \frac{k_z}{z-1+k_z} \tag{9}$$

The transfer functions between $r-y$ and $Q-y$ of the system in Fig.5 are given by

$$\begin{aligned} y &= \frac{(1+H_1)(1+H_2)}{(z+H_1)(z+H_2)} \\ &\times \frac{z-1+k_z}{z-1+k_z+k_z(-1+W_s(z))} W_s(z) r \end{aligned} \tag{10}$$

$$y = \frac{z-1}{z-1+k_z} \frac{z-1+k_z}{z-1+k_z W_s(z)} W_{Qy}(z) Q \tag{11}$$

where

$$W_s(z) \approx \frac{(1+H_3)(z-n_1)(z-n_2)}{(z+H_3)(1-n_1)(1-n_2)} \tag{12}$$

Here, if $W_s(z) \approx 1$, then Eqs.(10) and (11) become, respectively,

$$y \approx \frac{(1+H_1)(1+H_2)}{(z+H_1)(z+H_2)} r \tag{13}$$

(14)

$$y \approx \frac{z-1}{z-1+k_z} W_{Qy}(z) Q \tag{15}$$

From eqs.(13) and (14), it turns out that the characteristic from r to y can be specified with H_1 and H_2, and the characteristic from Q to y can be independently specified with k_z. That is, the system in Fig.5 is an approximate 2DOF system, and its sensitivity against disturbance, i.e., load change becomes lower with the increase of k_z.
If an equivalent conversion of the controller in Fig.5 is carried out, the good approximate 2DOF digital integral-type control systems will be obtained as shown in **Fig.6**. In Fig.6,

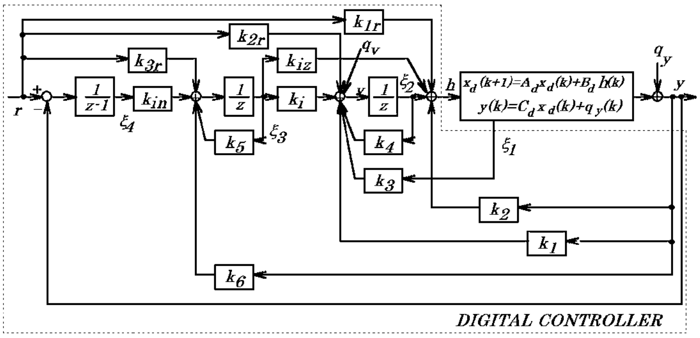

Fig. 6. Control system of approximate 2DOF digital integral type

the parameters of the controller are as follows :

$$\begin{aligned}
k_1 &= -F(1,1) - F(1,2)FF(1,1) + ((-F(1,4) \\
&- F(1,2)FF(1,4))(-F(1,2)/FF(1,2))) \\
&- (GH4 + GF_z)((1-n_a)k_z/((1+H1)(1+H2))) \\
k_2 &= -F(1,2)/FF(1,2) \\
&- G((1-n_a)k_z/((1+H1)(1+H2))) \\
k_3 &= -F(1,3) - F(1,2)(FF(1,3)) \\
k_4 &= F_z \quad k_5 = n_a \\
k_6 &= -(k_z(1-n_a)(1+H1+H2) \\
&+ n_a(1-n_a)k_z)/((1+H1)*(1+H2)) \\
k_i &= GH4 + GFz \quad k_{iz} = G \quad k_{in} = k_z(1-n_a) \\
k_{1r} &= G \quad k_{2r} = GH4 + GF_z \quad k_{3r} = k_z \qquad (16)
\end{aligned}$$

where

$$\begin{aligned}
FF(1,1) &= -A_d(1,1)/A_d(1,2) \\
FF(1,2) &= A_d(1,2) \\
FF(1,3) &= -A_d(1,3)/A_d(1,2) \\
FF(1,4) &= -B_d(1,1)/A_d(1,2) \\
F_z &= -F(1,4) - F(1,2)FF(1,4)
\end{aligned}$$

If k_z is large enough in eq.(10), the following equation holds at $W_s(z) \approx 1$, and feedforward parameters k_{1r}, k_{2r} and k_{3r} are not necessarily required in Fig. 6 :

$$\frac{z-1+k_z}{z-1+k_z} \approx \frac{k_z}{z-1+k_z} \qquad (17)$$

4. Experimental studies

The sampling period T are set at $3.3[\mu s]$ and the input dead time L_d is about $0.999T[\mu s]$. The nominal value of R_L is $0.33[\Omega]$. We are to design a control system so that all the specifications are satisfied. First of all, in order to satisfy the specification on the rising time of startup transient reponse, H_1, H_2, H_3 and H_4 are specified as

$$\begin{aligned}
H_1 = -0.83 \quad H_2 = -0.82 \\
H_3 = 0.3 \quad H_4 = -0.3 \qquad (18)
\end{aligned}$$

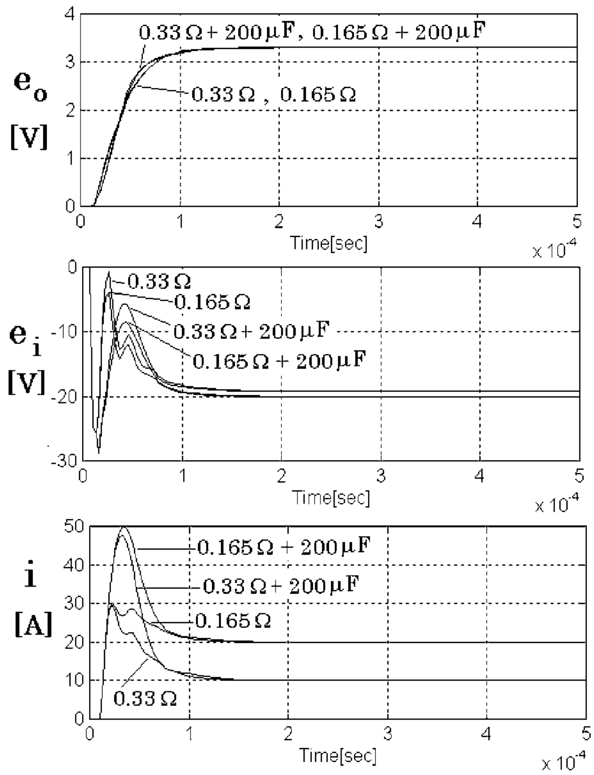

Fig. 7. Simulation results of startup responses at various loads

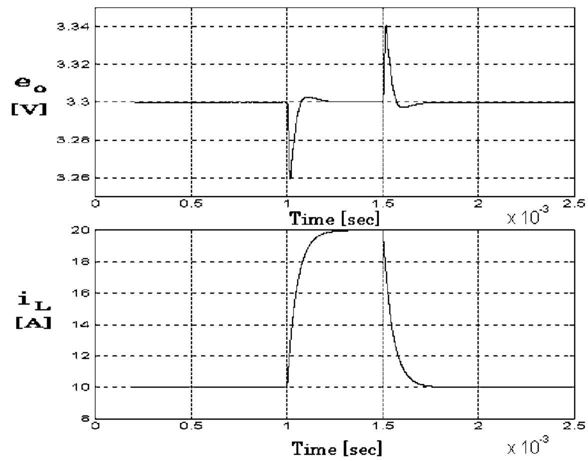

Fig. 8. Simulation result of dynamic load response at resistive load

The parameter k_z is selected as

$$k_z = 0.6 \qquad (19)$$

Then the parameters of controller become

$$\begin{aligned}
k_1 &= -194.88 \quad k_2 = 289.74 \quad k_3 = -0.045316 \\
k_4 &= -0.25781 \quad k_5 = -0.40000 \quad k_6 = 28.824 \\
k_i &= 4.9609 \quad k_{iz} = -8.8937 \quad k_{in} = 0.84000 \qquad (20)
\end{aligned}$$

k_{1r}, k_{2r} and k_{3r} are set at 0. The simulation results of the output voltage $y = e_o$, the input voltage $u = e_i$ and the

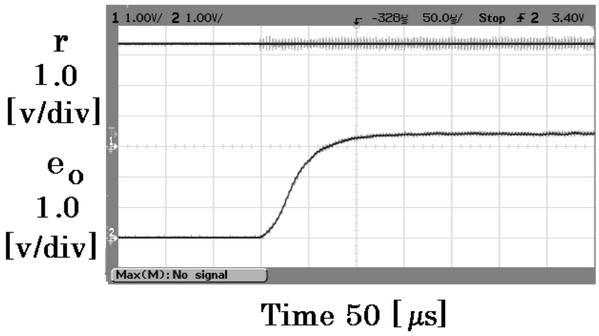

Fig. 9. Experimental result of startup response at resisitive load ($R_L = 0.33[\Omega]$)

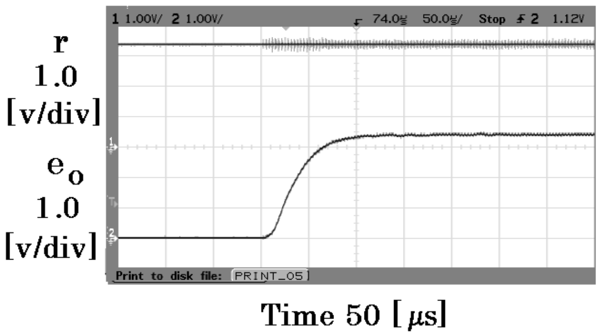

Fig. 10. Experimental result of startup response at resisitive load ($R_L = 0.165[\Omega]$)

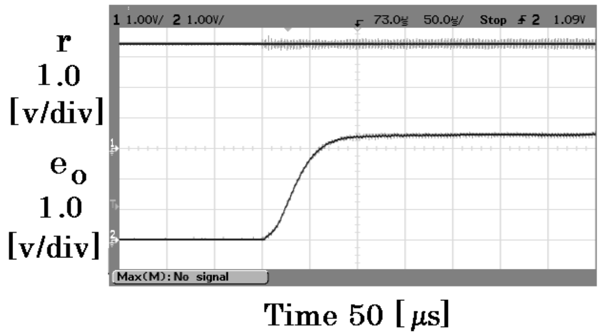

Fig. 11. Experimental result of startup response at parallel load of resistance ($R_L = 0.33[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$)

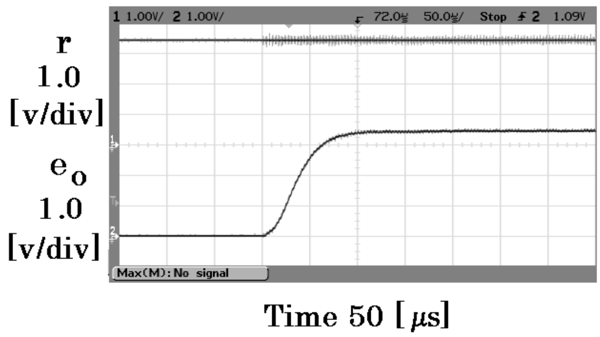

Fig. 12. Experimental result of startup response at the parallel load of resistance ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$)

current i are shown in **Fig.7**. From $y = e_o$ in this figure, it turns out that all the specifications are satisfied. It is checked that almost the same simulation results as Fig.8 are obtained when the input voltage V_i is changed by $\pm 20\%$. The simulation results of the dynamic load responses are shown in **Fig.8**. Fig.8 is for the case of the resistive load ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$). Although the load current changed suddenly from 20 [A] to 10 [A] or reverse, the output voltage change is very small and is suppressed within about 50 [mV]. It is checked that almost the same simulation result as **Fig.11** is obtained at the parallel load of resistance ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$). It turns out that all results satisfy the specifications.

Experimental results when realizing the digital controller with the parameters of eq.(20) by using the DSP, and connecting to the controlled object of eq.(1) are shown in **Figs.9-16**. Fig.9 shows a startup response at the resisitive load ($R_L = 0.33[\Omega]$). Fig.10 shows a startup response at the resisitive load ($R_L = 0.165[\Omega]$). Fig.11 shows a startup response at the parallel load of resistance ($R_L = 0.33[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$). Fig.11 shows a startup response at the parallel load of resistance ($R_L = 0.165[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$). From $y = e_o$ in these figure, it turns out tha almost the same exprimental results as the simulation results in Fig.7 are obtained and the specifications are satisfied. Fig.13 shows a startup response at the resisitive load ($R_L = 0.33[\Omega]$) when the input voltage is 58[V]. Fig.14 shows a startup response at the resisitive load ($R_L = 0.33[\Omega]$) when the input voltage is 38[V]. It turns out that the specifications are satisfied when the input voltage V_i is changed by $\pm 20\%$. Fig.15 shows dynamic load responses at the resistive load ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$). Fig.16 shows dynamic load responses at the resistive load ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$). Fig.15 shows dynamic load responses at the parallel load of resistance ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$) and capacity ($C_L = 200[\mu\text{F}]$). It turns out tha almost the same exprimental results as the simulation results in Fig.11 are obtained. Although the load current changed suddenly from 20 [A] to 10 [A] or reverse, the output voltage change is very small and is suppressed within about 50 [mV]. It turns out that all the specifications are satisfied.

5. Conclusion

In this paper, the concept of controller of a DC-DC converter to attain good robustness against an extensive load changes was given. The proposed digital controller was implemented on the DSP connecting to the controlled object (PWM switching part which consists of an electric power conversion part and an LC filter). It was shown from some simulations and experiments that a sufficiently robust digital controller

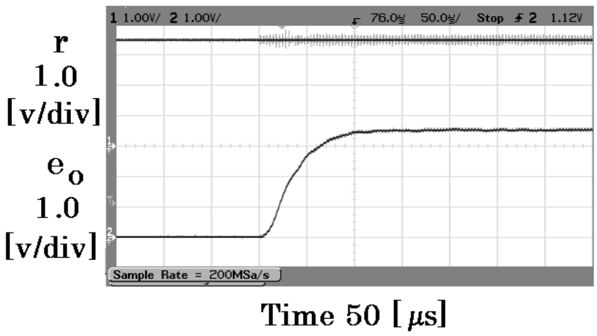

Fig. 13. Experimental result of startup response at resisitive load ($R_L = 0.33[\Omega]$) when the input voltage is 58[V]

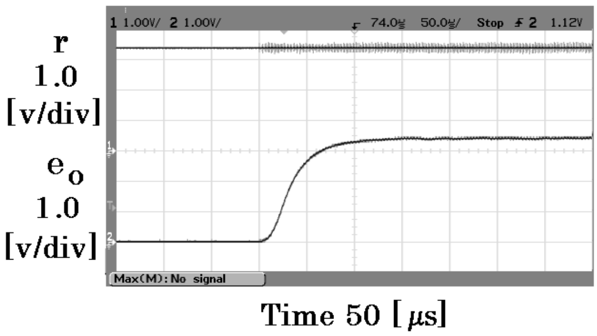

Fig. 14. Experimental result of startup response at resisitive load ($R_L = 0.33[\Omega]$) when the input voltage is 38[V]

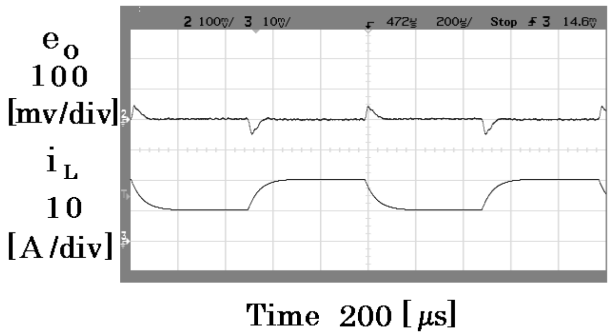

Fig. 15. Experimental result of dynamic load response at resistive load ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$)

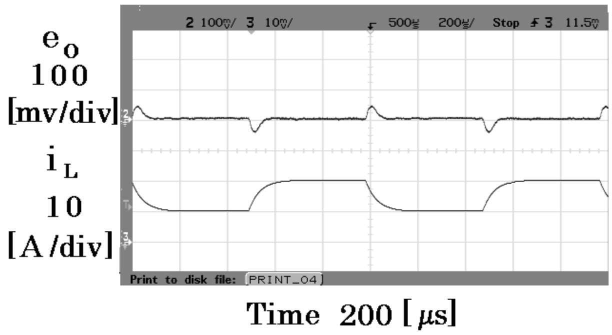

Fig. 16. Experimental result of dynamic load response at the parallel load of resistance and capacity ($R_L = 0.33 \leftrightarrow 0.165[\Omega]$)

is realizable. The characteristics of the startup transient response and the dynamic load response were improved by using the proposed good approximate 2DOF digital controller. An advanced control algorithm has been implemented with a short sampling time using the TI TMS320LF2401. This fact demonstrates the usefulness and practicality of our method. Experimental studies on sudden change in the input voltage are required for the future.

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