Accuracy of Free Space Path Loss and Matched Filter Gain Approximated by Using Passband Rectangular Pulse for Ultra Wideband Radio Systems

Pichaya Supanakoon†, Suchada Tanchotikul†, Prakit Tangtisanon†,
Sathaporn Promwong†,††, and Jun-ichi Takada††

†Department of Information Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang
Chalongkrung Rd., Ladkrabang, Bangkok 10520, Thailand
Email: {kspichay,s6068223,ktprakit,kpsathap}@kmitl.ac.th

††Graduate School of Science and Engineering,
Tokyo Institute of Technology
O-okayama Minami 6 Bldg., 2–12–1, O-okayama, Meguro-ku, Tokyo, 152–8552, Japan
Email: ken@ap.ide.titech.ac.jp

Abstract: This paper analyzes the accuracy of free space path loss and matched filter gain approximated by using a passband rectangular pulse for ultra wideband (UWB) radio systems. The example causal signal, a modulated Gaussian pulse with the same center frequency and frequency bandwidth of the passband rectangular pulse, is used to consider the accuracy. The path loss and matched filter gain of the modulated Gaussian pulse are simulated for the reference results. The UWB free space path loss is shown and is compared with that obtained from simulation and Friis' transmission formula. The UWB matched filter gain is shown and compared with simulation results. From the results, we can see that the UWB path loss formula is more accurate than the Friis' transmission formula. The results from the UWB free space path loss and matched filter gain formulas agree with the simulation. Then, these free space path loss and matched filter gain formulas approximated by using a passband rectangular pulse are appropriate for UWB system.

Keywords: Ultra wideband (UWB), Friis' transmission formula, free space path loss, matched filter gain

1. Introduction

Recently, ultra wideband (UWB) communications [1]-[3], are becoming an important topic for microwave communication. UWB is different from other radio wave (RF) technology. Instead of using a narrow carrier frequency, UWB transmit pulses of power in the range of ultra wide frequency spectrum. The Federal Communication Commission (FCC) [4], in US specifies that UWB has a frequency spectrum ranging from 3.1 to 10.6 GHz. The FCC defined UWB signal as those which have a fractional bandwidth greater than 0.20 or a bandwidth greater than 500 MHz measured at -10 dB points. The power density of UWB signal is considered to be noise for other communication systems because its power spectrum is below the noise level. The UWB receiver collects the power of the received signal to rebuild the pulse. Therefore, UWB radio technology can exist with other RF technologies without interference. UWB radio technology is an ideal candidate that can be utilized for commercial, short-range, low power, low cost indoor communication systems such as a wireless personal area network (WPAN) [5]-[7].
The Friis' transmission formula [8] is widely used to calculate the free space path loss for narrow band system. Complex form Friis' transmission formula and the use of matched filter are developed for UWB system [9]-[10]. The closed form expressions of the UWB path loss and matched filter gain for the free space channel are derived [11]. This paper analyzes the accuracy of free space path loss and matched filter gain approximated by using passband rectangular pulse for UWB-IR system. The example causal signal, a modulated Gaussian pulse with the same center frequency and frequency bandwidth of the passband rectangular pulse, is used to consider the accuracy. The path loss and matched filter gain of the modulated Gaussian pulse are simulated for the reference results. The UWB free space path loss is shown and is compared with simulation and the Friis' transmission formula. The UWB matched filter gain is shown and compared also with simulation.

2. Free Space Path Loss and Matched Filter Gain for Ultra Wideband System

The UWB transmitted signal is set to be a passband rectangular pulse. The expression of this pulse in time domain v_t and its spectral density function V_t are

$$v_\mathrm{t}(t) = \frac{1}{f_\mathrm{b}}\left[\begin{array}{c} f_\mathrm{max}\mathrm{sinc}(2\mathrm{f_{max}t}) \\ -f_\mathrm{min}\mathrm{sinc}(2\mathrm{f_{min}t}) \end{array}\right], \quad (1)$$

$$V_\mathrm{t}(f) = \begin{cases} \frac{1}{2f_\mathrm{b}} & ||f| - f_\mathrm{c}| \le \frac{f_\mathrm{b}}{2} \\ 0 & ||f| - f_\mathrm{c}| > \frac{f_\mathrm{b}}{2} \end{cases}, \quad (2)$$

where t is the time, f is the frequency, f_c is the center frequency, f_b is the spectral bandwidth, $f_\mathrm{min} = f_\mathrm{c} - f_\mathrm{b}/2$ is the minimum frequency, $f_\mathrm{max} = f_\mathrm{c} + f_\mathrm{b}/2$ is the maximum frequency and $\mathrm{sinc(x)} = \sin(\pi \mathrm{x})/(\pi \mathrm{x})$. This signal can be approximate to be an impulse function with constant spectral density from f_min to f_max Hz and the area is

$$\int_{-\infty}^{\infty} v_{\rm t}(t)dt = 1. \quad (3)$$

For the UWB free space channel, the complex form Friis' transmission formula is used. The transmitting (Tx) and Receiving (Rx) antennas are considered to be the isotropic antennas. Then, the free space transfer function $H_{\rm f}$ can be written as

$$\begin{aligned} H_{\rm f}(f,d) &= \frac{c}{4\pi|f|d}\exp(-j2\pi fd/c), \\ &= \frac{1}{4\pi|f|t_0}\exp(-j2\pi ft_0), \end{aligned} \quad (4)$$

where d is the transmitter-receiver (T-R) separation distance, c is the velocity of light and $t_0 = d/c$ is the delayed time.
The spectral density function $V_{\rm r,f}$ and the waveform in the time domain $v_{\rm r,f}$ of the received signal can be found from

$$\begin{aligned} &V_{\rm r,f(f,d)} \quad (5) \\ &= V_{\rm t}(f)\cdot H_{\rm f}(f,d), \\ &= \begin{cases} \frac{1}{8\pi f_{\rm b}|f|t_0}\exp(-j2\pi ft_0) & ||f|-f_{\rm c}| \le \frac{f_{\rm b}}{2} \\ 0 & ||f|-f_{\rm c}| > \frac{f_{\rm b}}{2} \end{cases}, \end{aligned}$$

$$\begin{aligned} &v_{\rm r,f}(f,d) \quad (6) \\ &= \mathcal{F}^{-1}\{V_{\rm r,f}(f,d)\}, \\ &= \begin{cases} \frac{1}{4\pi f_{\rm b}t_0}\ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right) & t = t_0 \\ \frac{1}{4\pi f_{\rm b}t_0}\begin{bmatrix} C_i(2\pi f_{\rm max}|t-t_0|) \\ -C_i(2\pi f_{\rm min}|t-t_0|) \end{bmatrix} & t \ne t_0 \end{cases}, \end{aligned}$$

where $\mathcal{F}^{-}1\{\cdot\}$ is the inverse Fourier transform operator and $C_i = \int_{\infty}^{x}\frac{\cos(\tau)}{\tau}d\tau$ is the cosine integral. This received signal equation can be defined as the impulse response of the free space channel.
At the receiver, the matched filter detection is introduced. Its frequency transfer function $H_{\rm MF,f}$ satisfies the following constant noise power condition between the input and output,

$$\int_{f_{\rm min}}^{f_{\rm max}} |H_{\rm MF,f}(f,d)|^2 df = f_{\rm b}. \quad (7)$$

For satisfying that condition, $H_{\rm MF,f}$ is given by

$$\begin{aligned} H_{\rm MF,f}(f,d) &= V^*_{\rm r,f}(f,d)\sqrt{\frac{f_{\rm b}}{\int_{f_{\rm min}}^{f_{\rm max}}|V_{\rm r,f}(f,d)|^2df}}, \quad (8) \\ &= \begin{cases} \frac{f_0}{|f|}\exp(j2\pi ft_0) & ||f|-f_{\rm c}| \le \frac{f_{\rm b}}{2} \\ 0 & ||f|-f_{\rm c}| > \frac{f_{\rm b}}{2} \end{cases}, \end{aligned}$$

where $f_0 = \sqrt{f_{\rm min}f_{\rm max}}$.
The spectral density function $V_{\rm MF,f}$ of the signal and time domain waveform $v_{\rm MF,f}$ at the matched filter output can be written as

$$\begin{aligned} V_{\rm MF,f} &= V_{\rm r,f}(f,d)\cdot H_{\rm MF,f}(f,d), \quad (9) \\ &= \begin{cases} \frac{f_0}{8\pi f_{\rm b}f^2t_0} & ||f|-f_{\rm c}| \le \frac{f_{\rm b}}{2} \\ 0 & ||f|-f_{\rm c}| > \frac{f_{\rm b}}{2} \end{cases}, \end{aligned}$$

$$\begin{aligned} v_{\rm MF,f} &= \mathcal{F}^{-1}\{V_{\rm MF,f}(f,d)\}, \quad (10) \\ &= \begin{cases} \frac{1}{4\pi df_0t_0} & t = 0 \\ \frac{f_0}{4\pi f_{\rm b}dt_0}\begin{bmatrix} \frac{\cos(2\pi f_{\rm min}t)}{f_{\rm min}} \\ -\frac{\cos(2\pi f_{\rm max}t)}{f_{\rm max}} \\ +2\pi tS_i(2\pi f_{\rm min}t) \\ -2\pi tS_i(2\pi f_{\rm max}t) \end{bmatrix} & t \ne 0 \end{cases}, \end{aligned}$$

where $S_i(x) = \int_0^x \frac{\sin(\tau)}{\tau}d\tau$ is the sine integral.
Defining the UWB free space path loss $PL_{\rm f}$ as the ratio between the maximum amplitude of the transmitted and received signal waveforms. Therefore, the UWB free space path loss in dB can be derived as shown below

$$\begin{aligned} PL_{\rm f}(d)[dB] &= 20\log\left[\frac{|v_{\rm t,f}(0)|}{|v_{\rm r,f}(t_0)|}\right], \quad (11) \\ &= 20\log\left[\frac{4\pi f_{\rm b}t_0}{\ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right)}\right]. \end{aligned}$$

The UWB matched filter gain of the free space channel $G_{\rm MF,f}$ is defined as the ratio between the maximum amplitude of the signal waveform at the output of the matched filter and that of the received signal waveform. It can be expressed as

$$\begin{aligned} G_{\rm MF,f}[dB] &= 20\log\left[\frac{v_{\rm MF,f}(0)}{v_{\rm r,f}(t_0)}\right], \quad (12) \\ &= 20\log\left[\frac{f_{\rm b}}{f_0\ln\left(\frac{f_{\rm max}}{f_{\rm min}}\right)}\right]. \end{aligned}$$

For the arbitrary causal signal $s(t)$, the free space path loss expressions is multiplied by the maximum amplitude $A_{\rm m}$ of the filtered signal waveform with spectrum ranging from $f_{\rm min}$ to $f_{\rm max}$. It can be calculated by using

$$\begin{aligned} &A_{\rm m} \quad (13) \\ &= \max\left|\int_{-f_{\rm max}}^{-f_{\rm min}} S(f)e^{j2\pi ft}df + \int_{f_{\rm min}}^{f_{\rm max}} S(f)e^{j2\pi ft}df\right|, \end{aligned}$$

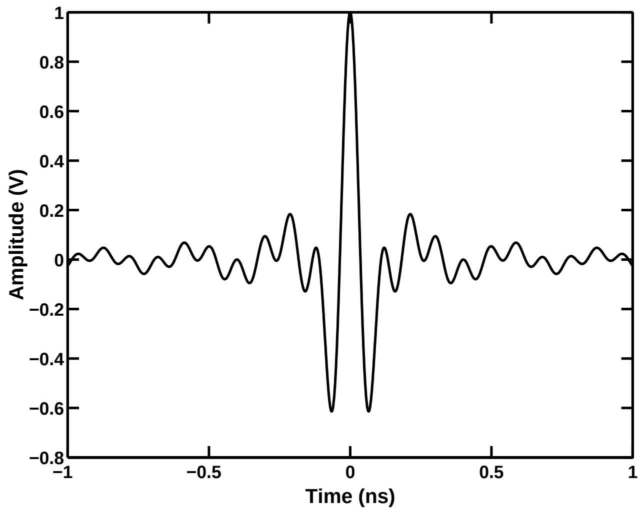

Fig. 1. Passband rectangular signal waveform.

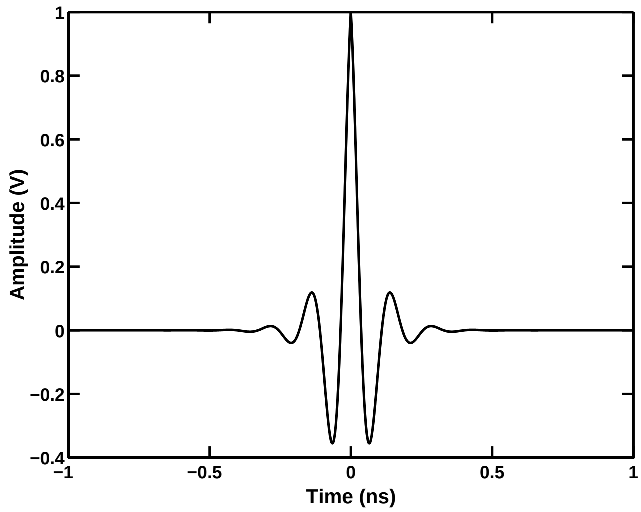

Fig. 2. Modulated Gaussian signal waveform.

where $S(f) = \mathcal{F}\{s(t)\}$ and $\mathcal{F}\{\cdot\}$ is the Fourier transform operator.

Then, the free space path loss for an arbitrary causal signal can be written as

$$PL_{\mathrm{f}}(d)[dB] \quad = \quad 20\log\left[\frac{4\pi f_{\mathrm{b}} t_0 A_{\mathrm{m}}}{\ln\left(\frac{f_{\max}}{f_{\min}}\right)}\right]. \tag{14}$$

3. Accurate Analysis

For accurate analysis, the passband rectangular pulse is set in the full UWB spectrum bandwidth. Figure 1 shows the passband rectangular waveform. The center frequency is $f_{\mathrm{c}} = 6.85$ GHz. The frequency bandwidth is $f_{\mathrm{b}} = 7.5$ GHz. Then, the minimum and maximum frequencies are $f_{\min} = 3.1$ GHz and $f_{\max} = 10.6$ GHz, respectively. The example causal signal, a modulated Gaussian pulse, is used to consider the accuracy.

To satisfy the carrier frequency and frequency bandwidth requirements, the carrier frequency is set to be f_c. The amplitude is decayed to $1/e$ when the time $t = \pm 1/f_{\mathrm{b}}$. The expression of this modulated Gaussian pulse is

$$s(t) = e^{-(f_{\mathrm{b}}t)^2}\cos(2\pi f_{\mathrm{c}} t). \tag{15}$$

The modulated Gaussian pulse is shown in Fig. 2. This pulse is filtered with passband frequency ranging from 3.1 GHz to 10.6 GHz for considering the A_{m}. The filtered pulses are shown in Fig. 3. From the figure, we can see that the A_{m} equals to 0.67.

For accurate analysis, the path loss and matched filter gain of the modulated Gaussian pulse are simulated for the reference results. The UWB free space path loss and Friis' transmission formulas are calculated. Figure 4 shows the UWB Path loss of the free space channel compared with simulation by using the modulated Gaussian pulse and Friis' transmission formula. From the figure we can see that the UWB free space path loss approximated by using the passband rectangular pulse is more accurate than the Friis' transmission formula. The Friis' transmission formula has different average value from the simulation of about 4.12 dB while the UWB path loss formula has average error of only 0.06 dB. The important notice is that the UWB path loss is less than the narrow band path loss obtained from the Friis' transmission formula.

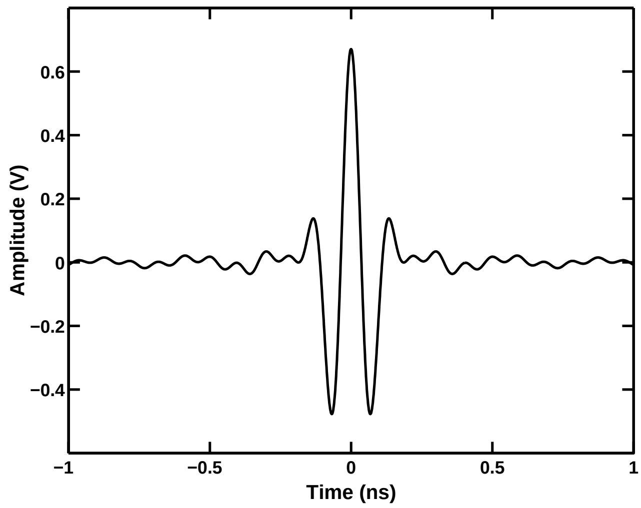

Fig. 3. Filtered signal waveform.

The UWB free space matched filter gain obtained from simulation and matched filter gain formula is listed in Table 1. The result from the matched filter gain formula is less than the simulation by only 0.03 dB.

Table 1. UWB free space matched filter gain obtained from simulation and matched filter gain formula.

Method	Matched filter gain [dB]
Simulation	0.57
Matched filter gain formula	0.54

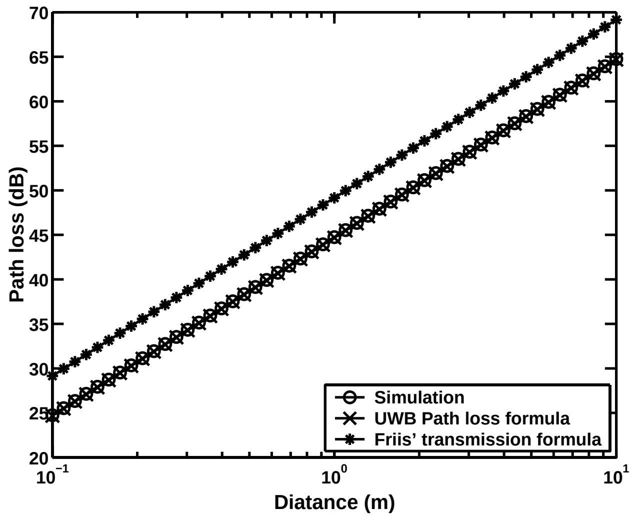

Fig. 4. UWB Path loss of the free space channel compared with simulation by using modulated Gaussian pulse and Friis' transmission formula.

4. Conclusion

This paper analyzes the accuracy of free space path loss and matched filter gain approximated by using a passband rectangular pulse for UWB systems. From the results, we can see that the UWB path loss formula is more accurate than the Friis' transmission formula. The results from the UWB free space path loss and matched filter gain formulas agree with the simulation. Then, these free space path loss and matched filter gain formulas approximated by using a passband rectangular pulse are appropriate for UWB systems.

References

[1] European Telecommunications Standards Institute (ETSI), Technical Report, "Technical characteristics for Short Range Devices communication equipment using Ultra-Wideband technology," TR 101 994-1, V1.0.0, August 2003 (http://www.etsi.org).

[2] M.Z.Win, R.A. Scholtz, "Impulse Radio: How it works," IEEE Comm. Letter, Vol.2, No.1, pp. 36-38, January 1998.

[3] R.A. Scholtz, R. Weaver, E. Homier, J. Lee, P. Hilmes, A. Taha, and R. Wilson, "UWB Radio Deployment Challenges," PIMRC, September 2000.

[4] Federal Communications Commission, "Revision of Part 15 of the Commission's Rules Regarding UWB Transmission Systems," First Report, FCC 02-48, Apr. 2002.

[5] K. Siwiak, "Ultra-Wide Radio: Introduccing a New Technology," 2001 Spring IEEE Veh. Tech. Conf. (VTC), Plenary session, pp. 1088-1093, May 2001.

[6] K. Siwiak, "Ultra-Wide Radio: The emergence of an Important RF Technology," Proc. 2001 Spring IEEE Veh. Tech. Conf. (VTC), pp. 1169-1172, May 2001.

[7] J. Farserotu, A. Hutter, F. Platbrood, J. Gerrits and A. Pollini, "UWB Transmission and MIMO Antenna Systems for Nomadic User and Mobile PAN," Wireless Personal Commun., no. 22, pp. 197-317, 2002.

[8] H. T. Friis, "A Note on a Simple Transmission Formula," *Proc. IRE*, Vol 34, no 5, pp. 254-256, May 1946.

[9] J. Takada, S. Promwong and W. Hachitani, "Extension of Friis' Transmission Formula for UWB Systems," IEICE Tech. Rep., May 2003.

[10] A.H. Mohammadian, A. Rajkotia, and S.S. Soliman, "Characterization of UWB Transmit-Receive Antenna System," Proc. IEEE Conf. Ultra Wideband Syst. Tech. (UWBST) 2003, Nov. 2003.

[11] P. Supanakoon, P. Tangtisanon, S. Promwong, and J. Takada, "Path Loss and Matched Filter Gain for UWB System," 2004 International Symposium on Antenna and Propagation (ISAP), August 2004. (to be presented)