

Boundary Control of Container Crane: Two-Stage Control of a Container Crane as Nonflexible and Flexible Cable

Hahn Park* and Keum-Shik Hong**

*Department of Mechanical and Intelligent Systems Engineering, Pusan National University, Busan, Korea
(Tel : +82-51-510-1481; E-mail: hpark97@pusan.ac.kr)

**School of Mechanical Engineering, Pusan National University, Busan, Korea
(Tel : +82-51-510-2454; E-mail: kshong@pusan.ac.kr)

Abstract: In this paper, we proposed a two-stage control of the container crane. The first stage control is time-optimal control for the purpose of fast trolley traveling. With suitable trolley velocity patterns, the sway which is generated during trolley moving is minimized. At the second stage control feedback control law is investigated for the quick suppression of residual vibration after the trolley motion. For more practical system, the container crane system is modeled as a partial differential equation (PDE) system with flexible cable. The dynamics of the cable is derived as a moving system with tension caused by payload using Hamilton's principle for the systems. A control law based upon the Lyapunov's method is derived. It is revealed that a time-varying control force and a suitable passive damping at the actuator can successfully suppress the transverse vibrations.

Keywords: Container crane, two-stage control strategies, PDE system, boundary control, Lyapunov's method, time-optimal control.

1. INTRODUCTION

The crane system, which transports a big object to certain place, is widely used in industrial field. As the industrial structure has grown and the higher work efficiency has been required, more improved crane system, which can transfer the object fast, has been needed. Especially in a harbor container crane system, the necessity is more important. The focus of harbor loading and unloading process is minimizing the moving time of container. However, if the velocity of container is increased in order to decrease the working time, the vibration of crane cable is generated in a loading/unloading process. This vibration makes some problems. We cannot achieve precise position of container and the working time is delayed. As a result, many research interested in searching methods that will eliminate the vibration of cable has been performed.

In former papers, various methods for a suppressing the sway of crane cable is investigated. There are generally two methods. One is the control method that reduces the sway of cable with a trolley velocity pattern as a control input shape. With the sway equation of motion of a linearized crane model, a sway feature for a specific velocity pattern is shown, and then a valuable velocity pattern is derived. Various results for certain velocity patterns are given in much paper, and the result is good. However, in this case, the model is not realistic. A real crane cable has flexibility. A real crane has long cables, huge containers and the disturbance like wind may be lead to flexible deformation of crane cable. Then we must consider the flexibility of crane cable.

In contrast with this method, the other is using the function like energy. This method is usually used in case that the crane cable is considered as a flexible string, not a rigid pendulum. With a stabilizing theory using the Lyapunov's method, the control law through the trolley motion is designed. But this method can not guarantee the fast trolley moving because this method is only related to the Lyapunov function candidate like an energy function.

For this reason, in this paper we propose a new control method which picks up the advantageous of two methods mentioned before. When a trolley moves fast, the sway due to the disturbance may be negligible because of a big system size

and container mass. In this case, the vibration is also negligible and the system can be regarded a rigid pendulum. And in a dissipating vibration portion there is no trolley moving. Then the influence of elastic deformation and control law reflect this must be considered. Moreover, when the trolley finishes the motion, the actuator operates to suppress the residual vibration. This control law is investigated following.

This paper is structured as follows: In Section 2, the equations of motion of flexible crane cable are derived by using the Hamilton's principle. And path planning and two-stage control strategies are investigated with the nonflexible crane model. In Section 3, the time-optimal control for the fast trolley moving is investigated. In Section 4, a stabilizing boundary control law that suppresses the vibration of crane cable is derived. Finally, conclusions are stated in Section 5.

2. SYSTEM CONTROL PROBLEM FORMULATION

2.1 System modeling

In many papers, the cable of container crane was modeled as nonflexible one shown in Fig.1 and Table 1. That is like a general pendulum. The following equations are used as nonflexible model (cable length is constant) [10]:

$$(m_1 + m_T)\ddot{x} + b_1\dot{x} = F_1, \quad x(0) = x_0, \quad \dot{x}(0) = 0, \quad (1)$$

$$(2m_2 + \frac{1}{2}m_L)\ddot{l} - \frac{1}{2}m_Lg = F_2, \quad l(0) = l_0, \quad \dot{l}(0) = 0, \quad (2)$$

$$\ddot{\phi}(t) + \frac{g}{l}\phi(t) = -\frac{\ddot{x}}{l}, \quad \phi(0) = \phi_0, \quad \dot{\phi}(0) = \dot{\phi}_0. \quad (3)$$

If we consider the variation of cable length, Eq. (3) may be changed as following.

$$\ddot{\phi}(t) + \frac{\dot{l}}{l}\dot{\phi}(t) + \frac{g}{l}\phi(t) = -\frac{\ddot{x}}{l}, \quad \phi(0) = \phi_0, \quad \dot{\phi}(0) = \dot{\phi}_0 \quad (4)$$

Note that if $\dot{l} > 0$, $\phi(t)$ become small and if $\dot{l} < 0$, $\phi(t)$ become large.

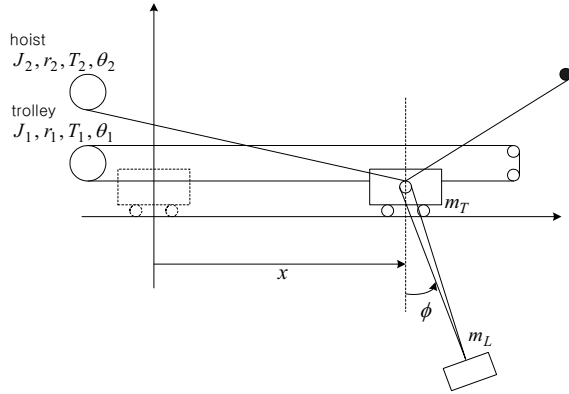


Fig. 1 A schematic diagram of the container crane (nonflexible model).

Table 1 Definitions of the variables.

Variables	Definition	Unit
b_1	d_1/r_1^2	kg/sec
b_2	d_2/r_2^2	kg/sec
d_1	$= r_1^2 b_1$, trolley damping	kgm^2/sec
d_2	$= r_2^2 b_2$, hoist damping	kgm^2/sec
$F_1 = F$	traction force of the trolley,	N
F_2	traction force of the hoist,	N
g	Acceleration of gravity (9.81)	m/sec^2
J_1	equivalent mass moment of inertia of the trolley drive	kgm^2
J_2	equivalent mass moment of inertia of the hoist drive	kgm^2
l	rope length, $l = r_2 \theta_2 / 2$	m
m_1	equivalent mass of the trolley drive,	kg
m_2	equivalent mass of the hoist drive,	kg
m_L	mass of a container including the spreader	kg
m_T	mass of trolley	kg
m	$m = m_1 + m_T$	kg
n_1	trolley motor gear ratio	
n_2	hoist motor gear ratio	
r_1	radius of trolley drum	m
r_2	radius of hoist drum	m
T_1	input torque at trolley drum,	$\text{N} \cdot \text{m}$
T_2	input torque at hoist drum,	$\text{N} \cdot \text{m}$
T_h	input torque of hoist motor	$\text{N} \cdot \text{m}$
T_t	input torque of trolley motor	$\text{N} \cdot \text{m}$
x	trolley displacement,	m
ϕ	sway angle of the container	rad
θ_1	angular displacement of the trolley drum	rad
θ_2	angular displacement of the hoist drum	rad

But in real container crane system, we can not neglect the flexibility of the cable easily. Because of the length of cable and the size of container, we have to consider the influence of disturbance like wind. However, during the fast trolley moving, the effect of disturbance is negligible. So we consider the system as nonflexible model in section of fast trolley moving, and in section of suppressing the residual vibration we consider the system as flexible model.

Fig. 2 shows a schematic diagram of the container crane considered as a flexible cable system. The cable is considered as an inextensible and vertically translating string of mass per unit length ρ with constant length l . We make assumption that the motion of cable in two dimensional plane. The difference of this modeling with other paper is using partial differential equation. In this paper, we consider that the cable length is constant and suppressing the vibration started after the trolley located at the desired position. Let t be the time, y be the spatial coordinate along the longitude of motion, and $P(y, t)$ be the tension caused by payload and mass of cable. $w(y, t)$ is the transversal displacement of the strip at spatial coordinate y and time t . And we suggest the actuator which is located under the trolley. This actuator is driven independently. So in Fig. 2, y_a is the actuator location.

Now, to derive the equation of motion, extended Hamilton's principle is used:

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W) dt = 0 \quad (5)$$

where T is the kinetic energy, V is the potential energy (in this case, strain energy) and W is the virtual work done by the control force $F_c(t)$. The tension of cable $P(y, t)$ is

$$P(y, t) = \{m_l + \rho(l - y)\}g \quad (6)$$

where m_l is the mass of the payload and g is the gravitational acceleration.

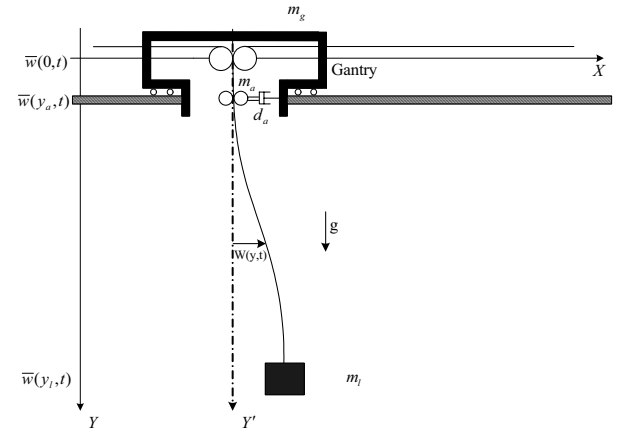


Fig. 2 A schematic diagram of the container crane (flexible model).

The kinetic energy of the cable, payload and actuator is

$$T = \frac{1}{2} \rho \int_0^{y_a} w_t^2 dy + \frac{1}{2} \rho \int_{y_a}^l w_t^2 dy + \frac{1}{2} m_a \left[\frac{Dw(y_a, t)}{Dt} \right]^2 + \frac{1}{2} m_l \left[\frac{Dw(l, t)}{Dt} \right]^2 \quad (7)$$

where $(\cdot)_t = \partial(\cdot)/\partial t$ and $(\cdot)_x = \partial(\cdot)/\partial x$ denote the partial derivatives, and $\frac{Dw(y, t)}{Dt} = \frac{\partial w(y, t)}{\partial t} + \frac{\partial w(y, t)}{\partial y} \frac{dy}{dt}$ is

material derivative. In this situation, the cable length does not vary. It means that $\frac{dy}{dt}$ is zero. So $\frac{Dw(y,t)}{Dt}$ is equal to $\frac{\partial w(y,t)}{\partial t}$. m_a is the mass of actuator. The first integral term is vibrational kinetic energy and other terms are translational kinetic energy due to transverse vibration of cable. The potential energy is

$$V = \int_0^{y_a^-} P(y,t) \varepsilon_y dy + \int_{y_a^+}^l P(y,t) \varepsilon_y dy$$

$$= \frac{1}{2} \int_0^{y_a^-} P(y,t) w_y^2 dy + \frac{1}{2} \int_{y_a^+}^l P(y,t) w_y^2 dy. \quad (8)$$

This potential energy is generated by the cable tension. ε_y is the strain and if the infinitesimal distance dy is replaced by the infinitesimal length ds , the strain ε_y can be approximated as [13]:

$$\varepsilon_y \approx \frac{1}{2} w_y^2. \quad (9)$$

The virtual work done by the control force and damping force is

$$\delta W = F_c(t) \delta w(y_a, t) - d_a w_t(y_a, t) \delta w(y_a, t), \quad (10)$$

where $F_c(t)$ is the control force.

The substitution of variation of Eqs. (7), (8) and (10) into Eq. (5) yields the governing equation as follows:

$$\rho w_{tt} - P_y(y,t) w_y - P(y,t) w_{yy} = 0, \quad y \neq y_a. \quad (11)$$

The boundary conditions are

$$w(0,t) = 0, \quad m_l w_{tt}(l,t) + P(l,t) w_y(l,t) = 0 \quad (12)$$

and the internal condition is

$$F_c(t) = m_a w_{tt}(y_a, t) + d_a w_t(y_a, t) + P(y_a^-, t) w_y(y_a^-, t) - P(y_a^+, t) w_y(y_a^+, t). \quad (13)$$

These results are similar to the result of [3] and [9]. In [3], the governing equation is equal, but the boundary condition is different. It is because of the difference of method which derives the equation. In [9], if we set $v = 0$, $\dot{v} = 0$ (it means that the length of cable is constant.) and $k_e = 0$ (it means that the end of cable is free.) of Eqs. (39)-(41) of [9], the results is same. The difference is that in this paper we consider the mass and the damping effect of actuator. In [9], the author did not consider that. Eq. (11) is partial differential equation expressing the transverse vibration.

Note that because of value of existing time-optimal control with nonflexible model, these governing equation, boundary condition and internal condition are to be used only to suppress the transverse vibration.

2.2 Path planning

In Fig. 3, the working sequence of the load is shown. This sequence consists of five stages. The load is lifted up from the initial elevation h_0 to elevation h_1 in the upward movement A-B. The elevation h_1 is above possible obstacles in the vicinity. The maximum lifting speed $v_{\max} = \dot{l}_{\max}$ is achieved at point B. The load is lifted up to elevation h_2 above any obstacles along the rest of path during the diagonal movement B-C. The trolley achieves its maximum speed at point C. The acceleration pattern during B-C is derived using the time-optimal control with zero sway terminal conditions. Along C-D, the trolley moves at its maximum speed. The maximum speed of the trolley is limited by mechanical

components and not by electrical components. During the diagonal movement D-E, the trolley is decelerated from its maximum speed to zero while the load is set down to elevation h_3 . The maximum downward speed of the load is achieved at E. In this paper, the target position where the residual vibration is suppressed is h_3 because we assume the PDE model that the length of crane cable is constant. In a real situation, through E-F the container is lowered further down to the target height h_4 .

2.3 Control strategy

According to the path planned as previously stated, a two-stage control is proposed. Note that we do not consider the movement of load from A to B. In the movement from B to E, the first stage control is a time-optimal control with feedback adjustment for the purpose of fast moving of the trolley. The trolley and hoist systems are independently controlled and the nonflexible simplified Eqs. (1)-(3) are used as a plant model. The second stage control, after section D-E, is a residual sway control. For the suppression of the residual vibration after section D-E, a feedback control law using Eqs. (11)-(13) and Lyapunov's method is proposed.

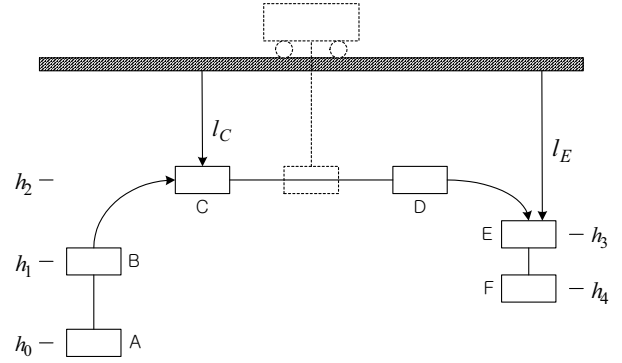


Fig. 3 Transportation sequence of the container.

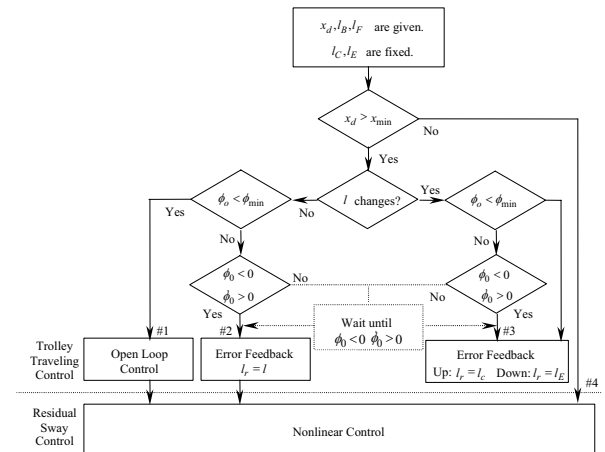


Fig. 4 Trolley traveling control strategies.

Fig. 4 shows various control strategies for a given distance and rope length. x_d denotes the target position and x_{\min} denotes the minimum distance needed to apply the time-optimal control. First, if the target distance x_d is smaller than the minimum distance x_{\min} needed to apply the bang/bang control (Fig. 6c) or any velocity patterns in Fig. 6(a)-(b), the nonlinear control strategy is directly applied from the start (#4). Second, assume that the rope length is

constant, which is the case that the load is lifted up to h_2 and then the trolley begins to move. If there is no wind and the initial sway angle is sufficiently small, i.e. $\phi_0 < \phi_{\min}$, then an open loop time-optimal control is recommended (#1). Note that the minimal angle ϕ_{\min} varies depending on the rope length. However, if the initial sway angle is too large, the open loop time-optimal control would not yield zero sway angle upon arrival at point C. Therefore, a modified time-optimal control with error feedback, which will be discussed in Section 3, is recommended (#2). If the rope length changes, like the diagonal movements B-C and D-E, l_C and l_E become the reference rope lengths for deriving the velocity patterns for acceleration and deceleration, respectively (#3). It is remarked that the assumption of constant rope length presumes that the trolley control begins at height h_2 in Fig. 3. Compared with the case that the trolley starts to move at height h_1 , the total traveling time gets longer if the trolley begins to move at height h_2 . However, since the control law, if assuming the constant rope length, is so simple, this constant rope length strategy can be wisely adopted if the traveling time is not so important.

3. TROLLEY MOVING CONTROL: TIME-OPTIMAL CONTROL

According to the path planned as previously stated, a two-stage control is proposed. Note that we do not consider the movement of load from A to B. In the movement from B to E, the first stage control is a time-optimal control with feedback adjustment for the purpose of fast moving of the trolley. The trolley and hoist systems are independently controlled and the nonflexible simplified Eqs. (1)-(3) are used as a plant model. The second stage control, after section D-E, is a residual sway control. For the suppression of the residual vibration after section D-E, a feedback control law using Eqs. (11)-(13) and Lyapunov's method is proposed.

Fig. 5 shows various control strategies for a given distance and rope length. x_d denotes the target position and x_{\min} denotes the minimum distance needed to apply the time-optimal control. First, if the target distance x_d is smaller than the minimum distance x_{\min} needed to apply the bang/bang control (Fig. 6c) or any velocity patterns in Fig. 6(a)-(b), the nonlinear control strategy is directly applied from the start (#4). Second, assume that the rope length is constant, which is the case that the load is lifted up to h_2 and then the trolley begins to move. If there is no wind and the initial sway angle is sufficiently small, i.e. $\phi_0 < \phi_{\min}$, then an open loop time-optimal control is recommended (#1). Note that the minimal angle ϕ_{\min} varies depending on the rope length. However, if the initial sway angle is too large, the open loop time-optimal control would not yield zero sway angle upon arrival at point C. Therefore, a modified time-optimal control with error feedback, which will be discussed in Section 3, is recommended (#2). If the rope length changes, like the diagonal movements B-C and D-E, l_C and l_E become the reference rope lengths for deriving the velocity patterns for acceleration and deceleration, respectively (#3). It is remarked that the assumption of constant rope length presumes that the trolley control begins at height h_2 in Fig. 3. Compared with the case that the trolley starts to move at height h_1 , the total traveling time gets longer if the trolley

begins to move at height h_2 . However, since the control law, if assuming the constant rope length, is so simple, this constant rope length strategy can be wisely adopted if the traveling time is not so important.

4. BOUNDARY CONTROL LAW

After the movement E-F in Fig. 3, the purpose of control is to dissipate the residual vibration of the crane cable. We define an energy-like (Lyapunov) function of time for container crane and denote it by $V(t)$.

$$V(t) = \frac{1}{2} \rho \int_0^{y_a^-} w_t^2 dy + \frac{1}{2} \rho \int_{y_a^+}^{y_l} w_t^2 dy + \frac{1}{2} \int_0^{y_a^-} P(y,t) w_y^2 dy + \frac{1}{2} \int_{y_a^+}^{y_l} P(y,t) w_y^2 dy + \frac{1}{2} m_a \left[\frac{Dw(y_a,t)}{Dt} \right]^2 + \frac{1}{2} m_l \left[\frac{Dw(l,t)}{Dt} \right]^2. \quad (16)$$

And we can rewrite Eq. (16) as follows

$$V(t) \leq \bar{V}(t) = \frac{1}{2} \rho \int_0^{y_a^-} w_t^2 dy + \frac{1}{2} \rho \int_{y_a^+}^{y_l} w_t^2 dy + \frac{1}{2} \int_0^{y_a^-} P_{\max} w_y^2 dy + \frac{1}{2} \int_{y_a^+}^{y_l} P_{\max} w_y^2 dy + \frac{1}{2} m_a \left[\frac{Dw(y_a,t)}{Dt} \right]^2 + \frac{1}{2} m_l \left[\frac{Dw(l,t)}{Dt} \right]^2. \quad (17)$$

Because the mass of load is efficiently bigger than the mass of unit length of the cable, we replace the tension $P(y,t)$ to $P_{\max} = (m_l + \rho l)g$.

$$\text{(e.g. } P_{\min} \leq [m_l + \rho(y-l)]g \leq P_{\max} \text{)}$$

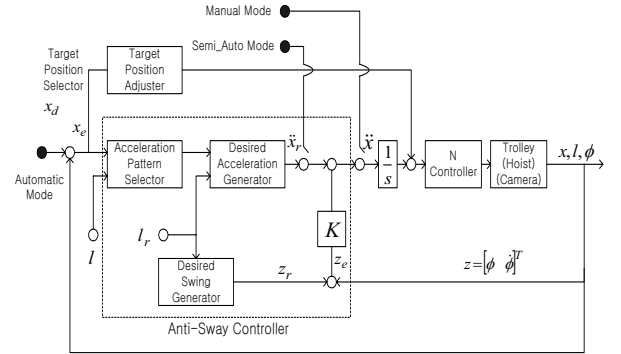


Fig. 5 Block diagram of the trolley traveling control.

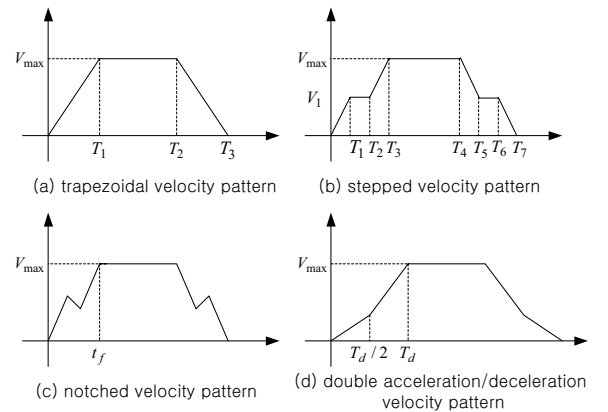


Fig. 6 Four different velocity patterns.

Now, the total derivative (or the material derivative) of Eq. (17) is evaluated. The time derivative of $\bar{V}(y,t)$ becomes

$$\frac{d}{dt}\bar{V}(y,t) = \frac{\partial}{\partial t}\bar{V}(y,t) + \frac{\partial}{\partial t}\bar{V}(y,t)\frac{\partial y}{\partial t} \quad (18)$$

where $\frac{\partial y}{\partial t} = \tilde{v}$, \tilde{v} is the speed of hoisting. But we consider

$\tilde{v} = 0$ condition. So Eq. (18) is

$$\frac{d}{dt}\bar{V}(y,t) = \frac{\partial}{\partial t}\bar{V}(y,t). \quad (19)$$

From Eqs. (11)-(13), the total derivative of Eq. (17) becomes

$$\begin{aligned} \frac{d}{dt}\bar{V}(y,t) &= \rho \int_0^{y_a^-} w_t w_{tt} dy + \rho \int_{y_a^+}^{y_l} w_t w_{tt} dy \\ &+ m_a \left[\frac{Dw(y_a,t)}{Dt} \right] \left[\frac{Dw^2(y_a,t)}{Dt^2} \right] \\ &+ m_l \left[\frac{Dw(l,t)}{Dt} \right] \left[\frac{D^2 w(l,t)}{Dt^2} \right] \\ &+ P_{\max} \int_0^{y_a^-} w_y w_{yt} dy + P_{\max} \int_{y_a^+}^{y_l} w_y w_{yt} dy \\ &= P_{\max} \int_0^{y_a^-} (w_{yy} w_t + w_y w_{yt}) dy + P_{\max} \int_{y_a^+}^{y_l} (w_{yy} w_t + w_y w_{yt}) dy \\ &\quad - P_{\max} w_y(y_l,t) w_t(y_l,t) \\ &\quad + \bar{w}_t(y_a,t) \{ F_c(t) - d_a w_t(y_a,t) - P_{\max} w_y(y_a^-,t) + P_{\max} w_y(y_a^+,t) \} \\ &= P_{\max} [w_y w_t]_0^{y_a^-} + P_{\max} [w_y w_t]_{y_a^+}^{y_l} - P_{\max} w_y(y_l,t) w_t(y_l,t) \\ &\quad + \bar{w}_t(y_a,t) \{ F_c(t) - d_a \bar{w}_t(y_a,t) - P_{\max} w_y(y_a^-,t) + P_{\max} w_y(y_a^+,t) \} \\ &= P_{\max} w_y(y_a^-,t) w_t(y_a^-,t) - P_{\max} w_y(y_a^+,t) w_t(y_a^+,t) \\ &\quad + w_t(y_a,t) \{ F_c(t) - d_a w_t(y_a,t) - P_{\max} w_y(y_a^-,t) + P_{\max} w_y(y_a^+,t) \}. \end{aligned} \quad (20)$$

The displacement of the string is continuous at $y = y_a$:

$$w(y_a^-,t) = w(y_a,t) = w(y_a^+,t). \quad (21)$$

Time derivative of Eq. (21) yields

$$w_t(y_a^-,t) = \frac{Dw(y_a,t)}{Dt} = w_t(y_a^+,t). \quad (22)$$

By using Eq. (22) the following feedback control law will make Eq. (20) negative semi-definite.

$$F_c(t) = -K w_t(y_a,t) \quad (23)$$

where $0 < K < l$ is a constant real number. Eq. (21) is also similar to the result of [9]. If we set $\dot{v} = 0$ (it means that the length of cable is constant.) of Eq. (52) in [9], the result is same.

Therefore, we can conclude that as we consider the container crane system Eqs. (11)-(13), the closed-loop system with the control law Eq. (23) is uniformly asymptotically stable.

5. CONCLUSIONS

In this paper, the time-optimal control to reduce the sway of crane cable and boundary feedback control to eliminate the residual vibration are investigated. Because the fast trolley moving is the main purpose, the various velocity pattern of trolley, which gives the acceleration to minimize the sway during the trolley moving is given. After the trolley moving, the control law using the actuator in order to suppress the residual vibration is suggested. These control methods have

advantages: minimizing the traveling time, reducing the residual vibration. But in this paper, we use two different models due to advantages of each model: nonflexible and flexible model. In a real working space, a flexible model is more realistic. And in the section E-F, there are no controls, because before section E-F the actuator suppresses the residual vibration. Because of the omission of the control in section E-F and the control method during the stoppage of hoisting, the working time may be longer. In a later research, at the section D-E and E-F in Fig. 3, the variation of length of crane cable will be considered as a flexible cable model. In this model, the same actuator investigated in this paper will control the sway and residual vibration of crane cable. This modeling method makes the motion of trolley not to be constraint in the velocity patterns mentioned in Section 3. Then we can design the crane system which has faster maximum speed of trolley. Next, using the flexible model (like a string) without using the nonflexible model (as a rigid pendulum) when we derive time-optimal control method with the velocity pattern, the control method using new velocity pattern without additional actuator will be derived.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Science and Technology of Korea under a program of the National Research Laboratory, grant number NRL M1-0302-00-0039 -03-J00-00-023-10.

REFERENCES

- [1] d'Andréa-Novél. B., Boustany, F., Conrad, F., "Control of an overhead crane: Stabilization of flexibilities," *Proceedings of the IFIP Boundary Control and Boundary Variation Workshop*, Sophia-Antipolis, 1990, pp. 1-26
- [2] d'Andréa-Novél, B., Boustany, F., Conrad, F., and Rao, B. P. , "Feedback stabilization of a hybrid PDE-ODE system: Application to an overhead crane," *MCSS journal*, vol. 7, pp. 1-22, 1994.
- [3] d'Andréa-Novél, B. and Coron, J.-M., "Exponential stabilization of an overhead crane with flexible cable via the cascade approach," *Proc. of the IFAC SYROCO'97 Conference*, Nantes, 1997.
- [4] d'Andréa-Novél, B. and Coron, J.-M., "Exponential stabilization of an overhead crane with flexible cable via a back-stepping approach," *Automatica*, vol. 36, pp. 587-593, 2000.
- [5] Boustany, F. and d'Andréa-Novél, B., "Adaptive control of an overhead crane using dynamic feedback linearization and estimation design," *Proc. of the 1992 IEEE: International Conference on Robotics and Automation*, Nice, France, pp. 1963-1968, 1992.
- [6] Hong, K. S., Park, B. J., and Lee, M. H., "Two-Stage Control for Container Cranes," *JSME International Journal*, Series C, Vol. 43, No. 2, pp.273-282, June 2000.
- [7] Hong, K. S., Sohn, S. H., and Lee, M. H., "Sway Control of a Container Crane (Part I): Modeling, Control Strategy, Error Feedback Control Via Reference Velocity Profiles," *Journal of Control, Automation and Systems Engineering*, Vol 3, No.1, pp.23-31, February 1997.
- [8] Hong, K. S., Sohn, S. C., and Lee, M. H., "Sway Control of a Container Crane (Part II): Regulation of the Pendulum Sway through Patternizing Trolley Moving Velocity," *Journal of Control, Automation and Systems Engineering*, Vol.3, No.2, pp.132-138, April

1997.

- [9] J. A. Wickert, "Non-linear Vibration of a Traveling Tensioned Beam, *International Journal of Non-linear Mechanics*, 27(3), pp 503-517, 1992.
- [10] Moustafa, K. A. F. and Ebeid, A. M., "Nonlinear modeling and control of overhead crane load sway," *Transaction of the ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 110, pp. 266-271, 1988.
- [11] Moustafa, K. A. F., "Feedback control of overhead cranes swing with variable rope length," *Proc. ACC*, pp. 691-695, 1994.
- [12] W. D. Zhu and J. Ni, "Energetics and stability of translating media with an arbitrarily varying length," *Journal of Vibration and Acoustics*, vol. 122 July 2000.
- [13] W. D. Zhu, J. Ni and J. Huang, "Active control of translating media with arbitrarily varying length," *Journal of Vibration and Acoustics*, vol. 123 July 2001.