# **Support-vector-machine Based Sensorless Control**

## of Permanent Magnet Synchronous Motor

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**Abstract**: Speed and torque control of PMSM(Permanent Magnet Synchronous Motor) are usually achieved by using position and speed sensors which require additional mounting space, reduce the reliability in harsh environments and increase the cost of a motor. Therefore, many studies have been performed for the elimination of speed and position sensors. In this paper, a novel speed sensorless control of a permanent magnet synchronous motor based on SVMR(Support Vector Machine Regression) is presented. The SVM regression method is an algorithm that estimates an unknown mapping between a system's input and outputs, from the available data or training data. Two well-known different voltage model is necessary to estimate the speed of a PMSM. The validity and the usefulness of proposed algorithm are thoroughly verified through numerical simulation.

**Keywords:** Permanent Magnet Synchronous Motor, Sensorless Control, Support Vector Machine Regression.

#### 1. INTRODUCTION

Recently, PMSMs have been noticed as variable speed motors with high-performance in many applications because of some merits such as high efficiency and high power factor. The vector control in the speed and torque controlled ac drive is widely used for a high performance application. The vector control of a permanent magnet synchronous motor is usually implemented through measuring the speed and position. However, speed and position sensors require additional mounting space, reduce the reliability in harsh environments and increase the cost of a motor. Various control algorithms have been proposed for the elimination of speed and position sensors: estimators using state equations, Luenberger or Kalman-filter observers, sliding mode control, Kalman-filter observers, sliding mode control, saliency effects, artificial intelligence, direct control of torque and flux, and so on[1-4]. In this paper, a novel speed sensorless control of a permanent magnet synchronous motor using SVMR(Support Vector Machine Regression) based on statistical learning theory is presented. Recently, a novel neural network algorithm, called SVM, was developed by Vapnik and his co-workers. Unlike most of the traditional neural network models which implement the empirical risk minimization principle, SVM implements the structural risk minimization principle which seeks to minimize an upper bound of the generalization error rather than the training error. This induction principle is based on the fact that the generalization error is bounded by the sum of the training error and a confidence interval term that depends on the Vapnik-Chervonenkis (VC) dimension. Based on this principle, SVM achieves an optimum network structure by striking a right balance between the empirical error and the VC-confidence interval. This eventually results in better generalization performance than other neural network models. Another merit of SVM lies in the training of SVM equivalent to solving a linearly constrained quadratic programming[6-7]. The proposed speed estimate algorithm is based on the SVMR using the stationary reference frame fixed to the stator voltage model and rotor reference frame with the rotating speed of  $\omega_r$  voltage model as two models for the back-EMF estimation. The validity and the usefulness of proposed algorithm are thoroughly verified through numerical simulation.

## 2. Mathematical modeling of PMSM

The proposed SVMR sensorless algorithm uses the stationary reference frame fixed to the stator voltage model and rotor reference frame with the rotating speed of  $\omega_r$  voltage model as two models for the back-EMF estimation. From the stator voltage equations in the real  $d_s^s$ -axes, and  $q_s^s$ -axis voltage equations in the stationary reference frame fixed to the stator can be expressed as

$$v_{ds}^{s} = R_{s}i_{ds}^{s} + L_{s}\frac{di_{ds}^{s}}{dt} + e_{ds}^{s}$$
 (1)

$$v_{qs}^{s} = R_{s}i_{qs}^{s} + L_{s}\frac{di_{qs}^{s}}{dt} + e_{qs}^{s}$$
 (2)

where  $e_{dqs}^{s}$  is the back-EMF.

$$e_{ds}^{s} = v_{ds}^{s} - R_{s}i_{ds}^{s} - L_{s}\frac{di_{ds}^{s}}{dt}$$

$$\tag{3}$$

$$e_{qs}^{s} = v_{qs}^{s} - R_{s}i_{qs}^{s} - L_{s}\frac{di_{qs}^{s}}{dt}$$
 (4)

From the stator voltage equations in the real  $d_r^s$ -axes, and  $q_r^s$ -axis voltage equations in the rotor reference frame fixed to the stator can be expressed as

$$v_{dr}^{s} = R_{s}i_{dr}^{s} + L_{s}\frac{di_{dr}^{s}}{dt} - \omega_{r}L_{s}i_{qr}^{s}$$

$$\tag{5}$$

$$v_{qr}^{s} = R_{s}i_{qr}^{s} + L_{s}\frac{di_{qr}^{s}}{dt} + \omega_{r}L_{s}i_{dr}^{s} + \omega_{r}K_{e}$$

$$\tag{6}$$

$$e_{dr}^{s} = v_{dr}^{s} - R_{s}i_{dr}^{s} - L_{s}\frac{di_{dr}^{s}}{dt}$$
 (7)

$$e_{qr}^{s} = v_{qr}^{s} - R_{s}i_{qr}^{s} - L_{s}\frac{di_{qr}^{s}}{dt} - +\omega_{r}K_{e}$$

$$\tag{8}$$

where  $K_e$ ,  $e_{dqr}^s$  are the back-EMF constant, back-EMF respectively  $e_{dqr}^s = \omega_r L_s i_{dqr}^s$ 

## 3. Support Vector Machine Regression

A regression method is an algorithm that estimates an unknown mapping between a system's input and outputs, from the available data or training data. Once such a dependency has been accurately estimated, it can be used for prediction of system outputs from the input values. The goal of regression is to select a function which approximates best the system's response. A function approximation problem can be formulated to obtain a function f from a set of observations,  $(y_1, x_1), ..., (y_N, x_N)$  with  $x \in R^m$  and  $y \in R$ , where N is the number of training data, x is the input vector, and y is

$$f(x,\omega) = \omega^T K(x) + b \tag{9}$$

the output data. The function in SVR gas the form of

where  $K(\cdot)$  is a mapping from  $R^m$  to so-called higher dimensional feature space  $F, \omega \in F$  is a weight vector to be identified in the function, and b is a bias term. To calculate the parameter vector  $\omega$ , the following cost function should be minimized [6]-[15].

Min 
$$\frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^{N} (\xi_i + \xi_i^*)$$

subject to

$$y_{i} - \omega x_{i} - b \le \varepsilon + \xi_{i}$$

$$\omega x_{i} + b - y_{i} \le \varepsilon + \xi_{i}^{*}$$
(10)

$$\xi_i \xi_i^* \ge 0, C > 0, i = 1,..., N$$

where C is a pre-specified value that controls the cost incurred by training errors and the slack variables,  $\xi_i \xi_i^*$  are introduced to accommodate error on the input training set.

With many reasonable choice of loss function,  $\xi$ , the solution will be characterized as the minimum of a convex function. The constraints also include a term,  $\varepsilon$ , which allows a margin of error without incurring any cost. The value of  $\varepsilon$  can affect the number of support vectors used to construct the regression function. The bigger  $\varepsilon$  is, the fewer support vectors are selected. Hence,  $\varepsilon$ -values affect model complexity.

Our goal is to find function  $f(x,\omega)$  that has at most  $\varepsilon$  deviation from the actually obtained targets  $y_i$  for all the training data, and at the same time, is as flat as possible for good generalization. In other words, we do not care about errors as long as they are less than  $\varepsilon$ , but will not accept any deviations larger than  $\varepsilon$ . This is equivalent to minimizing an upper bound on the generalization error, rather than minimizing training error.

The optimization problem in (10) can be transformed into the dual problem [11]-[15], and its solution is given by

$$f(x) = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) (K(x_i) \cdot K(x)) + b$$

$$s.t. \ 0 \le \alpha_i^* \le C, \ 0 \le \alpha_i \le C$$

$$(11)$$

In (11), the inner product  $(K(x_i) \cdot K(x))$  in the feature space is usually considered as a kernel function  $K(x_i, x)$ . Several choices for the kernel are possible to reflect special properties of approximating functions:

Linear kernel :  $K(x_i, x) = x^T x_i$ 

RBF kernel: 
$$K(x_i, x) = \exp(-\|x - x_i\|^2 / \sigma^2)$$
 (12)

The input data are projected to a higher dimensional feature space by mapping  $K(\cdot)$ . A linear regression is made in this higher dimensional feature space, responding to a nonlinear regression in the original input space of interest as shown in Fig. 1.

## 4. Speed estimation using SVMR

Target data and training data of SVMR use q axis stator the back-EMF model of (4) and rotor q axis rotor the back-EMF model of (8). In (8), if  $v_{qr}^s = v_{ds}^s$ ,  $i_{qr}^s = i_{qs}^s$ , then back-EMF model of (8) is expressed as

$$e_{qr}^{s} = v_{qs}^{s} - R_{s}i_{qs}^{s} - L_{s}\frac{di_{qs}^{s}}{dt} - \omega_{r}K_{e}$$

$$\tag{13}$$

The back-EMF model can be written in the form (9) with

$$y(t) = e_{qs}^{s}(t) \tag{14}$$

$$x = [v_{qs}^s, R_s i_{qs}^s, L_s \frac{di_{qs}^s}{dt}, K_e]$$
(15)

$$\boldsymbol{\omega}^T = [\boldsymbol{\omega}_r] \tag{16}$$

Using quadratic loss function, one has to find Lagrange multipliers  $\alpha_i, \alpha_i^* \cdots, l$ , that minimize the quadratic form

$$W(\alpha_{i}, \alpha_{i}^{*}) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{i} - \alpha_{i}^{*})(\alpha_{j} - \alpha_{j}^{*}) K(x_{i}, x_{j})$$

$$+ \frac{1}{2C} \sum_{i=1}^{N} (\alpha_{i}^{2} - \alpha_{i}^{*2}) - \sum_{i=1}^{N} y_{i}(\alpha_{i} - \alpha_{i}^{*})$$
(17)

The regression function is given by

$$\omega^{T} = \sum_{i=1}^{N} (\alpha_{i} - \alpha_{i}^{*}) K(x_{i}, x)$$
(18)

$$b = mean\left(\sum_{i=1}^{N} \left\{ y_i - (\alpha_i - \alpha_i^*) K(x_i, x) \right\} \right)$$
(19)

In order to solve this problem, one has to choose the parameters C and the value of  $\varepsilon$ . Parameters C and  $\varepsilon$  usually are selected by users based on a priori knowledge and/or user expertise. Obviously, this approach is not appropriate for non-expert users. It is well-known that the value of  $\varepsilon$  should be proportional to the input noise level that is difficult to estimate from data and the value of  $\varepsilon$  can effect the number of support vectors used to construct the regression function. In other words, SVR performance depends on both parameters C and the value C of  $\varepsilon$ . Unfortunately, SVR framework does not provide clear guidelines on how to select the value of C and  $\varepsilon$ . Hence, we applied the reference[11] in a simulation. Estimated motor speed is given by

$$\omega_r = K_p \sum_{i=1}^N \omega \tag{20}$$

where  $K_n$  is proportional gain.

Fig. 2. illustrates the structure of the proposed speed estimator of PMSM.

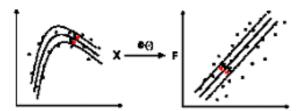


Fig.1. A feature map from input space to higher dimensional feature space.

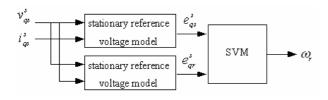


Fig.2. Structure of the speed estimator using SVMR

#### 5. Simulation and discussion

The simulation has been performed for the verification of the proposed control algorithm. Table 1 shows the specifications of PMSM used in the simulation.

Table 1 Motor specifications Number of pole 8 Rs  $0.423 \Omega$  Ls  $4.76 \mathrm{mH}$  Moment of inertia  $3.0 \times 10^{-2} Kg - m^2$  Back-EMF constant Nominal power  $0.75 \mathrm{Kw}$ 

Fig.3 shows the speed responses in the speed commands of 50rpm, 200rpm and 1000rpm without load. As shown in Fig. 3, the proposed algorithm has good speed response in the high speeds. However below 50rpm speed command has some bad speed response in the low speeds. The simulation results SVMR sensorless algorithm is confirmation of feasibility as the speed estimator.

## 6. Conclusions

This paper proposed a novel speed sensorless control algorithm of a permanent magnet synchronous motor. The proposed control algorithm is based on the SVMR using the stationary reference voltage model and rotor voltage model as two models for the back-EMF estimation. The simulation results SVMR sensorless algorithm is confirmation of feasibility as the speed estimator.

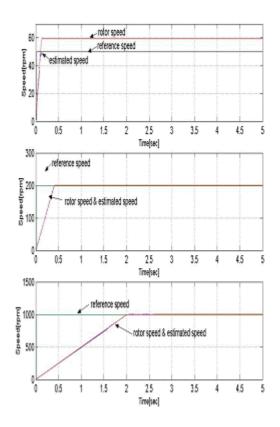


Fig.3. Speed responses in the speed command of 50rpm, 200rpm, and1000rpm.

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