### Robust Adaptive Control Simulation of Wire-Suspended Parallel Manipulator

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**Abstract**: This paper presents an adaptive control method based on parameter linearization for incompletely restrained wire-suspended mechanisms. The main purpose of this control method is utilizing it in a walking assist service robot for elderly people. This method is computationally simple and requires neither end-effector acceleration feedback nor inversion of estimated inertia matrix. In the proposed adaptive control law, mass, moment of inertia and external force and torque on the end-effector are considered as components of parameter adaptation vector. Nonlinear simulation for walking an elderly shows the effectiveness of the parameter adaptation law.

Keywords: Robust-adaptive control, wire-driven mechanism, parallel manipulator, walking assist

### 1. INTRODUCTION

Increasing the number of elderly people who live alone makes designing of service robots to assist them in walking and carrying their daily requirements very important. Many researchers think that mobile robots are supposed to be utilized in this purpose. However their mobility and payload capabilities are very limited, also they are very expensive and complicated. Beside mobile robots, other kind of crane-like robots mounted on a rail on roof have been suggested [1]. In these systems workspace is very limited to the rail direction and also they are heavy and expensive. A very good candidate mechanism for this kind of applications is wire-suspended mechanisms. This kind of mechanisms has large work space, light weight and high payload. Some researchers suggested wire-suspended mechanisms [2] which four wires suspend the robot main body from four upper corners of a room. The main disadvantage of this configuration is that with four wires it is impossible to have control on three position and three orientation parameters of the end-effector. We need a mechanism which we can have full control on end-effector position and orientation.

Generally the mechanisms which restrain an object by wires are called wire-suspended mechanisms. There are two kinds of wire-suspended mechanisms; completely restrained and incompletely restrained. Completely restrained wire suspended mechanisms restrain all degrees of freedom of the end-effector. There mechanisms can move the objects with high stiffness but the workspace is small. Incompletely restrained wire-suspended mechanisms do not restrain all degrees of freedom of their suspended object, so they need less number of wires and as result a larger workspace. The main disadvantage of incompletely restrained mechanisms is that their suspended object is easy to swing, so for describing the motion of their end-effector some dynamical conditions in addition kinematical constraints by wires are needed therefore for overcoming swinging problem developing a good control system for incompletely restrained mechanisms is very important.

The mechanism which has been chosen for walking assist system, as well as for carrying daily items after this we call it SpiderBot<sup>TM</sup>, is an incompletely restrained wire-suspended mechanism. The mechanism consists of three trolleys and three wires as shown in Fig. 1. Even though some degrees of

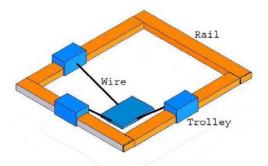


Figure 1. An incompletely restrained wire-suspended mechanism

freedom, say dawnward z-direction, are stiff and the others are flexible so that this mechanism is inherently safe for in-house utilization. The major characteristic of this mechanism is that three dimensional position and orientation of end-effector can be controlled by changing trolley positions and wire lengths.

Regarding the design, analysis, and control of this mechanism, Yamamoto et al. [3,4 and 5] proposed methods for inverse kinematics and dynamics analysis of this type of mechanism.

Many researches have proposed anti-sway control of this kind of mechanism. Yanai et al.[6] proposed a control method based on inverse dynamics calculation, in which the non-linearity of the mechanism is completely compensated by inverse dynamics calculation. They used an observer to get higher order derivatives of the position of the load which is needed for the inverse dynamics calculation. Yanai et al. [7] developed another anti-sway control method based on dynamics compensation. Yanai et al. [8] also developed a control method based on exact linearization using inverse dynamics of the system.

Because of special application of SpiderBot<sup>TM</sup> as a walking assistance system (for example, see Fig. 2) and also for carrying objects and furniture inside home, the control method must have some special properties. In each application scenario the mass and moment of inertia of the

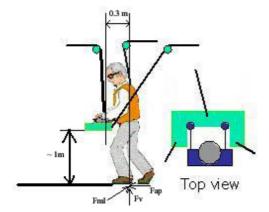


Figure 2. Walking assistance operation mode

end-effector may be very different. The resultant external force and external torques on the end-effector may also be very different. The best solution for overcoming this problem is an adaptive control system; the required system must have the ability to adapt itself with each operation mode. In this paper an adaptive control method based on parameter linearization [9,10] will be developed. Mass and moment of inertia of end-effector as well as external force and torque on the end-effector considered as components of parameter adaptation vector. A very important restriction on the parameter linearization method is that the parameter update vector components must be constant or slowly varying in time. Because of the main source of external force and torque on the end-effector is gait force and torque generated by them so they can be time varying. For this reason we categorized the unknown parameters into two groups: one group contains the parameters which are constant and estimated online and the other group contains the parameters not estimated online. The controller is made robust to the uncertainty on the parameters of second group rather than estimating them online. For the above mentioned reason we considered external force and torques as the members of second group.

In this paper first of all the dynamic model of SpiderBot<sup>TM</sup> will be discussed. Next section is about load conditions. In section 3 adaptive controller, robust issues and stability of the adaptive controller will be discussed. In the last section simulation results will show the effectiveness of the proposed adaptive controller.

# 2. LOAD CONDITIONS

In order to getting the best results from controller simulation, the loading conditions in simulation must be as similar as possible to loading conditions of robot in real working scenarios. SpiderBot<sup>TM</sup> is being developed for three major tasks: moving materials in home environment, active walking assist and passive walking assist. In the first case there is just one vertical force equal to weight of the moving object. The last two cases have more complicated loading conditions

**2.1 passive walking assist system:** In passive walking assist the robot moves the elderly body with no action of elderly himself. So there is mostly a vertical force in the end-effector equal to elderly weight. The lateral forces are assumed to be small

2.2: active walking assist system: In the case of active

walking assist system, there are three kinds of main forces

- 1) Vertical: which is maximum 140% of body weight
- 2) Anterior-Posterior: which is maximum 25% of body weight which acts on the direction of walking
- Medial-Lateral: which is maximum 2.5% of body weight and acts in the direction vertical to the direction of walking

The above mentioned gait forces have been estimated and designed by gait analysis[11] and act on the bottom of foot, since there is a distance between the ground and end-effector three torques also will act on the end-effector. The only solution for finding exact amount of torques on the end-effector is an experiment so here for simulation we use a rough estimation of the torque by multiplying the gait forces by one meter vertical and 0.3m horizontal distance between ground and end-effector. Fig. 2 shows a schematic view of walking assistance operation mode of SpiderBot<sup>TM</sup>.

### 3. DYNAMIC MODEL

For calculating wire tension vectors inverse dynamics must be solved. For finding three wire tension vectors we need nine equations. Newton's law gives three equations, Euler's law three more equations, and for three remaining equations we will utilize a geometrical property of the system. Yamamoto et al. [3,4,5] proposed the inverse dynamics of this mechanism. In their method external force and torque on the end-effector are not considered. Because of control of SpiderBot<sup>TM</sup> external forces and torques are very important so here we apply them on the dynamic model.

The notations below will be used in this section.

 $T_i$ : wire tension vector

 $\boldsymbol{X}$ : center of gravity of end-effector

 $oldsymbol{ heta}$  : orientation of end-effector

 $x_{ci}$ : wire connection vector

 $b_i$ : position vector of rail beginning point

 $S_i$ : vector parallel to rail

 $f_d$  : external force on end-effector

 $\tau_d$ : external torque on end-effector

Newton's translational quations of motion of the end-effector can be written as

$$T_1 + T_2 + T_3 = m(\ddot{x} - g) - f_d$$
 (1)

By denoting  $r_i$  as a vector from the center of gravity of the end-effector to the wire connection point to the end-effector and J as inertia tensor of the end-effector with respect to world coordinate system, we can write Euler's rotational equations of motion as

$$\sum_{i=1}^{3} r_i \times f_i = J\dot{\omega} + \omega \times J\omega - \tau_d$$
 (2)

It's obvious that the wire and so tension vector of each wire must be on a plan which passes through a rail and a wire connecting point on the end-effector corresponding to that rail. The normal vector of the above mentioned plan must be perpendicular to the wire tension vector. The normal vector of the plan can be calculated as below:

$$\boldsymbol{n}_i = (\boldsymbol{b}_i - \boldsymbol{x}_{ci}) \times \boldsymbol{s}_i \tag{3}$$

So we can write

$$\mathbf{n}_{i}.T_{i} = ((\mathbf{b}_{i} - \mathbf{x}_{ci}) \times \mathbf{s}_{i}).T_{i} = 0$$

$$\tag{4}$$

Equations (1),(2) and (4) can be rearrange in a matrix form as

$$\begin{bmatrix} m(\ddot{x}-g)-f_d \\ J\dot{\omega}+\omega\times J\omega-\tau_d \\ 0 \end{bmatrix} = Bu$$

$$u = \begin{bmatrix} T_1^T & T_2^T & T_3^T \end{bmatrix}$$
 (5)

$$B = \begin{bmatrix} I_3 & I_3 & I_3 \\ S(r_1) & S(r_2) & S(r_3) \\ n_1^T & \theta^T & \theta^T \\ \theta^T & n_2^T & \theta^T \\ \theta^T & \theta^T & n_3^T \end{bmatrix}$$

where  $I_3$  is  $3 \times 3$  identity matrix and  $\theta$  is  $3 \times 1$  zero vector and a skew-symmetric matrix  $S(r_i)$  is given as

$$S(r_i) = \begin{bmatrix} \theta & -r_{Iz} & r_{Iy} \\ r_{Iz} & \theta & -r_{Ix} \\ -r_{Iy} & r_{Ix} & \theta \end{bmatrix}$$
(6)

Here we write the equations of motion in terms of q coordinates in the following general form

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Bu \tag{7}$$

Note that  $\ddot{q} = (\ddot{x}, \dot{\omega})$ ,  $\dot{q} = (\dot{x}, \omega)$  and  $q = (x, \theta)$  where  $\theta$  represents orientation of the end-effector with roll-pitch-yaw angles,  $\dot{\theta}$  is related to the angular velocity,  $\omega$ , with the following equation:

$$\boldsymbol{\omega} = \boldsymbol{R}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{8}$$

where  $R(\theta)$  is  $3 \times 3$  matrix represented by angular parameter  $\theta$ .

In equation (7), H(q) is the inertia matrix for the system,  $C(q, \dot{q})\dot{q}$  is the vector of coriolis and centripetal terms, g(q) is the vector of resultant external force, torque and gravity. Where

$$H(q) = \begin{bmatrix} mI_3 & \theta_3 & \theta_3 \\ \theta_3 & I & \theta_3 \\ \theta_3 & \theta_3 & \theta_3 \end{bmatrix}$$
(9)

$$C(q,\dot{q}) = \begin{bmatrix} \theta_3 & \theta_3 & \theta_3 \\ \theta_3 & S(\omega) & \theta_3 \\ \theta_3 & \theta_3 & \theta_3 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} -f_d - mg \\ -\tau_d \\ \theta \end{bmatrix}$$
(10)

where  $o_3$  is  $3 \times 3$  zero matrix.

### 4. ADAPTIVE CONTROLLER

As mentioned in the introduction part, because of the special application of SpiderBot<sup>TM</sup> mass and moment of inertia tensor of the end-effector and also external force and torque on the end-effector maybe varying significantly in each operation mode. An adaptive control algorithm based on parameter linearization [9,10] will be used in this section.

Dynamics of the robot can be written in the following form as

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})p \tag{11}$$

where Y is  $9 \times 18$  matrix of  $\ddot{x}, \omega, \dot{\omega}$  components and p is a  $18 \times 1$  vector of m, J,  $f_d, \tau_d$ . For detailed Y and p components see Appendix of this paper.

We define:

$$\dot{q}_r = \dot{q}_d - \Lambda e$$

$$s = \dot{q} - \dot{q}_r = \dot{e} + \Lambda e$$
(12)

where  $e=q-q_d$  and  $q_d$ ,  $\dot{q}_d$ ,  $\ddot{q}_d$  are desired position, velocity and acceleration trajectories and  $\Lambda>0$  is a diagonal matrix.

Furthermore,

$$\hat{H}(q)\ddot{q}_{r} + \hat{C}(q,\dot{q})\dot{q}_{r} + \hat{g}(q) = Y(q,\dot{q},\dot{q}_{r},\ddot{q}_{r})\hat{p}$$
(13)

where hated symbols imply that the parameter estimates are used in the expression defined in (11).

Consider the control and adaptation law as below

$$Bu = \hat{H}(q)\ddot{q}_{r} + \hat{C}(q,\dot{q})\dot{q}_{r} + \hat{g}(q) - K_{D}s$$

$$\dot{\hat{p}} = -\Gamma^{-1}Y^{T}(q,\dot{q},\dot{q}_{r},\ddot{q}_{r})s$$
(14)

where  $K_D$  and  $\Gamma$  are symmetric positive definite matrices. Also we considered that  $\dot{\hat{p}}=\dot{\tilde{p}}$ , since the unknown parameter p is considered to be constant or at least slowly varying in time. It is very important to note that a constant p means we have to select parameter adaptation vector components in a way that they don't change during experiment. This is a very important restriction in this control law

One of the main advantages of this control law is that it

requires no feedback of end-effector acceleration.

We can demonstrate global convergence of the tracking by using the Lyapunov function candidate

$$V(t) = \frac{1}{2} s^{T} H s + \frac{1}{2} \widetilde{p}^{T} I \widetilde{p}$$
 (15)

where  $\tilde{p} = \hat{p}(t) - p$  denotes the parameter estimation error vector. Differentiating V yields

$$\dot{V}(t) = s^{T} H \dot{s} + \frac{1}{2} s^{T} \dot{H} \dot{s} + \tilde{p}^{T} \Gamma \dot{\tilde{p}}$$

$$= s^{T} (B u - C \dot{q} - g - H \ddot{q}_{r})$$

$$+ s^{T} (\frac{1}{2} (\dot{H} - 2C) + C) s + \tilde{p}^{T} \Gamma \dot{\tilde{p}}$$

$$= s^{T} (B u - H \ddot{q}_{r} - C \dot{q}_{r} - g) + \tilde{p}^{T} \Gamma \dot{\tilde{p}}$$
(16)

Note that using the skew-symmetry property of  $\dot{H} - 2C$  the term  $\frac{1}{2}s^T(\dot{H} - 2C)s$  will be eliminated.

Consider

$$\widetilde{H}(q) = \widehat{H}(q) - H(q)$$

$$\widetilde{C}(q, \dot{q}) = \widehat{C}(q, \dot{q}) - C(q, \dot{q})$$

$$\widetilde{g}(q) = \widehat{g}(q) - g(q)$$
(17)

By using equations (14),(16) and (17) we can write

$$\dot{V}(t) = s^{T} (\widetilde{H}(q) \ddot{q}_{r} + \widetilde{C}(q, \dot{q}) \dot{q}_{r} + \widetilde{g}(q) - K_{D} s) + \widetilde{p}^{T} I \dot{\widetilde{p}}$$
(18)

Using equation (11) into equation (18) yields

$$\dot{V}(t) = s^T K_D s + \tilde{p}^T (I \tilde{p} + Y^T s)$$
 (19)

substituting equation (14) into equation (19) yields

$$\dot{V}(t) = -s^T K_D s < 0 \tag{20}$$

so the control and adaptation law yield a globally stable adaptive controller.

In parameter adaptation vector components some parameters may already be known with reasonable precision so we may choose to make the controller robust to the uncertainty to this parameters rather than explicitly estimating them online.

Assume that the last 9 components of parameter adaptation vector p, which contains moment of inertia components, are to be estimated online. For the case of resultant external force bounds can be found from gait analysis data.  $f_{dz}$  variation range is between 140% and 70% of total body weight. And there force in direction of walking varies between 25% and -20% of body weight. This data will be used in simulation section. For vector p components see appendix of this paper.

$$p = [p_R^T \quad p_E^T]^T \tag{21}$$

with  $p_R = \{ p_j \}_{j=1,\dots,9}^T$ ,  $p_E = \{ p_j \}_{j=10,\dots,18}^T$  and let correspondingly,  $Y = [Y_R \ Y_E \ ]$ . Assume that the uncertainties on  $p_R$  are bounded:

$$\left| \tilde{p}_{j} \right| \le P_{j}, j = 1, \dots, 9 \tag{22}$$

Add a sliding control term to torque input (14):

$$Bu = \hat{H}(q)\dot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{g}(q) - K_D s - k sgn(s)$$

where the notation  $k \, sgn(s)$  stands for  $6 \times 1$  vector of components  $k_i \, sgn(s_i)$ , with the  $k_i$  yet to be specified. Using  $p_E$  and  $\Gamma_E$  in place of p and  $\Gamma$  in the Lyapunov function (15) we have

$$\dot{V}(t) = -s^T [K_D s + Y_R \tilde{p}_R - k \operatorname{sgn}(s)] 
+ \tilde{p}_E^T [\Gamma_E \dot{\tilde{p}}_E + Y_E^T s]$$
(23)

Since

$$Y_{R}\tilde{p}_{R} - k \, sgn(s) = \{ \sum_{i=1}^{9} Y_{ij} \tilde{a}_{j} - k_{i} sgn(s_{i}) \}_{i=1,\dots,6}^{T}$$
 (24)

we let

$$k_{i} = \sum_{j=1}^{9} |Y_{ij}| A_{j} + \eta_{i}, \quad i = 1, ..., 6$$

$$\hat{\hat{p}}_{E} = -\Gamma_{E}^{-1} Y_{E}^{T} s$$
(25)

where  $\eta_i$  are positive constants. So

$$\dot{V}(t) \le -\sum_{i=1}^{6} \eta_i \left| s_i \right| - s^T K_D s \le 0$$
 (26)

the system trajectories are thus guaranteed to reach sliding surface s=0, and therefore convergence of the tracking is achieved.

## 5. SIMULATION OF CONTROLLER

A simulation for the wire-suspended parallel manipulator was developed in Matlab Simulink to verify the adaptive controller concepts developed in Section 4.

In the controller model, tension values are found by multiplying  $B^{-1}$  in the left side of equation (14)

$$u = B^{-1}(Bu) \tag{27}$$

The geometrical and mechanical properties of the end-effector in SI units are: m=10kg, the diagonal elements of

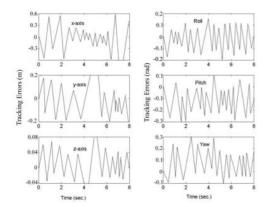


Figure 3. Tracking errors in adaptive controller without applying robustness

I (moment of inertia tensor of the end-effector with respect to its center of gravity)  $J_{II} = J_{22} = 0.225 kgm^2$  and  $J_{33} = 0.45 kgm^2$ , and off-diagonal elements are assumed to be zero.

Vertical component of resultant force on the end-effector considered to follow the following formula

$$f_{dz} = 1050 + 350 \sin(\pi t)$$
 N (28)

which refers to vertical gait forces of a 70 kg person.

For studying tracking performance of the control algorithm end-effector is moved from initial state  $q_{1d}(x_{1d},\theta_{1d})=(0.5,1,1,0,0,0)$  to terminal state  $q_{2d}(x_{2d},\theta_{2d})=(5,6,1,0.1,0.3,0.7)$  in 8 seconds. x-axis and y-axis in global coordinate system are parallel to ground and z-axis is downward. Roll, pitch and yaw are used to represent orientation of the end-effector. The external force equal to  $f_d=(250,25,1050+350\sin(\pi t))$  N and external torque  $\tau_d=(250,25,0)$  N.m are considered on the end-effector. This loading condition is corresponding to loading conditions in active walking assisting operation mode for a person with 1000 N weight according to loading conditions mentioned in Section 2.

Fig. 3 shows the simulation result of applying the above mentioned force condition to adaptive controller without robustness. Fig.4 shows the tracking errors of robot with the same load conditions but with applying robustness issues. It is obvious from results that the performance improves and tracking errors are smaller than pure adaptive controller.

### 6. CONCLUSION

In this paper after introducing SpiderBot<sup>TM</sup> and its load condition and operation mode an adaptive control algorithm based on parameter linearization suggested for the robot.

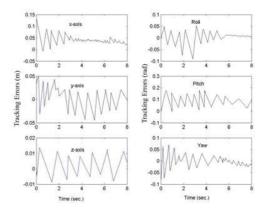


Figure 4. Tracking errors in adaptive controller after applying robustness

Because of the specific working condition of SpiderBot<sup>TM</sup> the best control method for it is adaptive control. The simulation results show that the parameter estimation performance and tracking performance are good and in a reasonable range.

Because most of the parameter estimation vector components vary in a limited range so it is possible to apply a robust-adaptive control algorithm to the system. As the simulation results show, applying robustness to the adaptive controller reduced tracking errors.

Construction of a prototype of SpiderBot<sup>TM</sup> is our future plan. The effectiveness of control algorithm will be tested in physical prototype.

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### **APPENDIX**

In this appendix the parameter linearzed form will be discussed. As mentioned in section 4, we have

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})p \tag{11}$$

where p is a  $18 \times 1$  vector of m, J,  $f_{d}$ ,  $\tau_{d}$  and Y is a  $9 \times 18$  matrix of  $\ddot{x}$ ,  $\omega$ ,  $\dot{\omega}$  components. Consider

$$f_{d} = \begin{vmatrix} f_{dx} & f_{dy} & f_{dz} \end{vmatrix}^{T}$$

$$\tau_{d} = \begin{vmatrix} \tau_{dx} & \tau_{dy} & \tau_{dz} \end{vmatrix}^{T}$$
(29)

and also  $J_{\theta ij}$  as  $ij^{th}$  component of  $J_{\theta}$  the inertia tensor of the end-effector with respect to end-effector coordinate system. If J be the inertia tensor of the end-effector with respect to world coordinates system and T the translational matrix from the world coordinate system to the end-effector coordinate system then we have

$$J = TJ_{\theta}T^{T} \tag{30}$$

It is very important to note that in parameter linearization method the parameter adaptation vector components must be constant, or slowly varying in time so just  $J_{\theta}$  components can be considered as adaptation component because J is variable.

 $\boldsymbol{Y}$  and  $\boldsymbol{p}$  components are :

(31)