Modified Quasi Newton algorithm for boundary estimation in Electrical Impedance Tomography

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Abstract: In boundary estimation in Electrical Impedance Tomography (EIT), conventional method is the modified Newton Raphson (mNR) method .The mNR is famous for good method since has good convergence and robustness against noisy data. But the mNR is low efficiency to get and update Jacobian matrix. So, the mNR become very slow algorithm. We propose the Quasi Newton (QN) method to improve efficiency which will lead to speed up in boundary estimation. The QN can improve a low efficiency by using estimated Jacobian matrix contrary to using exactly calculated Jacobian matrix, this used by the mNR. And finally, we propose the modified Quasi Newton (mQN) method because the QN has some problems such as bad early convergence rate and instability of 'divided by zero'. For the verification of the propose method, numerical experiments are conducted and the results show a good performance.

Keywords: Electrical impedance Tomography, Boundary estimation, Quasi Newton method, Newton Raphson method

1. INTRODUCTION

In electrical impedance tomography (EIT) an array of electrodes is attached around the object and small alternating currents are injected via these electrodes and the resulting voltages are measured. Based on the imposed currents and the measured voltages, an approximation for the internal impedance distribution is computed. Usually the object is discretized into a number of small pixels, in each of which impedance is assumed to be constant. The image reconstruction in EIT will be the inverse problem to estimate the impedance of each pixel.

Inexpensive hard-ware, negligible or no health consideration, and high time resolution of EIT could imply a good possibility in the practical applications to many areas including medical diagnosis, engineering process monitoring, non-destructive detection and so on. Due to the well-known ill-posed nature of EIT, however, the reconstructed image is very susceptible to the noise in the imposed and measured electrical signals and regularization should be introduced to mitigate the ill-posedness. This results in poor spatial resolution and the boundary of anomalies is hardly achieved. Hence, some researchers are interested in the direct estimation of the boundary rather than the impedance distribution when anomalies and background have different but constant impedance values [1-5].

In most papers dealing with the boundary estimation, the boundary is expressed as Fourier series and the inverse problem is to find the Fourier coefficients instead of the impedances of pixels. The method to solve the inverse problem is the modified Newton Raphson (mNR) method. The mNR uses most widely because it is strong against noise and it has superior convergence. But the mNR spends most time calculating Jacobian matrix. This becomes the reason of low efficiency when mNR applied to boundary estimation in EIT.

So we propose the Quasi Newton (QN)method in EIT. This method has the advantage about efficiency of calculating jacobian matrix as are equal to mNR's convergence boundary. But the QN has some problems such as bad early convergence rate which is caused by using estimated jacobian matrix and instability of 'divided by zero' which is caused by development on QN algorithm. So we propose the modified Quasi Newton (mQN) method to improve the QN's some problems. To increase the QN's early convergence rate, the mQN suggest the concept of the orthogonal which guarantee the independent of data. And to improve 'divided by zero' instability, the mQN add some constant to a denominator. For the verification of the proposed method, we conducted numerical experiments.

2. METHOD

The image reconstruction algorithm in EIT constructed two parts as likely fig.1. One is forward problem part. This part calculates the voltages which are calculated by function of impedance distribution. The other part is inverse problem part. This part calculates the Fourier coefficient which is calculated by minimization method. Most used forward solver is the finite element method (FEM) and minimization method is the mNR.

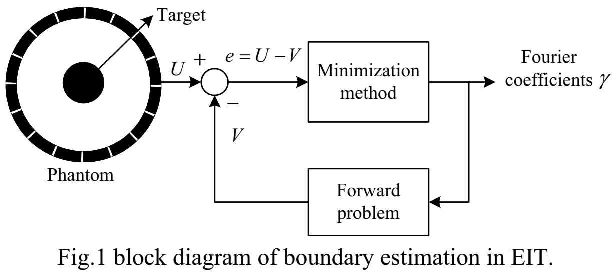


Fig.1 block diagram of boundary estimation in EIT.

2.1 Forward problem

In the forward problem of EIT we calculate the potentials on the boundary of an object given the resistivity distribution and injected currents on the boundary. When electrical currents $I_l (l = 1,2,\cdots,L)$ are injected into the object $\Omega \in \Re^2$ through the electrodes $e_l (l = 1,2,\cdots,L)$ attached on the boundary and the resistivity distribution $\rho(x, y)$ is known for the Ω, the corresponding electrical potential $u(x, y)$ on the Ω can be determined uniquely from the following partial differential equation, which can be derived from the Maxwell equations:

$$\nabla \cdot (\rho^{-1} \nabla u) = 0 \quad \text{in } \Omega . \tag{1}$$

Subject to the boundary conditions

$$\int_{e_l} \rho^{-1} \frac{\partial u}{\partial n} dS = I_l , \quad l = 1,2,\cdots,L \tag{2}$$

$$\rho^{-1} \frac{\partial u}{\partial n} = 0 \quad \text{on} \quad \partial\Omega \setminus \bigcup_{l=1}^{L} e_l . \tag{3}$$

From the complete electrode model (CEM), the boundary potential at the electrodes is expressed as:

$$u + z_l \rho^{-1} \frac{\partial u}{\partial n} = U_l \quad \text{on } e_l , \quad l = 1,2,\cdots,L \tag{4}$$

Where z_l is the effective contact impedance between *l*-th electrode and object, U_l is the potential on the *l*-th electrode, e_l is *l*-th electrode, n is outward unit normal vector, and L is the number of electrodes. The CEM takes into account the shunting effect (i.e. the voltage U_l is constant over the electrode e_l) and the additional voltage drop due to the contact impedance.

In addition, we must ensure the following two constraints for the injected currents and the measured voltages from the conservation of electrical charge and the uniqueness of the solution, respectively.

$$\sum_{l=1}^{L} I_l = 0 , \tag{5}$$

$$\sum_{l=1}^{L} U_l = 0 . \tag{6}$$

Since the forward problem cannot be solved analytically for arbitrary geometries we have to resort to the numerical method. In this paper, we used the finite element method (FEM) to obtain the numerical method. In the FEM, the object area is discretized into small elements having a node at each corner. It is assumed that the resistivity distribution is constant within element. The potential at each node is calculated by discretizing Eq. (1) into $Yv = c$, where $Y \in \Re^{N \times N}$ is the admittance matrix that is a function of resistivity and c represents the current injected into the object. The number of FEM nodes is denoted by N [6-7].

2.2 Modified Newton Raphson Method

Most people who research to boundary estimation in EIT express object boundary by the Fourier coefficient. Conventional method to find the Fourier coefficient is the mNR. The mNR minimized difference between measured voltages and computed voltages from forward solver. The mNR's cost function is as follow

$$\Phi(\gamma) = \frac{1}{2}[V_m - V(\gamma)]^T [V_m - V(\gamma)] + \frac{1}{2}\alpha\gamma R^T R\gamma \tag{6}$$

Where $V_m \in \Re^{L \times (L-1) \times 1}$ is the measured voltages on each electrode about current pattern, $V(\gamma) \in \Re^{L \times (L-1) \times 1}$ is the computed voltages by forward solver, $R^T R = diag(J^T J)$ and α are regularization parameters. Solution of minimized the voltage difference as follow

$$\gamma_{i+1} = \gamma_i + \left(J_i^T J_i\right)^{-1} J_i^T \left(V_m - V(\gamma_i)\right) \tag{7}$$

Where J is Jacobian matrix at $\gamma = \gamma_i$ form as

$$J \equiv \frac{\partial V(\gamma_i)}{\partial \gamma} \tag{8}$$

The mNR is famous for good method since good convergence and robustness against noisy data. But the mNR is low efficiency to get and update Jacobian matrix. Because the mNR uses an analytical calculation to get and update Jacobian matrix. So the mNR becomes very slowly algorithm in EIT.

3. MODIFIED QUASI NEWTON METHOD

We apply the QN to improve efficiency to get and update Jacobian matrix. The QN use estimated Jacobian matrix contrary to exact calculated Jacobian matrix, it used by the mNR.

Proposed the QN is as follow,

(a) γ, Fourier coefficient, express the object boundary
(b) $V(\gamma)$, voltages obtained by forward solver at γ
(c) V_m, the measured voltage on each electrode
(d) $F = V - V_m$,
(e) $\lambda_i = F_{i+1} - F_i$,
(f) $\Delta\gamma = \gamma_{i+1} - \gamma_i$.

The QN find the γ, which satisfied to F=0. Its solution is as follow,

$$\gamma_{i+1} = \gamma_i + B_i^{-1} F_i \tag{9}$$

Where $B \in \Re^{L \times (L-1) \times 1}$ is the Jacobian matrix, which is written

$$B_{i+1} = B_i + \frac{\lambda_i - B_i \Delta\gamma_i}{\Delta\gamma_i^T \Delta\gamma_i} \Delta\gamma_i^T \tag{10}$$

The B is updated every iteration and it is made by rank-one update algorithm [8-9].

We can get the Jacobian matrix but the QN use eventually the inverse Jacobian matrix. Similarly the mNR use inverse jacobian matrix. In the mNR case which obtain by recalculate from jacobian matrix. This performance spends much cost. In QN case which easily obtain by using the matrix inversion lemma. The Matrix inversion Lemma is given as,

$$(A+xy^T)^{-1} = A^{-1} - \frac{A^{-1}xy^T A^{-1}}{1+y^T A^{-1}x} \tag{16}$$

And $H_i = B_i^{-1}$ inverse Jacobian is gotten by lemma as,

$$H_{i+1} = H_i + \frac{(\Delta\gamma_i - H_i\lambda_i)\Delta\gamma_i H_i}{\Delta\gamma_i^T H_i \lambda_i} \tag{17}$$

Then the QN's solution is represented from Eq. (9) as,

$$\gamma_{i+1} = \gamma_i + H_i^{-1} F_i \tag{18}$$

The QN improve the mNR by changing to Jacobian matrix' update algorithm. The QN becomes faster algorithm than the mNR. But considering the QN we can find some problems. One is bad early convergence rate. Contrary the mNR use exact Jacobian matrix, this problem caused by using estimated Jacobian matrix. The other is H's instability caused by 'divided by zero'. As proceed iteration, H's denominator goes to zero. Due to this situation the QN has instability. So we propose the modified Quasi Newton (mQN) method to solve the QN's such problem. The mQN is gotten by change of H matrix. The H is newly expressed as follow,

$$H_{i+1} = H_i + \frac{(\Delta\gamma_i - H_i\lambda_i)\Delta\gamma_i P_{i-1} H_i}{c + \Delta\gamma_i^T P_{i-1} H_i \lambda_i} \tag{19}$$

$$P_{i-1} = P_{i-2} - \frac{P_{i-2}\Delta\gamma_{i-1}\Delta\gamma_{i-1}^T P_{i-2}}{\Delta\gamma_{i-1}^T P_{i-2}\Delta\gamma_{i-1}} \tag{20}$$

We apply $\Delta\gamma_i P_{i-1}$ instead of $\Delta\gamma_i$. This action makes that $\Delta\gamma_i P_{i-1}$ is orthogonal of $\Delta\gamma_{i-1} P_{i-2}$. It means that they have independence each other. It makes that early convergence rate is batter than the QN. Figure.1 shows comparison of both. And we solve to 'divided by zero' problem by adding constant c to a denominator. As adding c, c's size is very important. If c is very large, c cause bad impact to Jacobian matrix. If c is very small, c has also bad influence to Jacobian matrix. In this paper we select optimum value $c = 0.01$ by experiments.

4. Numerical Experiments

For the verification of the proposed algorithm we conduct numerical experiments. The main focus of our numerical experiment is the computation time, robustness and accuracy of the mQN, mNR. At first, we will compare the proposed method with conventional method at point of speed of algorithm as estimated to object boundary. Secondly, we will compare the two methods in noise free case and noisy data case. In the experiment, we estimate γ, and we define the RMS error as following to evaluate the quality of solution,

$$e_{rms} = \sqrt{\frac{(V_m - V(\gamma))^T (V_m - V(\gamma))}{V_m^T V_m}} \tag{21}$$

We consider a circular phantom of radius 140mm which has 32 electrodes along the boundary. The domain is discretized into 2736 triangular element in the finite element calculation. The resistivity value of the anomaly and the background are set to 6000 Ωcm and 300 Ωcm, respectively. And the simulations are carried out with a 2.4 GHz Pentium4 personal computer with 512Mb RAM based on MATLAB 6.5.

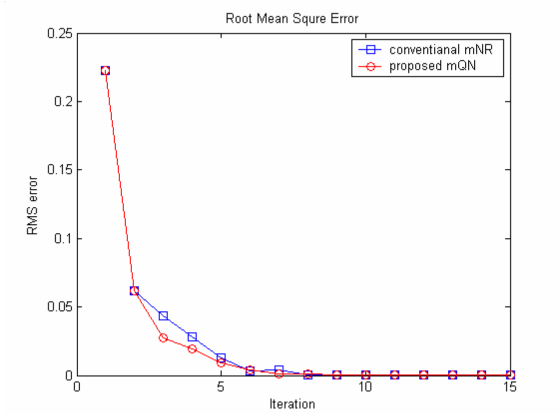
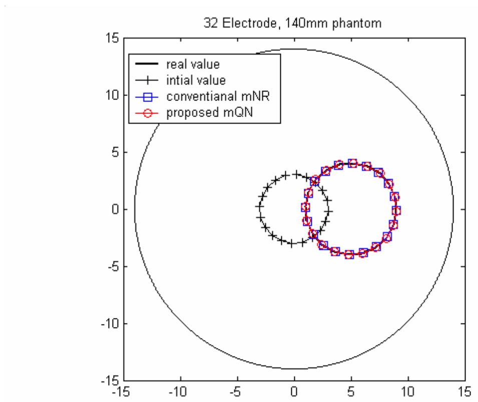
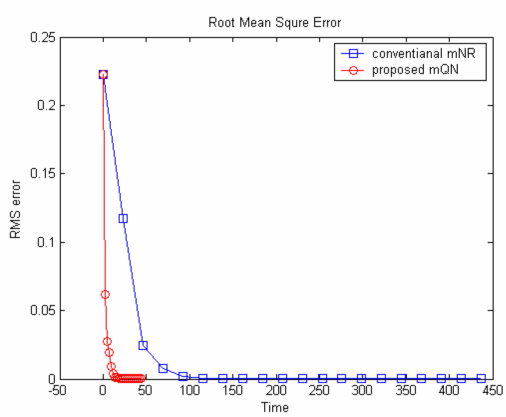


Fig. 1 early convergence rate compared mQN with QN.

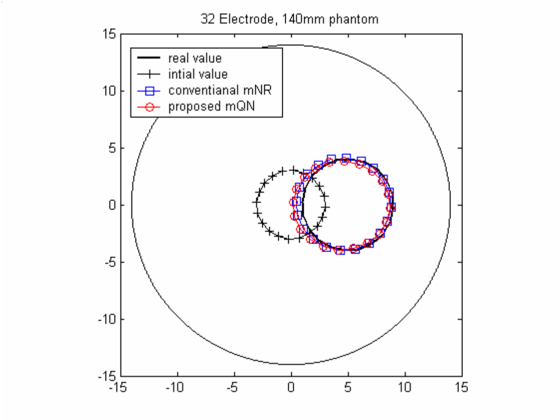


(a) Boundary estimation

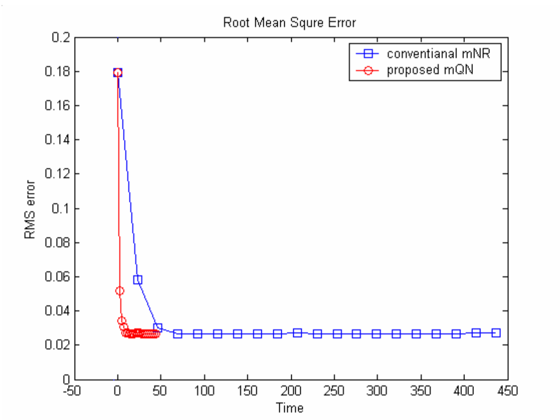


(b) RMS error
Fig. 2 noise free.

Figure.2 (a) shows that the mQN has a good performance equal to conventional the mNR in noise free measured data. Regardless of the use estimated Jacobian matrix, the result agreed with the object very well. Figure.2 (b) shows RMS error of both proposed the mQN and conventional the mNR which change per cpu time. While the mNR spends about 23sec per iteration, the mQN spends about 2.3sec per iteration. When the mNR is going to 2~3 iteration, the mQN has already converged. Finally the mQN is faster than the mNR. Also they have a same RMS error about 10^{-4} order RMS error.



(a) Boundary estimation



(b) RMS error
Fig.3 3% noise.

Figgure.3 shows that the mQN can be applied to noisy measured data. The proposed algorithm has robustness in the boundary estimation even with noisy measured data.

5. CONCLUSION

In this paper, we propose a new method to estimate the boundary of anomaly in the electrical impedance imaging. Because conventional the mNR has a low efficiency to get and update Jacobian matrix, although it is famous for good method since has good convergence and robustness against noisy data. Its low efficiency leads to slow speed during running algorithm. The proposed mQN has an efficiency to get and update Jacobian matrix. The mQN uses the estimated Jacobian matrix instead of the exact Jacobian matrix which used the mNR. Its efficiency leads to faster speed than the mNR. The proposed method modified to the QN which is also suggested in this paper. As the QN has some problems, which is such as bad early convergence rate and instability of 'divided by zero', so we might modify it to solve its problems.

For the verification of the proposed method, this is the mQN; we performed numerical experiments with time comparison in noise free data and noise 3% data. The results show excellent agreements with efficiency and the true boundary.

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