A New Design Tool for PID and First Order Controllers in Parameter Space (ICCAS 2004)

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Abstract: In the paper, we present a practical design tool which allows us to design PID types and the first order lead/lag compensators incorporating the step response requirements in the stability. For a given plant transfer function of arbitrary order, this program provides a set of controller gains that satisfies both stability and time domain performances. This implies that even if one selects any controller parameters inside the set, the closed loop system with the controller gains meets the design objectives. This tool based on Matlab and GUI will be very useful for control design in the controller parameter space, especially when time domain requirement is given.

Keywords : PID, first order controller, time domain specification, CRA

1. INTRODUCTION

One of the most important requirements in control system design is the time domain specifications such as overshoot and speed of response. Nevertheless, since the precise relationship between a system and its transient response is not known yet except for the case of second order all-pole system, the method that is mainly occupied by practical control engineers is still of the frequency domain design based on gain and phase margins, bandwidth. On the other hand, the modern approaches address robust stability and attempt to get a feedback controller that optimize a closed loop performance measure. Despite the theoretical successes of these methods, these are rarely used in industry because modern methods give unnecessarily high order controller. The main drawback of such controllers is that the designers is forced to carry out tuning, analysis, and modifications in higher dimensional design space. In addition, higher order controllers in general have acute sensitivity with respect to parameter variations.

This is best reinforced by the fact that about 90% of controllers in actual use in industry are of PID and the first order types. In most cases, design of PID type controllers still relies on classical control methods such as the Ziegler and Nichols tuning method, frequency response method via loop shaping. Recently, some other methods using fuzzy neural network, linear matrix inequality(LMI), and objective sensitivity function have been proposed, see [1]. However, there are few methods of designing PID types and lower order controllers subject to the time response specifications.

Motivated by these considerations, in the paper we show a practical design tool which allows us to design PID types and the first order lead/lag compensators to meet the step response requirements in the stability. If we select any controller parameters in this set, the closed loop system with selected controller gains meets the design specifications. In this work, we apply some very useful results recently obtained by Datta et al.[3] and Tantaris et al.[4], which are a new linear programming characterization of all stabilizing PID controllers and a numerical computation of all stabilizing first order controllers for a given but arbitrary linear time invariant(LTI) continuous time system. It is noted that both methods give the complete set of stabilizing controllers in the controller parameter space. The other result that we have used in this tool relates to the so-called *characteristic ratios* and *a generalized time constant* which are defined by coefficients of the denominator of the transfer function. It has been shown in [5] that the damping of LTI system is closely connected with the *characteristic ratio*s and that the speed of response is exactly controlled by the *generalized time constant*. Based on this properties, Kim proposed the so-called characteristic ratio assignment (CRA) for the sake of transient response control. They have also showed in [6] that the CRA can be extended to the PID design problem that shall be met time response requirements such as overshoot and settling time.

Three preliminary results above lead to the proposed design tool. The tool consists of three steps: For a given plant, (i) the all stabilizing set of controllers is first determined by using either Datta's or Tantaris' algorithm, (ii) the next step is to extract a subset of first order controllers satisfying the gain/phase margins requirements from the all stabilizing controllers by using Tantaris' algorithm[7], (iii) the third step is to take a intersection of subset to meet given time response specifications from stabilizing controllers.

Basically, this tool is applicable to the feedback system with any fixed structure of controller wherein the controllers should be limited by the forms of PI, PD, PID, and first order lead/lag. Here we have assumed that the design specifications are given by (1) the closed loop stability, (2) the gain and phase margins in the case of first order controller, (3) the maximum overshoot of step response less than a pre-specified value, and (4) the upper/lower bounds of settling time. These are the most popular requirements in design process. Also, suppose that the plant be a LTI system and the model transfer function is given. Say again, the tool has been developed to provide several information which are (a) a whole set of controller parameters that meet the specifications (that is, both performance and stability conditions), (b) the complete set of all stabilizing controllers which are depicted in the form of 3D-visual graphics in the controller parameter space, (c) a specific numeric values of the controller that represents a point in the graph when it is arbitrarily selected by clicking the mouse button of PC, and so on. Finally, a example is given for the sake of demonstrations.

2. Preliminaries

In this section, we briefly review of CRA properties for applying to select a subset of controller gains from all stabilizing controllers in the controller parameter space which stabilize a fixed LTI plant. The procedure of determining all stabilizing controllers is founded in [3,4,7].

2.1 Characteristic ratios and time response

Consider a following characteristic polynomial with real positive coefficients:

$$\delta(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0. \ \forall a_i > 0. \quad (1)$$

The *characteristic ratios* are defined as :

$$\alpha_1 := \frac{a_1^2}{a_0 a_2}, \ \alpha_2 := \frac{a_2^2}{a_1 a_3}, \ \cdots, \ \alpha_{n-1} := \frac{a_{n-1}^2}{a_{n-2} a_n} \quad (2)$$

and the *generalized time constant* is defined as

$$\tau := \frac{a_1}{a_0}. \quad (3)$$

It was shown in [5] that τ represents the speed of the response of a system with denominator $\delta(s)$. As an important result, it says that the speed of response of a linear all-pole system) can be controlled, while maintaining the exact shape of response, by adjusting the value of τ if its $\alpha_i s$ can be kept the same. Now, we consider two systems which have their generalized time constant τ_1 and τ_2 respectively, having the same characteristic ratios. The step response of each system, $y_1(t)$, $y_2(t)$, has the following relationship[5].

$$y_1(t) = y_2\left(\frac{\tau_1}{\tau_2} \cdot t\right), \ \forall t \geq 0 \quad (4)$$

The coefficients a_i of $\delta(s)$ may also be represented in terms of α_i and τ as follows:

$$a_1 = a_0 \tau$$
$$a_i = \frac{a_0 \tau^i}{\alpha_{i-1} \alpha_{i-2}^2 \alpha_{i-3}^3 \cdots \alpha_2^{i-2} \alpha_1^{i-1}}, \ i = 2, 3, \cdots, n \quad (5)$$

We see that for a given set of values $\alpha_i s$, τ and a_0 the corresponding polynomial $\delta(s)$ is completely determined. We set $\alpha_1 = \alpha$ and call it the *principle characteristic ratio*. Kim[5] also proposed the method to obtain the desired closed loop polynomial as followings.

(i) $\alpha_1 > 2$,

(ii) $$\alpha_k = \frac{\sin\left(\frac{k\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)}{2\sin\left(\frac{k\pi}{n}\right)} \cdot \alpha_1, \ \ k = 2, \cdots, n-1 . (6)$$

The construction mechanism of eq.(6), the so-called *K-polynomial*, involves only α_1 which must have a value greater than 2. Kim also showed that the overshoot of step response can be adjusted by selecting only α_1.

We also showed, for fixed τ, the three characteristic ratios, α_1, α_2 and α_3, dominantly affect the step response regardless of the order of denominator higher than 4 in [8]. As a result, a criterion to find a set of controller gains satisfying the time domain specification within a stabilizing one's is given by the following inequalities :

$$\alpha_1 \geq \alpha_1^*, \ \alpha_2 \geq \alpha_2^*, \ \alpha_3 \geq \alpha_3^*, \ \tau^- \leq \tau \leq \tau^+ . \quad (7)$$

Where, α_1^*, α_2^*, α_3^*, $\tau^-$and τ^+ are a target set of parameters that corresponds to a closed loop system satisfying the desired time response specifications such as the maximum percent overshoot and the range of permissible settling time. A value of τ^- is used to limit overshoot in the plant with zeros that increase inevitably the overshoot and τ^+ is used for the settling time.

2.2 Choosing controller gain with time response specifications

The Matlab-based controller design tool constructed in this paper provides designer to get subset of controller gains to meet time domain specification, maintaining stability. For first order controller case, It also guarantees controller gains to satisfy gain and phase margin requirement.

Consider a feedback control system with PID and first order controller shown in Fig. 1 and Fig. 2, respectively. These configuration have the advantage that the zeros of the controller are not added to the zeros of the overall system, and the configuration is common in practice. Where K_{dc} is the compensator for dc gain of a closed loop system.

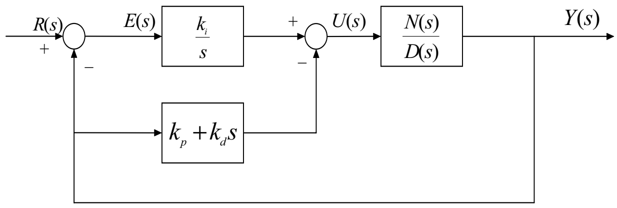


Fig. 1 Feedback control system with I-PD structure

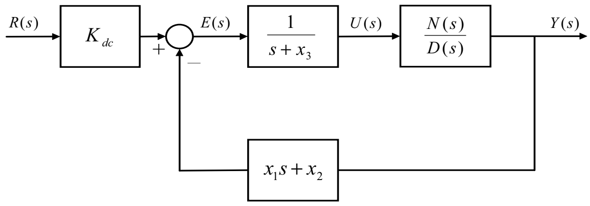


Fig. 2 Feedback control system with two-parameter configuration

The closed-loop characteristic polynomial for PID is

$$\delta(s, k_p, k_i, k_d) = sD(s) + (k_i + k_p s + k_d s^2)N(s) \qquad (8)$$

We can easily compute the characteristic ratios of eq. (2) and the generalized time constant of eq. (3), see [6]. To find out PID gains to meet time domain specifications, the regions included eq. (7) are redefined as follows:

$$\begin{aligned} K_1 &:= \{(k_p,\ k_i,\ k_d),\ \ \alpha_1 \geq \alpha_1^*\} \\ K_2 &:= \{(k_p,\ k_i,\ k_d),\ \ \alpha_2 \geq \alpha_2^*\} \\ K_3 &:= \{(k_p,\ k_i,\ k_d),\ \ \alpha_3 \geq \alpha_3^*\} \\ K_\tau &:= \{(k_p,\ k_i,\ k_d),\ \ \tau^- \leq \tau \leq \tau^+\}. \end{aligned} \qquad (9)$$

We denote all set of stbilizing PID gains by K. Then the region of interest which satisfies both performance and stability requirements is:

$$K^* := K_1 \cap K_2 \cap K_3 \cap K_\tau \cap K. \qquad (10)$$

The inequalities in (9) can be expressed in terms of controller parameters by quadratic form for fixed k_p as following :

$$f_j(k_i, k_d) := xQ_j x^T + 2q_j x + r_j \leq 0 \quad \text{for} \quad j = 1, 2, 3 \qquad (11)$$

Where, the relationship x, Q_j, q_j, r_j can be found in [6]. Similarly, The last inequality in eq.(7), the τ condition appears as a linear function of (k_p, k_i, k_d) as :

$$\tau^- \leq \left(k_p + \frac{d_0}{n_0}\right)\frac{1}{k_i} + \frac{n_1}{n_0} \leq \tau^+. \qquad (12)$$

The solution of (k_i, k_d) in eq.(11) has a linear or quadratic inequality equation form as an order of $P(s)$. The constraints eq.(11) and (12) can be displayed on the (k_i, k_d) plane for a fixed k_p as shown in Fig. 3. We can see visually that an admissible subset of (k_i, k_d) can be easily determined graphically or alternatively by nonlinear programming methods. By sweeping k_p over all admissible values in the stabilizing set and repeating the above procedure at every k_p, the complete subset of PID gains satisfactory specifications is obtained.

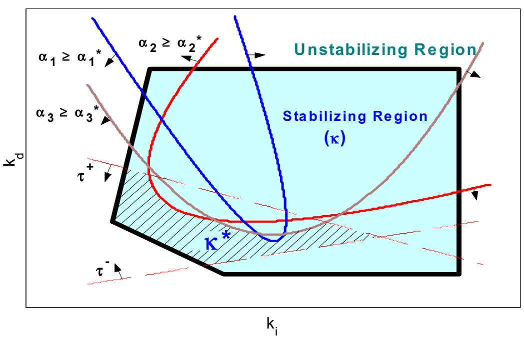


Fig. 3 An admissible set of (k_i, k_d) pairs under a prescribed $\alpha_1{}^*$, $\alpha_2{}^*$, $\alpha_3{}^*$, $\tau^-$and τ^+ at fixed k_p

The case of first order type is same as PID if parameters replace (k_p, k_i, k_d) by (x_3, x_1, x_2). see result of [7]. Moreover, since PI and PD controller is a special case of the PID controller, the algorithm of computing its parameter set to meet time domain specification is very similar to PID. So the tool presented this paper includes PI and PD controller as well as PID and first order.

3. Description of Controller Design Toolbox

The implementation of this design tool starts in the environment of Matlab software for GUI(graphics user interface), which can be easily used by field engineer who has no profound knowledge of control theory. There are four design step in the tool. (1) selection of controller, input of plant parameter and specification, (2) choosing simulation condition, (3) obtaining of all set stabilizing controller and extracting of subset to meet given specification from result of step 2, (4) display on the final design report.

The initial screen as shown in fig. 4 is created by user's start command.

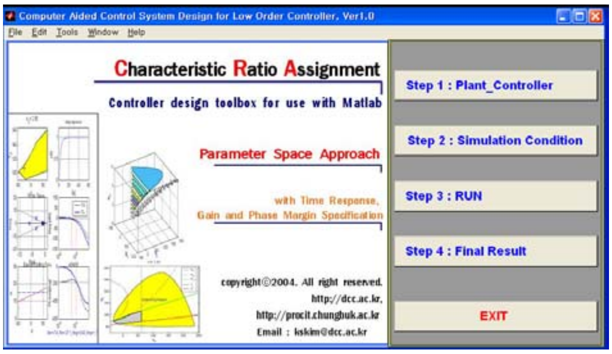


Fig. 4 Initial screen of toolbox

Step 1 : Choose Step 1 button from Fig. 4 to display the following dialog box. A user should input plant parameter and specification, and then select controller type and simulation style in dialog box of fig. 5. The case of gain and phase margin can only apply to first order type in the current version of this tool. We consider at next version of this tool that include the gain and phase margin requirement for PID, PI and PD controller type. if click 'OK' button, data store to variables and return to Fig. 4 screen.

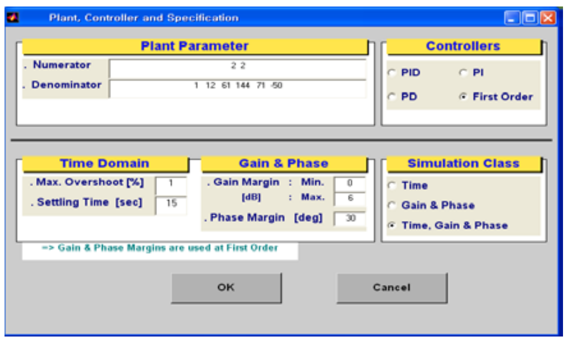


Fig. 5 Input screen for plant, controller, simulation class, specification.

Step 2 : Click Step 2 button from Fig. 4, if user selected PID controller in Step 1, then following dialog box will be displayed. PI, PD or first order case are similar to PID type.

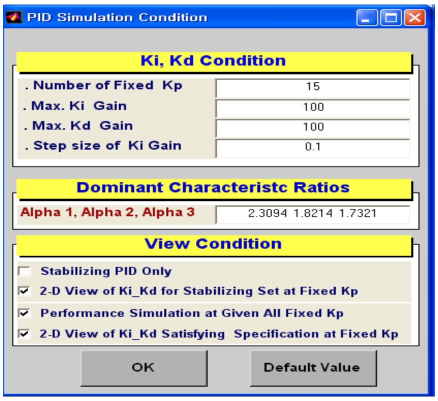
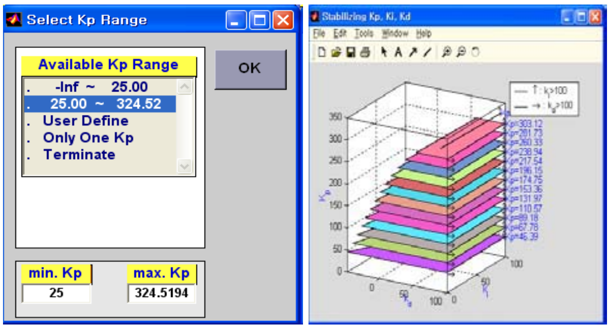


Fig. 6 screen of simulation conditions.

For the PID, PD and PD controller, since k_i and k_d gains to stabilize system can have quit large or infinite value, limited maximum k_i and k_d values are required in order to simulate. To obtain all stabilizing first order controllers, Tantaris[2] determined stabilizing (x_1, x_2) gain by solving linear equations for the boundary crossing points as function of frequency in closed form. So maximum frequency to be simulated is required. Many Experiences say that if this frequency value put at (6~10) [rad/sec], it is enough to get all stabilizing first order controller gains. From Step 1, we can get the order of closed loop transfer function for each controller. If fixed α_1 is given, other α_i values are calculated by using eq.(6) for target closed loop characteristic polynomial with this order. A dominant characteristic ratios are calculated using step response of target transfer function to meet maximum overshoot requirement. This tool supplies values of dominant characteristic ratios automatically. But user can change this value too. This tool provides four sort of view condition. User chooses following option.

- only set of stabilizing Controller.
- roots locations of a stabilizing controller.
- whether take performance simulation all given fixed gain or only one gain
- performance output for a pair of parameter that is point clicked mouse of PC by user - roots locations, step response and bode plot-.

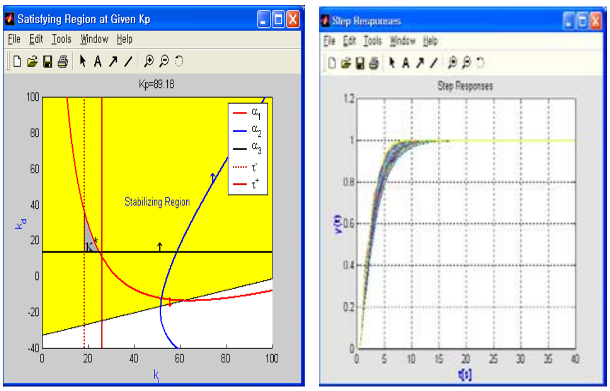
Step 3 : if user choose "Run" button from Fig. 4, following dialog to input fixed gain will display. User can put into a fixed gain value such as k_p range for PID, x_3 range for first order type. Where, an available fixed gains are calculated using breakaway point of root locus idea and Hurwitz necessary condition for stability of polynomial. Click 'OK' button, All stabilizing controller gains display as shown in fig. 7(b). If gain and phase margin requirement are given, a subset of controller which meet given requirements is depicted in form of 2D-visual graphics inside of stabilizing region. After obtaining all stabilizing controllers, user selects a fixed gain inside of set and click mouse on 2-D parameter screen to observe not only roots location but also bode plot if requirement include gain and phase margin. In next process, avaliable generalized time constant range$(\tau^- \sim \tau^+)$ are calculated using eq.(4) and given settling time requirement.



(a) Selection of fixed gain (b) All stabilizing controllers.

Fig. 7 All stabilizing gains at given Fixed gain.

Using inequality equations of eq.(9) and (10) with dominant characteristic ratios and avaliable generalized time constant range, we get subset of satisfying design requirement at each fixed gain as following fig. 8(a). Computations involved are intersections of two-dimensional sets satisfying linear or quadratic inequalities in controller parameter space and can be conveniently displayed to the designer. Since generalized time constant range is necessary condition, if settling time exceed given requirement, user can change generalized time constant range$(\tau^- \sim \tau^+)$ during simulation at fixed gain, observing step responses of fig. 8(b) displayed at the same time as fig. 8(a).



(a) An admissible subset (b) Step responses

Fig. 8 An admissible set satisfying eq. (7) and its step responses at fixed gain

For prescribed all fixed gain, repeat above mentioned process. then we can get all subset of controller parameters to meet specification in maintaining stability. Finally, we want to choose a controller parameter which has more robustness. This tool supplies an available solution for it. See following figure that displays stabilizing region(K) as well as satisfying performance region(K^*), Control parameter values as far as possible from left-bottom corner are more robust. So designer can easily choose a controller parameter, seeing 2-D graphics in parameter space. This tool also offers other screen to display parameter value, roots location, bode plot for gain and phase margin, step response and expected settling time when user clicks mouse button of PC on K^* region. Detail display screen will show at the example of next chapter.

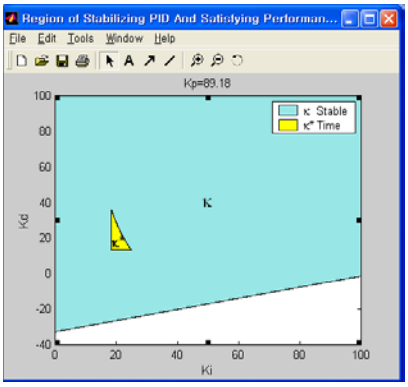


Fig. 9 Satisfying performance and stabilizing controller region

Step 4 : When controller design is finished, to get finial design report or print its result, select 'Final Result' button of fig. 4. The final design report displayed are

- controller type
- simulated date and time and given plant
- simulated fixed gain range
- applied dominant characteristic ratios
- applied tau range
- designed controller gain.
- settling time, maximum overshoot of time response
- gain and phase margin of frequency response
- pole locations

4. Illustrative Examples

In this section, we consider example to show how this tool applies to the problem of determining an admissible set of first order controllers which meets given time and frequency domain specification, in particular, the overshoot and the settling time of step response and gain and phase margin, simultaneously.

Example : Consider the feedback system with first order controller and the following unstable plant that pole locations are $-3.12 \pm j3.35$, -4.81, -1.33 and 0.37.

$$P(s) = \frac{2s+2}{s^5+12s^4+61s^3+144s^2+71s-50}$$

The objective is to design a set of first order controllers such that the closed loop step responses have following design specification

- maximum overshoot of step response : 1%
- settling time of step response : 10 sec.
- gain margin : 6[dB].
- phase margin : 30°

We first obtain the set of all first order controller gains to meet gain phase margin in maintaining stability with $x_3 \in [-2.36, \infty]$. Fig. 10 shows this region when $x_3 \in [1, 20]$.

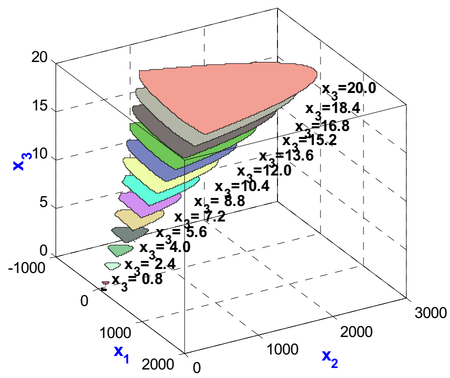


Fig. 10 Region of satisfying gain and phase margin maintaining stability

Dominant characteristic ratios of 6th order target closed loop system with 1 % maximum overshoot of step response are $\alpha_i^* = \{2.3094, \ 1.8214, \ 1.7321\}$ that calculated using eq. (5),(6) and result of step response. Avaliable generalized time constant range $\tau \in [2.5, 3.5]$ calculated using eq.(4) and given settling time requirement. Fig. 11 shows the subset of controller gains satisfying eq. (9) and (10) with the dominant characteristic ratios and the generalized time constant.

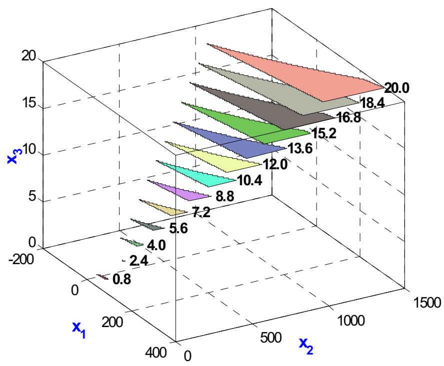


Fig. 11 First order gains set with satisfactory given all specifications

We select a fixed $x_3 = 20.0$ from Fig. 11 to get one pair of (x_1, x_2, x_3). It is important to know location of subset for satisfying time domain performance within a set of stabilizing controllers. Fig. 12 shows these regions at fixed x_3 - stabilizing controller in outside closed curved surface, satisfying gains and phase margin in middle area and set of satisfying time domain performance at inner closed section.

Region of satisfying time domain performance in Fig. 12 zooms in Fig. 13 to get select (x_1, x_2) and to confirm performance.

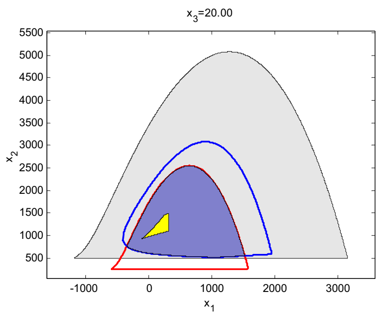


Fig. 12 Location of subset for satisfying time domain performance

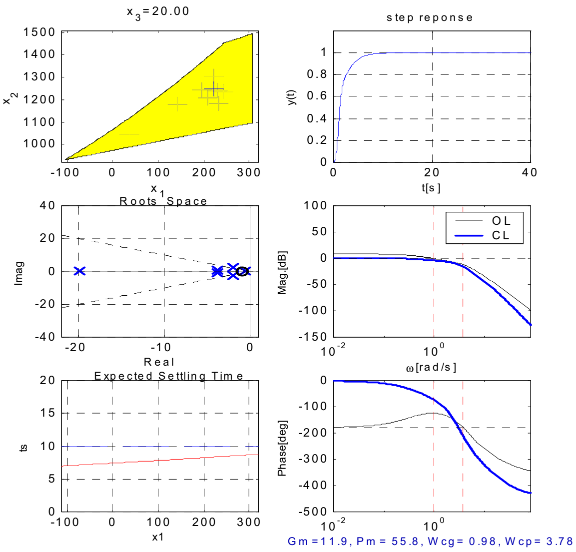


Fig. 13 Closed loop system performance for designed controller parameter

Final design report are as followings

Design Report for First Order Controller : 10-Jun-2004,

(1) Design Parameter

- Simulated x_3 Range = -2.36 ~ 20.00
- Applied α_k Values = 2.31, 1.82, 1.73
- Applied tau Range = 2.50 ~ 3.50

- First order gain(x_1, x_2, x_3) = 223.16, 1250.05, 20.00

(2) Performance Data

- Settling Time = 7.12[s]
- Maximum Overshoot= 0.000[%]
- Gm=11.9[dB], Pm= 55.8[deg], Wcg= 0.98[rad/s], Wcp= 3.78[rad/s]
- Pole Locations

  $-19.928, \ -0.5219,$

  $-1.9239 \pm j2.3815, \ -3.8512 \pm j0.74660$

5. Conclusions

In this paper, we propose a new computer aided control system design (CACSD) tool for PID and first order lead/lag controllers in which the stability margin and the time response specifications as like overshoot and speed of response have to be satisfied and all design processes are carried out in the controller parameter space and in the environment of Matlab-based and GUI .

Exploiting the recent result on finding the set of all stabilizing controller gains for a given LTI plant, we propose a design scheme based on characteristic ratio and generalized time constant assignment. As a result, it is important to note that the computations of this algorithm involved in are intersections of two dimensional sets satisfying linear or quadratic inequalities in the controller parameter spaces and can be conveniently displayed to the designer. The graphical display of sets of feasible solutions using 2-D or 3D graphics is an attractive feature of the design method and would allow the imposition of further design requirements. We convince ourselves that the proposed this tool is valuable in the practical sense, since it guarantees that any of the overshoot and the settling time specifications can be satisfied almost completely.

Acknowledgments

This work was supported by grant No. R01-2003-000-11738-0 from the Basic Research Program of the Korea Science & Engineering Foundation

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