Pole Assignment for Structural Active Control

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Abstract: Significant progress has been achieved in the active control of civil-engineering structures, not only in the control algorithm, but also in control testing of the scaled model and full-scale building. At the present time, most algorithms used in the active control of civil engineering structures are based on the various active control techniques. In this paper represents active control method, by using pole assignment for reducing structural vibration under excited load. Numerical simulations are performed to assess the effectiveness of pole assignment control system. The relative displacement of structure system is significantly reduced.

Keywords: Pole Assignment, Active Control, Wind Load, Earthquake Load

1. INTRODUCTION

Active control systems for seismic – excited civil engineering structures has been developed to the degree where practical implementations have been made in a number of buildings [e.g., Soong (1990), Soong and Reinhorn (1991), Reinhorn and Soong (1992) , Kobori and Kamagata (1991), and are the subject of Proceedings (1993)]. For protecting the safety and integrity of the building subjected to strong earthquakes, the reduction in the interstory drifts is most critical. Most of the literature in active control mainly deals with this subject. For protecting the integrity of the structure and to prevent injury to the occupants and damage to the contents.

Various active control techniques have been investigated for applications to civil engineering structures. This paper presents pole assignment approach to the design close-loop control structural system. Pole assignment algorithms have been studied extensively in the general control literature [Abdel – Rohman M and Leipholz 1978]. Its application to the study of civil engineering structural control has been fruitful when only a few vibrational modes contribute significantly to the response. The difficult for this control is the way to find an efficient method of generating the control gain. The procedure described in this paper used Ackermann's Technique.

2. FORMULATION

Consider a building structure modeled by an n-degree – of – freedom system

$$M\ddot{x}(t) + C\dot{x}(t) + K\,x(t) = D\,u(t) + E\,f(t) \quad \text{........(1)}$$

Where x(t) = n – dimensional displacement vector ; and M,C, and K = n x n mass, damping, and stiffness matrices, respectively. The vector f(t) represents the r – dimensional external excitation vector, and u(t) = m – dimensional control force vector. D = n x m control force location matrix, and E = n x r excitation location matrix.
Eq. (1) can be written as follows

$$\dot{Z}(t) = A\,Z(t) + B\,u(t) + H\,f(t) \quad \text{..................(2)}$$

Where the 2n – dimensional state vector Z(t) is expressed as

$$Z(t) = \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix} \quad \text{..............................(3)}$$

with initial conditions

$$Z(0) = 0 \quad \text{................................(4)}$$

The 2n x 2n system matrix A, the 2 n x m controller location matrix B, and the 2n x r external excitation location matrix H are expressed as :

$$A = \begin{bmatrix} O & I \\ M^{-1}K & -M^{-1}C \end{bmatrix} \quad \text{..................(5)}$$

$$B = \begin{bmatrix} O \\ -M^{-1}D \end{bmatrix};\ H = \begin{bmatrix} O \\ M^{-1}E \end{bmatrix} \quad \text{.........(6,7)}$$

Consider the state – space equation (2) , The system matrix A defines the open – loop system dynamics and its eigenvalues provide modal damping and stiffness characteristic. Let the control force be determined by linear state feedback.

$$u(t) = -Gz(t) \quad \text{...............................(8)}$$

Where G is a constant gain matrix. Then , the close – loop system thus takes the form.

$$\dot{Z}(t) = (A - BG)\,Z(t) + Hf(t)\ ,\ Z(0) = 0 \quad \text{..........(9)}$$

This modification of system matrix through active control alters modal damping ratios and frequencies. This is reflected by the fact that the eigenvalues of A-BG are generally different form those of A. For structural system, these eigenvalues, which denote by μ_i , are related to the model frequencies ω_i and damping ζ_i in complex conjugate pairs by

$$\mu_i = \varsigma_i\ \omega_i \pm j\omega\sqrt{1-\zeta^2}\ ,\ j = \sqrt{-1} \quad \text{.............(10)}$$

Since these closed – loop eigenvalues define the controlled system behavior, a feasible control strategy is to choose the control gain G in such a way that the μ_i take a set of valves prescribed by the designer. Control algorithms developed base on this procedure are generally referred to as pole assignment techniques.

In control design using the pole assignment need to finding as efficient method of generating the control gain G. The procedure described below is relatively simple and is due by using Matlab.

3. ACKERMANN'S FORMULA

The system equation and control equation are give in Eq. (2) and Eq. (3) respectively. By substituting Eq.(8) into Eq. (2) we obtain Eq. (9) let define

$$\hat{A} = A - BG \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The desired characteristic equation is

$$|sI - A + BG| = |sI - \hat{A}| = (s-\mu_1)(s-\mu_2)\ldots\ldots\ldots(s-\mu_n)$$
$$= s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n = 0 \ldots\ldots\ldots\ldots (12)$$

Define

$$\phi(S) = S^n + \alpha_1 S^{n-1} + \ldots + \alpha_{n-1} S + \alpha_n$$

then

$$\phi(A) = A^n + \alpha_1 A^{n-1} + \ldots + \alpha_{n-1} A + \alpha_n I \ldots\ldots\ldots (13)$$

Since the Cayley – Hamilton theorem states that $\hat{A}$ satisfies its own characteristic equation, then

$$\phi(\hat{A}) = \hat{A}^n + \alpha_1 \hat{A}^{n-1} + \ldots + \alpha_{n-1}\hat{A} + \alpha_n I = 0 \ldots\ldots (14)$$

Use Eq.(14) to derive Ackermann's formula. To simplify the derivation, let consider the case where n = 3. Consider the following identities:

$$I = I$$
$$\hat{A} = A - BG$$
$$\hat{A}^2 = (A - BG)^2 = A^2 - ABG - BG\hat{A}$$
$$\hat{A}^3 = (A - BG)^3 = \hat{A}^3 - A^2BG - ABG\hat{A} - BG\hat{A}^2$$

Multiplying the preceding equations in order by α_3 , α_2, α_1 , α_0 (Where $\alpha_0 = 1$), Respectively, and adding the results then obtain

$$\alpha_3 I + \alpha_2\hat{A} + \alpha_1\hat{A}^2 + \hat{A}^3$$
$$= \alpha_3 I + \alpha_2(A - BG) + \alpha_1(A^2 + ABG - BG\hat{A}) + A^3$$
$$- A^2BG - ABG\hat{A} - BG\hat{A}^2$$
$$= \alpha_3 I + \alpha_2 A + \alpha_1\hat{A}^2 + \hat{A}^3 - \alpha_2 BG - \alpha_1 ABG$$
$$- \alpha_1 BG\hat{A} - A^2BG - ABG\hat{A} - BG\hat{A}^2 \ldots\ldots\ldots\ldots\ldots (15)$$

Referring to Eq.(14) then

$$\alpha_3 I + \alpha_2\hat{A} + \alpha_1\hat{A}^2 + \hat{A}^3 = \phi(\hat{A}) = 0$$

Also

$$\alpha_3 I + \alpha_2\hat{A} + \alpha_1\hat{A}^2 + \hat{A}^3 = \phi(A) \neq 0$$

Substituting the last two equations into Eq. (15) , get

$$\phi(\hat{A}) = \phi(A) - \alpha_2 BG - \alpha_1 BG\hat{A} - BG\hat{A}^2 - \alpha_1 ABG$$
$$- ABG\hat{A} - \hat{A}^2 BG$$

Since $\phi(\hat{A}) = 0$ then

$$\phi(A) = B(\alpha_2 G + \alpha_1 G\hat{A} + G\hat{A}^2) + AB(\alpha_1 G + G\hat{A})$$
$$+ A^2 BG$$
$$= [B \vdots AB \vdots A^2B]\begin{bmatrix} a_2 G + a_1 G\hat{A} + G\hat{A}^2 \\ aG + G\hat{A} \\ G \end{bmatrix}$$
$$\ldots\ldots\ldots (16)$$

Since the system is completely state controllable, the inverse of controllability matrix

$$[B \vdots AB \vdots A^2 B]$$

exists. Premultiplying the inverse of the controllability matrix to both sides of Eq.(16),
Then

$$[B \vdots AB \vdots A^2B]^{-1}\phi(A) = \begin{bmatrix} a_2 G + a_1 G\hat{A} + G\hat{A}^2 \\ a_1 G + G\hat{A} \\ G \end{bmatrix}$$

Premultiplying both sides of this last equation by [0 0 1], will obtain

$$[0 \;\; 0 \;\; 1][B \vdots AB \vdots A^2B]^{-1}\begin{bmatrix} a_2 G + a_1 G\hat{A} + G\hat{A}^2 \\ a_1 G + G\hat{A} \\ G \end{bmatrix} = G$$

or

$$G = [0 \; 0 \; 1][B \vdots AB \vdots A^2B]^{-1}\phi(A) \qquad \ldots\ldots\ldots\ldots (17)$$

Eq. (17) gives the required state feedback gain Matrix G. For an arbitrary positive integer n, we have

$$G = [0 \; 0 \ldots 1]\begin{bmatrix} B \vdots AB \vdots \ldots \vdots A^{n-1}B \end{bmatrix}^{-1}\phi(A)$$
$$= [0 \; 0 \ldots 1] J^{-1}\phi(A) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

where

$$J = \begin{bmatrix} B \vdots AB \vdots \ldots \vdots A^{n-1}B \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Eq.(18) is know as Ackermann's formula for the determination of the state feedback gain matrix G.

4. NUMERICAL EXAMPLE

To illustrate the application of the pole assignment and their performances, simulation results for a liner building are presented in this section. A six-story scaled buildings model studied by Schmitedorf et al. (1993a, b, 1994). The mass of each floor, and the stiffness and damping coefficients of each story unit are m=345.6 metric ton; $k_i = 3.404 \times 10^5$ kN/m; and c_i = 2,937 kNs/m (sec Fig. 1 for a schematic).

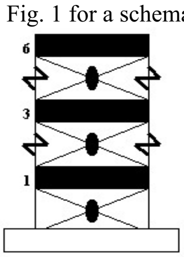

Fig 1. Model of Full – Scale Building with Active

Bracing System

4.1 FOR WIND LOAD

The wind load used for the simulation study is similar to one used in Soong (1990) as

$$f(t) = 3 \sin \omega t + 50 \sin 2\omega t + 7 \sin\omega t + 4 \sin\omega t \ldots\ldots\ldots(20)$$

In Eg. (20) the value of ω is chosen to be 2.5. This choice of ω (which is close to the first frequency of the structure is $0.0252 \pm 2.5j$. In the simulation, the value of the first pole is chosen to be 0.0252 ± 6.0. The time histories of the relative displacement of each floor for the system with and without control are shown in Fig.2 for comparisons.

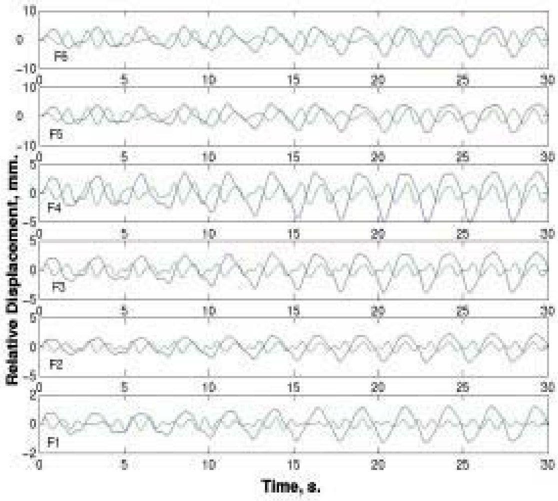

Fig. 2 The time history of the relative displacement of each floor, case Wind Load.

4.2 FOR EARTHQUAKE LOAD

The earthquake excitation used for the simulation study is E1 – Centro 1940 NS. The maximum acceleration of the earthquake is 0.3 g. (as see on the Fig.3)

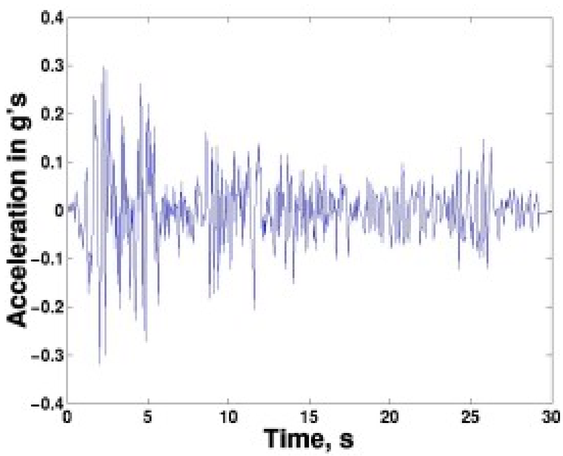

Fig 3. Ground Acceleration

The first frequency of the building is $0.0252 \pm 2.5j$. In the simulation, the value of the first pole is chosen to be $0.0252 \pm 4.0j$. The time history of the relative displacement of each floor for the system with and without control is shown in Fig.4 for comparisons. It can be seen from Fig.2 and Fig.4 that the proposed control law could reduce the responses of the system.

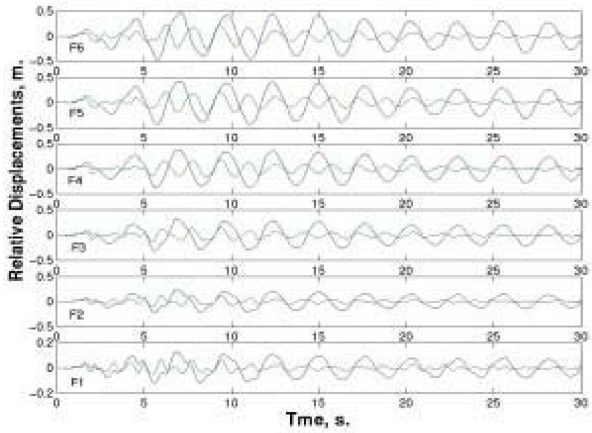

Fig 4. The time history of the relative displacement of each floor, case Earthquake Load.

5. CONCLUSIONS

The pole assignment method for excited load by using Ackermann's technique have been presented. Simulation results using a six-story building indicate that the control method presented are effective in reducing the effect of excited wind and earth quake load on the relatives response of the building.

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