Assessment of Sinkhole Occurrences Using Fuzzy Reasoning Techniques

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Abstract

Underground mining causes surface subsidence long after the mining operation had been ceased. Surface subsidence can be in the form of saucer-shaped depression or collapsed chimneys or sinkholes. Sinkhole formations are predominant over shallow-depth room and pillar mines having weak overburden strata. In this study, occurrences of sinkholes due to mining activity are assessed based on local geological conditions and mining parameters using fuzzy reasoning techniques. All input and output parameters are represented with linguistic hedges. Numerous fuzzy rules are developed to relate sinkhole occurrences with input parameters using fuzzy relational matrix. Based on the combined fuzzy rules, possibility of sinkhole occurrences can be ascertained once the geological and mining parameters of any area are known.

Keywords: Sinkhole subsidence, fuzzy relation, fuzzy rules, room and pillar mining

1. Introduction

Abandoned mine workings exist in many countries including in Korea and sometimes the extent of working may not be known. Most of the old mines are worked with room and pillar method of mining and possesses great potential of surface subsidence by roof fall/pillar failure. In UK, more than 70,000 old mine workings are reported and some of them may be three centuries old (Whittaker et al., 1989). In the USA, 354 subsidence incidents were reported over Pittsburgh Coal bed most of which in the form of sinkholes (Gray et al, 1977). In 1985, Marino et al reported that both trough and sinkhole type subsidence occurred in Illinois (Whittaker et al., 1989) over shallow depth room and pillar mines. After extensive study of surface subsidence in the USA, Gray et al commented that the most prevalent subsidence features over abandoned mined land are sinkholes, with depth of sinkhole more than 3 ft, and trough or sags less than 3 ft (Peng, 1992). After studying subsidence incidents in Germany, Kratzsch commented that sudden cave-ins and irregular depressions in the form of sinkholes over near-surface abandoned mines possess a serious risk to the populated area nearby (Kratzsch, 1983). His study also suggests that size of the effected area must be established based on statistical investigations, taking into account type of workings, strength and thickness of roof, fluctuation of ground water and others. In Korea, more than 300 coal mines have been ceased their operations with the economic structural adjustments in early 1990s. With lots of abandoned coal mines, the surface subsidence problem has been coming up to the public, known as one of geohazards (Choi et al, 2004). Their study suggests that several parameters should be emphasized for evaluating the surface subsidence in coal mine area, and these parameters should be dealt with adding an extra weight for realistic analysis. In addition, the sinkhole type subsidence in urban area, whatever it caused by gangway collapse in coal mines or in metal mines, can be a death-blow to many structures as well as public welfare. The surface subsidence in the Boopyung graveyard was one example for this. It happened by the roof fall and pillar failure in near-surface openings and was restored by pumping the sand slurry into those openings (KIGAM report, 1993).

All of the above mentioned literatures suggest that sinkhole development is a process of collapsed junction having weak/fractured roof and then progression of collapse chimney up to the surface at shallow depth of cover. In general, sinkhole is in the form of conical depression/cavity suddenly appears on the surface. The major factors which contribute to sinkhole formation are width of mine opening/gallery (W), height of opening (M), depth of cover (H), rock type and thickness of roof, water condition, pillar strength, time elapsed after mining operation was ceased, inclination of the seam and others. Whittaker et al (1989) reported that width of gallery, depth of cover and roof conditions are the primary factors for the determination of sinkhole occurrences and higher chances are associated with lower H, higher W and weak roof conditions. Peng (1992) had also commented on the similar factors which contributed in sinkhole development in the Pittsburgh Coal bed. However, most of the reported sinkhole occurrences are not statistically related to these factors. The major difficulty lies in the representation of W, H, roof conditions, water and time factors numerically and then trying to correlate with sinkhole dimension such as depth and diameter. As for example, an abandoned mine roof can be better represented as "weak roof" or "strong roof" rather than quantifying with single numerical value. Thus representation of these parameters using linguistic terminology provides more realistic approach to apprehend the complex nature of roof geology. In a mine environment, a linguistic definition of roof classification or any parametric evaluation is more appropriate and representative rather than providing a numerical value such as W is 3 m or H is 6 m and so on. This paper outlines the analysis of vagueness in data using fuzzy reasoning techniques and establishes relations between inputs and output based on fuzzy rules designed using field data.

Fuzzy set theory is being used in every engineering and science disciplines where data cannot be represented using crisp set. It is said that one way of simplifying a complex system is to allow some degree of uncertainty in its description (Klir et al., 1988). As for example, if the width of gallery or roof condition is to be represented with linguistics prefix as "High", "Low" or "Medium", the crisp set theory and classical statistical techniques cannot be used to analyze these data. For this purpose, fuzzy membership function for each linguistic hedge has to be determined and operations of fuzzy relational matrix have to be adopted. Jiang et al, has effectively applied fuzzy set theories for the classification of longwall roof (Jiang et al., 1996) using field measured data. Applications of fuzzy sets and fuzzy logics are well established in mineral processing and other geo-mining fields such as mine subsidence analysis (Liao, 1993) and estimating roof fall rating (Deb, 2003). Recently, Mamdani's fuzzy influence technique was applied to Geological Strength Index (GSI) for the assessment of slope stability (Sonmez et al., 2004).

Four parameters, W, H/M, pillar strength factor (P/M) and roof Index (R) are found to be directly related to occurrences of sinkhole. Each of these parameters are classified into three linguistic hedge groups of "Low", "Medium", and "High" based on their respective values or range of values. The possibility of sinkhole formation (S) is grouped with five linguistic hedges with additional "Very Low" and "Very High" hedges. For each parameter, fuzzy membership grades are assigned for each group within that parameter. Based on the data reported data in various literatures, fifteen (15) fuzzy rules are formulated using IF-THEN statements with linguistics hedges. The W, H/M, P/M and R are the cause of each statement and S is assigned as the result. These rules are used to generate fuzzy relational matrices using Mamdani's principles. All of these matrices are then grouped together to develop the final relational matrix signifying the relationship between input parameters and the output. Once this relationship is developed between W, H/M P/M and R with S forecasted value of S can be obtained if the linguistic hedges of the input parameters are known. The output, S can be obtained using linguistic terminology or can be reduced to a representative numerical value.

2. Fuzzy Memberships of Mining and Geological Parameters

2.1. Definition of Fuzzy Set

Consider a set X which has N number of variables or parameters as follows:

$$X = \{X_1, X_2, \cdots, X_N\}$$

The fuzzy set A is defined by assigning to each individual variable a value between 0 and 1, called membership grades, 0 being absolute uncertainty and 1 being complete certainty. In mathematical term, the fuzzy set A will be

$$A = \{\mu_A(X_1), \mu_A(X_2), \cdots, \mu_A(X_N)\}$$

where $\mu_A(X_i)$ is the membership grade of the variable Xi and defined as

$$\mu_A(X_i) \rightarrow [0,1] \qquad i = 1, 2, \ldots, N$$

2.2. Fuzzy Membership Grades of Gallery Width (W)

Gallery width or extension of unsupported roof span is an important parameter for development of collapsed chimney or sinkhole. In general, wider gallery will be more favorable for sinkhole development since higher tensile stress develops in the middle of the span. Theoretically, limiting tensile stress is directly proportional to the square of the unsupported roof span or gallery width. Diameter of the sinkhole is directly related to opening span and was equated to W or $W\sqrt{2}$, where W is the gallery width (Whittaker et al., 1989). Many abandoned mines are left with irregular shaped pillars and thus dimension of opening is not uniform everywhere. Apart from that type of junctions, 3-way or 4-way also influence the effective width of opening. In this study, average gallery width is considered to develop the fuzzy membership grade.

Recorded data of sinkholes from USA, UK and other country show that possibility of sinkholes is great if gallery width exceeds about 7~8 m. The occurrences of sinkhole diminish if the gallery width is below 4 m. Based on these data, fuzzy membership grade of gallery width is developed as shown in Figure 1. It is noted that a "Low W" is defined with a membership grade of 1.0 at W = 3.5 m or less and then it gradually decline to 0 at W = 6.5 m and above. On the other hand, a "High W" means a membership grade of 0 for W = 5 m or below and gradually increases to 1.0 at W = 8 m or above. A "Medium W" signifies membership grade of 1.0 at W = 5.5 m and then it reduces once W exceeds or recedes from 5.5 m as shown in the Figure 1.

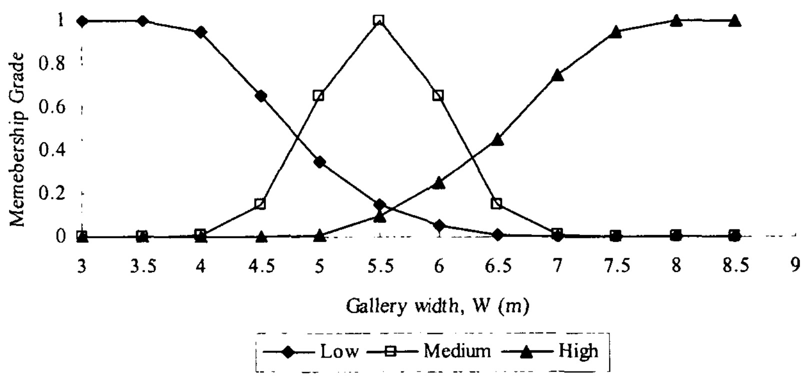

Figure 1. membership grade of gallery width, W

2.3. Fuzzy Membership Grades of H/M Ratio

Overburden (cover) depth is one of the major factors which ultimately determine sinkhole appearance on the surface. Peng (1992) reported that ratio of cover depth to opening height (H/M) below 4~5 are favorable for sinkhole formation with a maximum value of 11. One study in UK referred that maximum height of collapse can be ten times of mining height with an average value of 3~5M (Whittaker et al., 1989). Based on these and other studies, twelve values of H/M ranging from 1 to 12 are considered for the definition of fuzzy membership grades of three linguistics hedges as mentioned above. Figure 2 shows the membership grades of these three hedges based on different values of H/M. In this case, a "Low H/M" is defined with the membership grade of 1.0 when the H/M value is 2 or less and this grade decreases to 0 at the H/M value 7 or above. On the other hand, a "High H/M" means a membership grade of 1.0 at H/M value of 10 or above and 0 at H/M value of 4 or less. Similarly "medium H/M", "is defined as before and given in the Figure 2.

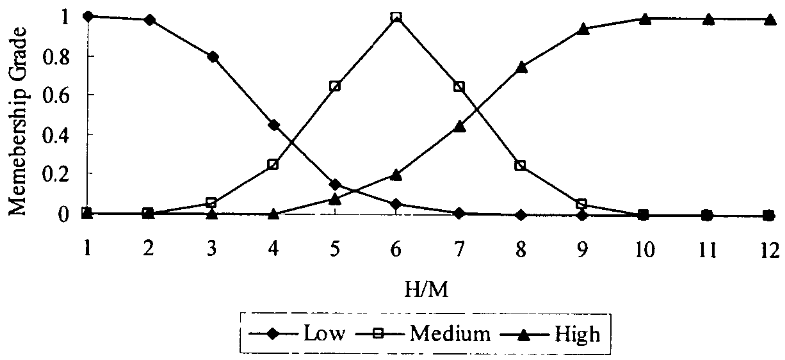

Figure 2. Membership grade of depth to mining height ratio (H/M)

2.4. Fuzzy Membership Grades of P/M Ratio

The ratio of pillar width (P) to mining height (M) is a factor signifying pillar strength. In general, higher ratio implies more load bearing capacity of the pillar and may be deterrent for sinkhole development. Recorded data shows that irregular pillars may exists in the abandoned mines width ranging from 2~3 m to 20 m. Thus a "High P/M" ratio is assigned a membership grade of 1.0 at P/M of 6 and above as shown in Figure 3. A "Low P/M" ratio means a membership grade of 1.0 for P/M less than 1 and this value diminishes to 0 at P/M of 4 or more.

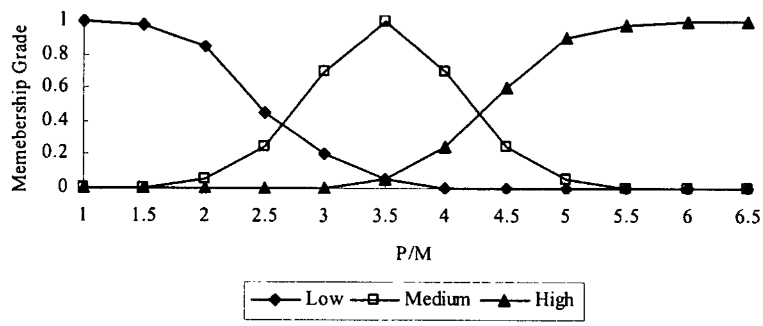

Figure 3. Fuzzy membership grade of P/M ratio

2.5. Fuzzy Membership Grades of Roof Index (R)

Geological conditions of rock strata can be approximated using Geological Strength Index (GSI) or Rock Mass Rating (RMR) or Q value or shear strength. However, it requires underground exposure of roof strata or drill cores of the same. In many cases, underground exposure is not possible due to safety conditions of abandoned mine workings. Hence, drill cores or the observation of surface cracks and depression may lead to some understanding of the overburden characteristics. In this study, a parameter Roof Index (R) is defined to quantify roof conditions based on geological strength index (GSI), thickness of the rock strata, water factor and time factor as given below:

$$R = \frac{\sum_{i=1}^{N} (GSI \times t_i) \Big/ \sum_{i=1}^{N} t_i}{WF \times TF}$$

where,

GSI = geological strength index of ith roof strata

t_i = thickness of ith roof strata

WF = water factor: values: dry – 1.0, partially full – 1.5, completely full – 2.0

TF = time factor: values: 0~10 years – 1.0, 10~30 years – 2.0, more than 30 years – 2.5

N = number of rock strata above the worked seam

The parameter, R is a rough measure of competency of the roof. Higher GSI of rock strata signify competent rock mass and causes higher value of R. Studies show that higher subsidence potential exists within 30 years of mine closure although subsidence may happen after 100 years (Whittaker et al., 1989, Peng et al., 1992, Kratzsch, 1985). Water can wash out broken rock material through the cavity to mine voids and can also deteriorate rock strength. Hence both of these factors adversely affect the roof condition. Practically the value of R as defined above can range from 10 to 60. Figure 4 provides the membership grades of the “Low R”, “Medium R” and “High R”. The membership grade of “Low R” is 1.0 when the value of R is 15 or less and it gradually decreases to 0 when the PRSUP value is 40 or over. On the contrary, a “High R” is defined with the membership grade of 1.0 when the value of R is 55 or above and 0 when the same is 30 or less.

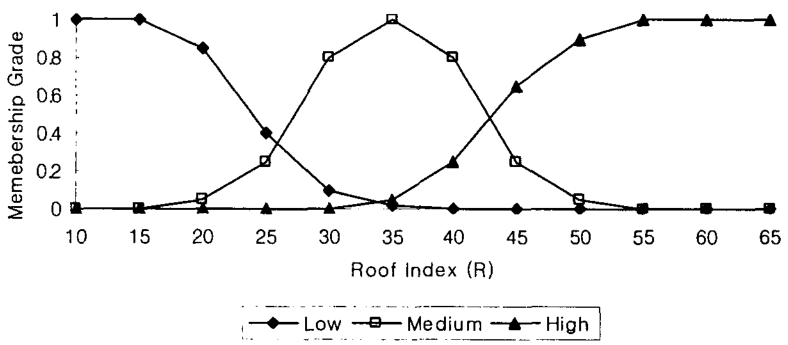

Figure 4. Membership grade of Roof Index, R

2.6. Fuzzy Membership Grades of Sinkhole Possibility (S)

Possibility of sinkhole (S) is defined as an index between 0 to 100, 0 being no possibility and 100 meaning absolute chance of sinkhole occurrences. A possibility index of more than 70 is considered to be in the higher side.

One the contrary, a S value less than 20 signifies lower possibility of sinkhole occurrences. Figure 5 describes five linguistic hedges "Very Low", "Low", "Medium", "High" and "Very High" possibility of sinkhole occurrences. The "Low S" is defined with membership grade of 1.0 when the value of S is 10 or less and that of 0.0 when the S value is 50 and more. On the other hand, "High S" means a membership grade of 1.0 when the S value is 90 or over and that of 0.0 when the S value is 45 or less. The membership grade of "Medium S" is defined as 1.0 for a value of 50 and this grade declines both sides and becomes 0 at S value of 20 and 80. The "Very Low" and "Very High" membership grades are estimated from "Low" and "High" grades as below:

$$\mu_{verylow}(S) = \mu_{low}(S)^3$$
$$\mu_{veryhigh}(S) = \mu_{high}(S)^3$$

2.7. Fuzzy Reasoning and Fuzzy Relational Matrix

This generalized modus ponens is expressed in IF-THEN form as follows:

Premise 1: IF x is A and y is B THEN z is C

Premise 2: x is A' and y is B'

Consequence: z is C'

where, A, B, C, A', B' and C' are fuzzy sets. Here, the first premise or rule establishes the relationship between fuzzy sets A and B with the output fuzzy set C. The premise 2 describes different fuzzy sets A' and B' which can be different from the fuzzy set A and B respectively. Using fuzzy reasoning algorithm, the consequence fuzzy set C' can be obtained. However, for fuzzy reasoning technique, multiple rules in premise 1 are required to perform a flexible reasoning. Based on the recorded data in US, UK, and European mines, following fifteen fuzzy rules are developed as given in Table 1:

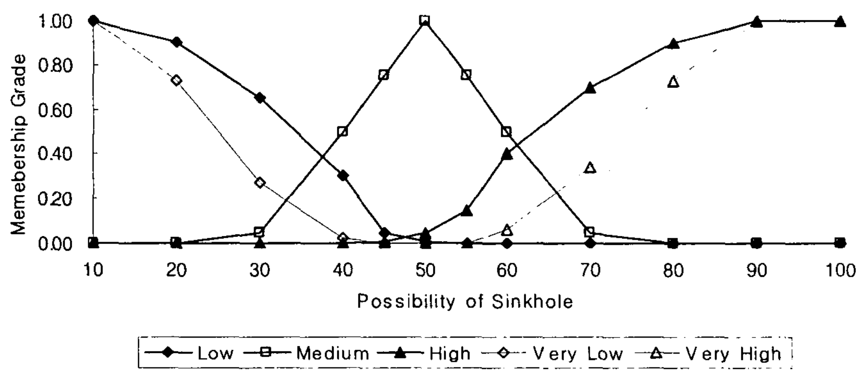

Figure 5. Membership grade of sinkhole possibility

In general, these rules state that for high W, low H/M and low R, possibility of sinkhole occurrences is high. Moreover, higher W and lower P/M may also yield higher possibility of sinkhole index. On the contrary, a low S is expected when lower W, higher H/M and higher R is obtained.

In order to analyze these rules mathematically, each of these rules is transformed into a fuzzy relational matrix. A fuzzy relational matrix, R is derived using Mamdani's method as follows:

Table 1. Fuzzy rules for determining possibility of sinkhole occurrences

No.	IF	W	AND	H/M	AND	P/M	AND	R	THEN	S
1	*If*	High	*and*	Low	*and*	High	*and*	Low	*then*	High
2	*If*	High	*and*	Low	*and*	Low	*and*	Low	*then*	Very High
3	*If*	High	*and*	Low	*and*	Medium	*and*	Low	*then*	Very High
4	*If*	Low	*and*	High	*and*	High	*and*	High	*then*	Very Low
5	*If*	Low	*and*	High	*and*	Medium	*and*	High	*then*	Very Low
6	*If*	Low	*and*	High	*and*	Low	*and*	High	*then*	Low
7	*If*	Low	*and*	Low	*and*	High	*and*	Low	*then*	High
8	*If*	Low	*and*	Low	*and*	Low	*and*	Low	*then*	High
9	*If*	High	*and*	Low	*and*	Medium	*and*	High	*then*	Medium
10	*If*	Medium	*and*	Medium	*and*	Medium	*and*	Medium	*then*	Low
11	*If*	High	*and*	Low	*and*	Low	*and*	Medium	*then*	High
12	*If*	Low	*and*	Medium	*and*	High	*and*	Medium	*then*	Low
13	*If*	Medium	*and*	Low	*and*	Medium	*and*	Medium	*then*	Medium
14	*If*	Medium	*and*	Medium	*and*	High	*and*	Medium	*then*	Low
15	*If*	High	*and*	High	*and*	High	*and*	High	*then*	Low

$F = \text{A and B and C and D} \rightarrow \text{E} = \text{A} \times \text{B} \times \text{C} \times \text{D} \times \text{E}$

$\mu_F(p,q,r,s,w) = \mu_A(p) \wedge \mu_B(q) \wedge \mu_C(r) \wedge \mu_D(s) \wedge \mu_E(w) = \min(\mu_A(p), \mu_B(q), \mu_C(r), \mu_D(s), \mu_E(w))$

where A, B, C and D represent fuzzy set of W, H/M, P/M and R, respectively and E signifies the fuzzy set of S. The variables p, q, r, s and w are the representative values of W, H/M, P/M, R and S respectively. Here the membership grade of fuzzy relational matrix is obtained by selecting the minimum membership grade among the fuzzy sets A, B, C, D and E. If the dimension of the fuzzy set A, B, C, D and E is n then that of the fuzzy relational matrix will be n^5. In this case twelve (12) values are considered for each parameter and thus the dimension of each fuzzy relational matrix is $12^5 = 248{,}832$.

For each fuzzy rule, a fuzzy relational matrix is developed using the technique as mentioned above. Thus for m number of rules, F_i (i = 1, m) fuzzy relational matrices are developed. Then all these fuzzy relational matrices are compiled together to form a global fuzzy relational matrix F as follows:

$$F = F_1 \cup F_2 \cup \ldots \cup F_m = \bigcup_{i=1}^{m} F_i$$

In this technique, for each index in n^5 dimension, the maximum membership grade is selected among the m fuzzy relational matrices. In this process, all the rules are compiled into a single fuzzy relational matrix. Now lets assume the premise 2 is found to be as follows

W is A' and H/M is B' and P/M is C' and R is D'

Where A', B', C' and D' are fuzzy sets. Using these fuzzy sets, the fuzzy set of S, E' can be obtained from the relational matrix, F using max-min operation (Deb, 2003). In mathematical term, it can be expressed as follows:

$$E' = (A' \text{and B'and C'and D'}) \circ F = A' \circ (B' \circ (C' \circ [D' \circ F]))$$

where, $D' \circ F = \max_{z \in Z} \{\min[\mu_D(s), \mu_F(p,q,r,s,w)]\}$ and so on.

For each $z \in Z$, this max-min composition corresponds to the fuzzy conditional statement "if D' then find the output fuzzy matrix by R". The fuzzy membership grade of D' represents the degree of certainty of the input values for R. The fuzzy relational matrix, F signifies the knowledge about the possibility of sinkhole occurrences based on above mention four parameters. Once the entire max-min operation is computed the fuzzy membership grade of S will be obtained.

After the fuzzy set E' is estimated, defuzzyfication of this set can be obtained using a suitable α-cut (α is any value between 0 to 1) and weighted mean of membership grades. This will provide real values of S based on the different α-cuts. If w_i represents the ith value of S and if $\mu_{E'}(w_i)$ is greater than or equal to α then S is expressed as

$$S_\alpha = \frac{\sum_i (w_i \times \mu_{E'}(w_i))}{\sum_i \mu_{E'}(w_i)}$$

3. Results and Discussions

A C++ computer program has been developed to construct fuzzy relational matrix based on the above theories and fuzzy rules. This program also analyzes any premises and provide membership grade of S for twelve different values ranging from 10 to 100. Defuzzyfication value of S for any given value of α can also be obtained from this program.

Based on the fuzzy rules described earlier, the following three premises or conditions of mine are analyzed to show the applicability of fuzzy set theories for the estimation of S:

1. W is *Low* and H/M is *High* and P/M is *Low* and R is *High*
2. W is *Very High* and H/M is *Very Low* and P/M is *High* and R is *Very Low*
3. W is *Low* and H/M is *Low* and P/M is *Medium* and R is *Low*

The mine condition 1 mentioned above is the same as fuzzy rules 6 and those of 2 and 3 represent the variation from any of the fuzzy rules. The results of these analysis is given in Figure 6 and Table 2 shows the defuzzyfication of S for α = 0, 0.5, and 0.9.

The mine condition 1 shows that possibility of sinkhole occurrences will be "Very Low" although rule 6 suggests it to be "Low". This is due to the fact that rules 4 and 5 are little variations of rule 6 and thus has dominated the outcome of mine condition 1. The interaction between fuzzy rules has generated due to the development of fuzzy relational matrix. Thus, fuzzy relational matrix is the combined effect of all fifteen fuzzy rules. The average numerical value of S for 0 α-cut is found to be 27.62. As the value of α-cut increases the average numerical value of S decreases giving 10.0 at α = 0.9 as shown in Table 2. A higher value of α-cut means that we are interested in more specific value of S and a lower α-cut provides average value of S.

The mine condition-2 is the variation of rule 1 having "Very High" W, "Very Low" H/M and "Very Low" R. The obvious outcome will be "Very High" S. The result shows that possibility of sinkhole occurrences is 95 out of 100 point scale for α-cut 0.9. This value is extremely high and thus high possibility exists for sinkhole development. Mine condition-3 is little variation of rules 7 and 8 stating "Medium P/M". This condition is assumed to verify the interaction of fuzzy rules in the relational matrix. The expected outcome will be "High" possibility of sinkhole occurrences. The result shows the same with possibility index of sinkhole ranging from 69.13 to 90.34 for a-cuts 0.0 to 0.9 respectively.

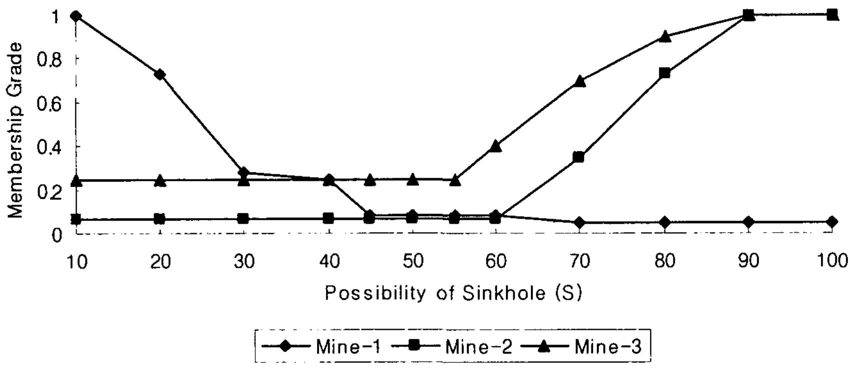

Figure 6. Fuzzy membership grades of S for three different conditions of mines

Table 2. Defuzzification value of S

α-cut	0	0.5	0.9
Mine-1	27.62	14.22	10.00
Mine-2	81.52	90.99	95.00
Mine-3	69.13	86.39	90.34

This analysis proves that it is possible to express complex geological and mining information into fuzzy sets and then fuzzy relational matrix can be built to analyze sinkhole occurrences. This paper outlined the concept of this analysis. However, it requires further investigation by generating more fuzzy rules and verification with field data or numerical modeling.

4. Conclusions

It is no doubt that occurrences of sinkholes can be better expressed using linguistic terminologies rather than numerical numbers. Numerical representation of geology and mining conditions may mean something to an engineer or a scientist who is actively involved in ground control research or in the application of subsidence engineering. For a layman or a field engineer, he is more interested to know whether possibility of sinkhole occurrences at a particular location is "low", or "high" or "medium". In the same token, W, H/M, P/M and R or any other parameter should also be represented using linguistic terminologies which are more accepted in the field.

Fuzzy set theory and fuzzy logic provide strong mathematical and analytical foundation for the analysis of vagueness in data which are represented by "low", "high", "medium", etc. In this paper, an outline is given for the analysis of sinkhole occurrences using four parameters which are represented using three linguistics hedges. This paper also shows that fuzzy reasoning techniques can be used for the analysis of sinkhole occurrences and may provide better understanding of relationship between input parameters to the output variable.

Three different mine conditions are analyzed using the final fuzzy relational matrix developed from the fuzzy rules. Out of these, one condition is exactly the same as one fuzzy rule. It is found that under this circumstance, the output from fuzzy relational matrix is even better and shows correct interaction between fuzzy rules.. Other two roof conditions are different and cannot be matched with any fuzzy rules. In these two cases, forecasting of S is also possible and provides with reasonable accuracy. However, more fuzzy rules have to be incorporated to obtain more accurate results. It is also possible to have contradictory rules when large number of fuzzy rules is incorporated in the analysis. Fuzzy set theory can also analyze such rules in an effective manner although it is better not to have contradiction between the rules.

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