An Economic Design of the Integrated Process Control Procedure with Repeated Adjustments and EWMA Monitoring[1)]

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Abstract

Statistical process control (SPC) and engineering process control (EPC) are based on different strategies for process quality improvement. SPC reduces process variability by detecting and eliminating special causes of process variation, while EPC reduces process variability by adjusting compensatory variables to keep the quality variable close to target. Recently there has been need for an integrated process control (IPC) procedure which combines the two strategies. This article considers a scheme that simultaneously applies SPC and EPC techniques to reduce the variation of a process.

The process disturbance model under consideration is an IMA(1,1) model with a location shift. The EPC part of the scheme adjusts the process, while the SPC part of the scheme detects the occurrence of a special cause. For adjusting the process repeated adjustment is applied by compensating the predicted deviation from target. For detecting special causes the two kinds of exponentially weighted moving average (EWMA) control chart are applied to the observed deviations: One for detecting location shift and the other for detecting increment of variability. It was assumed that the adjustment of the process under the presence of a special cause may change any of the process parameters as well as the system gain.

The effectiveness of the IPC scheme is evaluated in the context of the average cost per unit time (ACU) during the operation of the scheme. One major objective of this article is to investigate the effects of the process parameters to the ACU. Another major objective is to give a practical guide for the efficient selection of the parameters of the two EWMA control charts.

Keywords : SPC, EPC, IMA(1,1), EWMA, average cost per unit time.

1. Introduction

In process industries, the process control problem starts from the fact that quality observations are wandering about due to the inherent disturbance. In order to solve such

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problem we need a process control procedure. The aim of the process control is to obtain the mean value closest to target with smallest variation.

There are two main causes of variation : inherent disturbance and special cause. The inherent disturbance arises from naturally occurring phenomena, or due to unknown cause, and can not be removed economically or technologically. However, it can be controlled by adjustments and such an activity is often called as engineering process control (EPC). A special cause or an assignable cause occurs by change in: plant, processes, operators, materials and other factors. The effect of a special cause is usually large in magnitude, but can be eliminated if detected by monitoring. Such an activity is called as the statistical process control (SPC)

2. Adjustments

Assume the followings.

Z_t ; disturbance from target without adjustment

X_t ; input compensatory variable

Y_t ; output compensation ($= bX_t$, b: system gain)

e_t ; observed deviation from target ($= Y_t + Z_t$)

For controlling the disturbance, we adjust X_t to keep e_t close to 0. Then this kind of adjustment, which adjust the process after observing the output deviation, corresponds to the feedback adjustment. If the adjustment is done at every fixed number of sampling intervals, it is called as the repeated adjustment.

In a repeated adjustment, the adjustment policy to set

$$Y_t = -\hat{Z}_t$$

so that

$$\begin{aligned} e_t &= Z_t + Y_t \\ &= Z_t - \hat{Z}_t \end{aligned}$$

Thus the optimal adjustment policy will be to select $\hat{Z}_t$ as the minimum mean squared error forecast. The adjustment activity is concerned with the statistical estimation.

3 Monitoring

The objective of the monitoring is to detect a SC as early as possible when it has been occurred. The most commonly used control charts are the Shewhart, cumulative cum (CUSUM), and the exponentially weighted moving average (EWMA) charts. We consider two states of control : in-control (IC) state and the out-of-control (OC) state.

4 The IPC procedure

4.1 Disturbance and SC

When both of the disturbance and the special cause can occur in the process, we implement the adjustments and monitoring simultanously. This activity is called as the integrated process control (IPC), in which the adjustments are done by observing the predicted deviations, whereas monitoring is done by the observed deviations. We consider the repeated adjustments and the EWMA control chart for the IPC procedure.

Let Z_t be the unadjusted process disturbance from target during the IC period and be defined as an IMA(1,1) model such as

$$Z_t = Z_{t-1} + a_t - \theta a_{t-1}\ , t = 1,2,3,\cdots$$

where $a_t \sim iid\, N(0,\sigma_a^2)$, and $0 \leq \theta < 1$. For $Z_0 = a_0 = 0$, we have

$$Z_t = \lambda \sum_{j=1}^{t-1} a_j + a_t$$

where $\lambda = 1 - \theta$.

$\hat{Z}_t$: forecast of Z_t

$$\begin{aligned}\hat{Z}_t &= \hat{Z}_{t-1} + \lambda a_{t-1} \\ &= \lambda \sum_{j=1}^{t-1} a_j\end{aligned}$$

The recurrence relation of the forecast is as follows.

$$\hat{Z}_t = \hat{Z}_{t-1} + \lambda e_{t-1}$$

In the OC period, we assume that a special cause occurs at some random time u, and the effects of adjustments during OC period for $t = u + k,\ k = 1,2,3,\cdots$, are considered as follows.

disturbance parameter	$\theta \rightarrow d\theta\, (d \neq 1)$
system gain	$Y_t \rightarrow g Y_t\, (g < 1)$
variability	$a_t \rightarrow v_k a_t\, (v_k > 1)$
disturbance level	$Z_t \rightarrow Z_t + \delta_k \sigma_a\, (\delta_k \neq 0)$

For, $t = u + k,\ k = 1,2,3,\cdots$, the distribution of e_t is obtained as

$$
\begin{aligned}
e_{u+k} &= Z_{u+k} + Y_u + g(Y_{u+k} - Y_u) \\
&= (1 - d\theta - \lambda g)\sum_{j=0}^{k-1} v_k \cdot a_{u+j}(1-\lambda g)^{k-j-1} \quad (v_0 = 1) \\
&+ v_k \cdot a_{u+k} \\
&+ \sigma_a \sum_{j=0}^{k-1} \delta_{j+1}(1-\lambda g)^{k-j-1}
\end{aligned}
$$

For simplicity, we set

$$v_k = v, \text{ for } k = 1,2,\cdots$$

Set δ_k as

$$\delta_k = \begin{cases} \delta, & k = 1 \\ 0, & k > 1 \end{cases}$$

for a step shift, or

$$\delta_k = \delta,\ k = 1,2,3,\cdots$$

for a drifting shift.

The mean of the observed deviation is derived as

$$E(e_{u+k}) = \begin{cases} \sigma_a \cdot \delta \cdot (1-\lambda g)^{k-1}, & \textit{for a step shift} \\ \sigma_a \cdot \delta \cdot \dfrac{1-(1-\lambda g)^k}{\lambda}, & \textit{for a drifting shift} \end{cases}$$

$$\begin{cases} 0 \\ \sigma_a \cdot \dfrac{\delta}{\lambda g} \end{cases}$$

The variance of the observed deviation is also obtained as

$$Var(e_{u+k}) = \begin{cases} \sigma_a^2\left[(1-d\theta-\lambda g)^2\left\{(1-\lambda g)^{2(k-1)} + v^2\dfrac{1-(1-\lambda g)^{2(k-1)}}{1-(1-\lambda g)^2}\right\}\right. \\ \left. + v^2 + \delta^2(1-\lambda g)^{2(k-1)}\right], \textit{ for a step shift} \\ \\ \sigma_a^2\left[(1-d\theta-\lambda g)^2\left\{(1-\lambda g)^{2(k-1)} + v^2\dfrac{1-(1-\lambda g)^{2(k-1)}}{1-(1-\lambda g)^2}\right\}\right. \\ \left. + v^2 + \delta^2\dfrac{1-(1-\lambda g)^{2k}}{1-(1-\lambda g)^2}\right], \textit{ for a drifting shift} \end{cases}$$

$$\begin{cases} \sigma_a^2\left[v^2\left\{(1-d\theta-\lambda g)^2 \cdot \dfrac{1}{1-(1-\lambda g)^2}+1\right\}\right] \\ \sigma_a^2\left[v^2\left\{(1-d\theta-\lambda g)^2 \cdot \dfrac{1}{1-(1-\lambda g)^2}+1\right\}+\delta^2\dfrac{1}{1-(1-\lambda g)^2}\right] \end{cases}$$

4.2 Implementation of EWMA

The EWMA for the mean shift and for the variance increase are

$$E_t^X = r \cdot e_t + (1-r)E_{t-1}^X$$

for $t = 1,2,\cdots$ with $E_0^X = 0$, $0 < r \leq 1$, and

$$E_t^{X^2} = r \cdot e_t^2 + (1-r)max\left\{E_{t-1}^{X^2}, \sigma_a^2\right\}$$

for $t = 1,2,\cdots$ with $E_0^{X^2} = \sigma_a^2$, $0 < r \leq 1$.

An out-of-control signal will be given if

$$|E_t^X| \geq \sigma_a \cdot h_X \cdot \sqrt{\frac{r}{2-r}}$$

or

$$E_t^{X^2} \geq \sigma_a^2\left(1 + h_{X^2} \cdot \sqrt{\frac{2r}{2-r}}\right)$$

4.3 Efficiency of the IPC procedure

The following process and cost parameters are defined for deriving the efficiency.

T_0 : the time which a SC occurs
N_F : no. of false alarms
T_1 : OC run length
S_1 : squared deviations durings OC period
C_M : monitoring cost
C_A : adjustment cost
C_F : false alarm cost
C_T : off-target cost/ square deviation (in unit of σ_a^2)

Then we have the expected increased cost per unit time (ICU)

$$ICU = \frac{R_f \cdot E(N_F) + \frac{E(S_1)}{\sigma_a^2} - E(T_1)}{\frac{1}{p} + E(T_1)}$$

where $R_f = C_F/C_T$

5. Conclusions

The main contribution of this study is to consider various possible changes in the parameters when the process is adjusted under the presence of a special cause. The distribution of e_t leads to the use of the EWMA charts for the mean shift and the variance increase together. A practical guide to (h_X, h_{X^2}) and r are shown to be

$$(h_X, h_{X^2}) = \begin{cases} (3.8, 5.0) \ if \ R_f = 10 \\ (4.2, 6.6) \ if \ R_f = 100 \end{cases} \quad \text{and } r = 0.2 \ or \ 0.3$$

The weight r works uniformly optimal for various values of δ.

For further research, the behavior of IPU for the drifting shift is considered. Also the CUSUM chart approach instead of the EWMA is recommended.