Determination of Physical Camera Parameters from DLT Parameters

Soo Jeong, Changno Lee, Jaehong Oh
Spatial Information Research Team, ETRI
161 Gajeong-dong, Yuseong-gu, Daejeon 305-301, Korea
{soo, changno, ojh}@etri.re.kr

Abstract: In this study, we analyzed the accuracy of the conversion from DLT parameters to physical camera parameters and optimized the use of DLT model for non-metric cameras in photogrammetric tasks.

Using the simulated data, we computed two sets of physical camera parameters from DLT parameters and Bundle adjustment for various cases. Comparing two results based on the RMSE values of check points, we optimized the arrangement of GCPs for DLT.

Keywords: DLT(Direct Linear Transformation), non-metric camera, physical camera parameters

1. Introduction

Recently, non-metric cameras have been used in various photogrammetric tasks, in which low accuracy is allowed. In case of photogrammetry with non-metric cameras, it's not easy to use a rigorous physical model unlike metric cameras because the camera parameters of non-metric cameras cannot be accurately determined. So, in case of non-metric cameras, DLT(Direct Linear Transformation) as an abstract model has been used.

DLT parameters can be converted to physical camera parameters. However, the accuracy of the converted camera parameters is not guaranteed. So, in most cases, the converted camera parameters are used as initial values for the adjustment of physical model.

In this study, we analyzed the accuracy of the conversion from DLT parameters to camera parameters and optimized the use of DLT model for non-metric cameras in photogrammetric tasks.

2. Methodology

1) The determination of DLT parameters

The DLT models the geometric relationship between the image coordinates and the object space coordinates as a linear function (Abdel-Aziz and Karara, 1971; Marzan and Karara, 1975).

The DLT equations can be expressed as follows (Mikhail. et al., 2001):

$$x = \frac{L_1 X_P + L_2 Y_P + L_3 Z_P + L_4}{L_9 X_P + L_{10} Y_P + L_{11} Z_P + 1} \qquad y = \frac{L_5 X_P + L_6 Y_P + L_7 Z_P + L_8}{L_9 X_P + L_{10} Y_P + L_{11} Z_P + 1} \tag{1}$$

Where x and y are the image coordinates; X_P, Y_P and Z_P are the object space coordinates of the point.

The linear solution of DLT requires at least six non-coplanar control points with all three coordinates known. Unlike a non-linear rigorous photogrammetric solution, the image and the object coordinates should be normalized for a reliable solution.

The parameters can be obtained by the following matrix equation.

$$\begin{bmatrix} L_1 \\ \cdots \\ L_{11} \end{bmatrix} = \left(\begin{bmatrix} A_1 \\ \mathrm{M} \\ A_N \end{bmatrix}^T \begin{bmatrix} A_1 \\ \mathrm{M} \\ A_N \end{bmatrix} \right)^{-1} \begin{bmatrix} A_1 \\ \mathrm{M} \\ A_N \end{bmatrix}^T \begin{bmatrix} x_1 \\ y_1 \\ \mathrm{M} \\ y_N \end{bmatrix} \tag{2}$$

where, $N \geq 6$

$$A_i = \begin{bmatrix} X_i\ Y_i\ Z_i\ 1 & 0 & 0 & 0 & 0 & -x_i X_i & -x_i Y_i & -x_i Z_i \\ 0 & 0 & 0 & 0 & X_i\ Y_i\ Z_i\ 1 & -y_i X_i & -y_i Y_i & -y_i Z_i \end{bmatrix}$$

X_i, Y_i, Z_i : Normalized object coordinates

x_i, y_i : Normalized image coordinates

2) Relationship between the DLT parameters and the physical camera parameters

In order to relate the DLT parameters with the physical camera parameters, we first write the projective equations using physical parameters with homogeneous coordinates(Mikhail. et al., 2001, Faugeras, 1999)

$$[x' \ y' \ w]^T = \underset{3\times4}{T} [X' \ Y' \ Z' \ W]^T = T_I \cdot T_E [X' \ Y' \ Z' \ W]^T$$

$$= \begin{bmatrix} 1 & -\cot\theta & -\frac{x_0}{c} & 0 \\ 0 & \frac{K_y}{\sin\theta} & -\frac{y_0}{c} & 0 \\ 0 & 0 & -\frac{1}{c} & 0 \end{bmatrix} \begin{bmatrix} & \underset{3\times3}{M} & & -T_X \\ & & & -T_Y \\ & & & -T_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \\ W \end{bmatrix} \tag{3}$$

where, T_E is the exterior orientation matrix, T_I is the interior orientation matrix, $[x \ y]^T \cong [x' \ y' \ w]^T$

$x=\frac{x'}{w}, y=\frac{y'}{w}$, $[T_X\ T_Y\ T_Z]^T = M[X_C\ Y_C\ Z_C]^T$, θ is nonorthogonality of the image coordinate axes, K_y is a scale difference between the axes, x_0, y_0 are the principal point coordinates, c is the principal distance(focal length), M is a rotation matrix, and X_C, Y_C, Z_C are the coordinates of the perspective center in the object space.

Similarly, the DLT can be expressed in homogeneous coordinates.

$$\begin{bmatrix} x'' \\ y'' \\ w'' \end{bmatrix} = \begin{bmatrix} L_1'' & L_2'' & L_3'' & L_4'' \\ L_5'' & L_6'' & L_7'' & L_8'' \\ L_9'' & L_{10}'' & L_{11}'' & 1 \end{bmatrix} \begin{bmatrix} X'' \\ Y'' \\ Z'' \\ W'' \end{bmatrix} = T' \begin{bmatrix} X'' \\ Y'' \\ Z'' \\ W'' \end{bmatrix} \quad (4)$$

X'', Y'', Z'' : Normalized object coordinates

x'', y'' : Normalized image coordinates

DLT parameters are determined from the normalized coordinates. For directly calculating the physical parameters of original coordinates, we need to compute DLT parameters of original coordinate system(T) using *scale* and *offset*.

$$[x''\ y''\ w'']^T = P[x\ y\ w]^T,$$
$$[X''\ Y''\ Z''\ W'']^T = O[X\ Y\ Z\ W]^T \quad (5)$$
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = T \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = P^{-1}T'O \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{s_x} & 0 & -\frac{o_x}{s_x} \\ 0 & \frac{1}{s_y} & -\frac{o_y}{s_y} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} L_1'' & L_2'' & L_3'' & L_4'' \\ L_5'' & L_6'' & L_7'' & L_8'' \\ L_9'' & L_{10}'' & L_{11}'' & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s_X} & 0 & 0 & -\frac{o_X}{s_X} \\ 0 & \frac{1}{s_Y} & 0 & -\frac{o_Y}{s_Y} \\ 0 & 0 & \frac{1}{s_Z} & -\frac{o_Z}{s_Z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$
$$= \begin{bmatrix} L_1 & L_2 & L_3 & L_4 \\ L_5 & L_6 & L_7 & L_8 \\ L_9 & L_{10} & L_{11} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

Where, $s = scale$, $o = offset$

3) Derivation of the interior orientation parameter

The interior orientation parameters can be derived from the DLT parameters using the relationships between the rows of the orthogonal matrix M as following equation:

$$L^2 = L_9^2 + L_{10}^2 + L_{11}^2$$
$$x_0 = \frac{L_1L_9 + L_2L_{10} + L_3L_{11}}{L^2}$$
$$y_0 = \frac{L_5L_9 + L_6L_{10} + L_7L_{11}}{L^2}$$
$$K_y = \frac{L_5^2 + L_6^2 + L_7^2 - y_0^2L^2}{L_1^2 + L_2^2 + L_3^2 - x_0^2L^2} \quad (6)$$
$$\theta = \frac{L_1L_5 + L_2L_6 + L_3L_7 - x_0y_0L^2}{K_y(L_1^2 + L_2^2 + L_3^2 - x_0^2L^2)}$$
$$c = \sqrt{\sin^2\theta \frac{L_1^2 + L_2^2 + L_3^2}{L^2} - x_0^2}$$

4) Derivation of the exterior orientation parameter

Since the interior orientation parameters were calculated, interior orientation matrix T_I can be formed.

And then, the exterior orientation matrix T_E is

$$T_E = T_I^{-1}T \quad (7)$$

The rotation matrix M can be extracted from the exterior orientation matrix T_E using characteristics of an orthogonal matrix. Orientation angles can be derived from the rotation matrix M.

The camera position is calculated from

$$\begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix} = -\begin{bmatrix} L_1 & L_2 & L_3 \\ L_5 & L_6 & L_7 \\ L_9 & L_{10} & L_{11} \end{bmatrix}^{-1} \begin{bmatrix} L_4 \\ L_8 \\ 1 \end{bmatrix} \quad (8)$$

3. Experimental Results

1) Data

Every measurement inevitably contains errors from various sources. If we use the real measured data, it's not easy to separate the transformation error from the total errors from various sources. So, we made our experiment using simulation data to clarify the source of errors.

The simulated data are made based on the camera parameters using real aerial photograph. As shown in Fig. 1., 25 GCPs and 16 Check points are evenly positioned on the photographs. The image coordinate of GCPs and c check points are simulated using the camera parameters as shown in Table 1. At first, photo coordinates are computed from the 3 dimensional coordinate of GCPs and Check points using the collinearity equation. Then we assumed the photograph is scanned with resolution of 1000 DPI. So, we get the image coordinates of GCPs and check points.

2) Experiments

In this study, we computed camera parameters from DLT parameters for the 7 cases, the number of GCPs and arrangements of which are shown in Fig. 2. As a result of the computation, the camera parameters derived from DLT parameter of 7 cases were all the same with the simulated camera parameters of Table 1. within the errors range of numerical analysis. So, we found that the conversion of DLT parameters to camera parameters would not be affected by the number or the arrangement of GCPs. Also, we found that our conversion method is suitable.

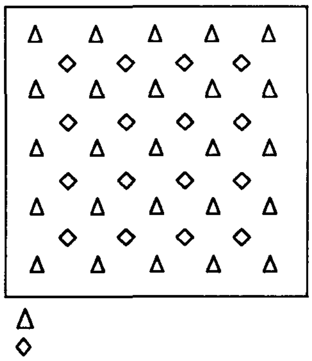

Fig. 1. arrangement of GCPs and Check points

Table 1. simulated camera parameters

parameters	value	Remarks
Ky	1	scale of two image coordinate axes
Theta(Deg)	90	angle of two image coordinate axes
xo(mm)	0.013	photo coordinate center from fiducial center in x-axis
yo(mm)	-0.015	photo coordinate center from fiducial center in y-axis
f(mm)	303.1	camera constant
Xc(m)	173610	lens center position in X-axis
Yc(m)	190930	lens center position in Y-axis
Zc(m)	950	lens center position in Z-axis
w(Deg)	0.5	rotation angle of camera around X-axis
p(Deg)	0.4	rotation angle of camera around Y-axis
k(Deg)	-92	rotation angle of camera around Z-axis

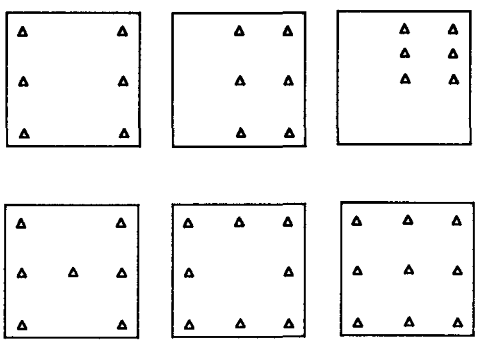

Fig. 2. Cases of GCP number and arrangement

In actual case, every measurement contains observation errors. So, we added observation errors in the image coordinates of GCPs. The observation errors were added by generating random numbers which have normal distribution and standard deviation of 0.5 pixels. To prevent that observation errors were partially concentrated, generation of observation errors and computations were repeated 5 times for each cases of Fig. 2. For each case, camera parameters were computed not only from DLT parameters but also using Bundle adjustments. Then, we computed RMSE value of image points of evenly distributed 16 check points in Fig. 1 using two camera parameters from DLT and Bundle adjustment. Table 2 and Fig. 3 show the results of the computations.

Table 2. RMSE values using two camera parameters from DLT and Bundle adjustment (unit : pixels)

case		1	2	3	4	5	6	7
DLT	Δx	0.91	0.69	9.13	0.30	0.52	0.29	0.18
	Δy	0.99	0.97	8.69	0.30	0.55	0.36	0.18
	Δp	1.35	1.19	12.60	0.42	0.76	0.47	0.26
Bundle	Δx	0.42	0.47	1.42	0.26	0.36	0.25	0.18
	Δy	0.37	0.48	1.50	0.27	0.28	0.38	0.16
	Δp	0.56	0.67	2.07	0.37	0.46	0.45	0.24

※ $\Delta p = \sqrt{\Delta x^2 + \Delta y^2}$

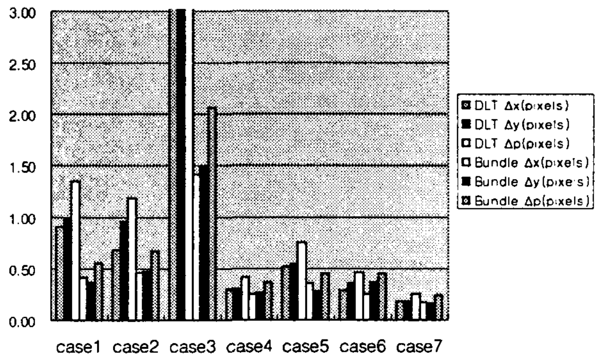

Fig. 3. RMSE values using two camera parameters from DLT and Bundle adjustment (unit : pixels)

Comparing Case 1 to Case 2, we found that RMSEs of Case 2 were better than those of Case 1. Case 1 had 6 GCPs which were evenly distributed only in left and right side, but in Case 2, 6 GCPs were partially distributed in half right side but one GCP was placed in the center of photo.

In Case 3, GCPs were concentrated on right upper side and RMSE values were deviated from allowable ranges.

From Case 4 to Case 7, all RMSE values from Bundle adjustment were less than 0.5 pixels which was the standard deviation value of observation error added in the simulated image coordinates. However, in case of DLT, RMSE values of Case 5 exceeded 0.5 pixels. Comparing

Case 5 to Case 4, Case 5 had one more GCP but it had no GCP in center of photo.

Comparing Case 6 to Case 7, we found, more GCPs, less RMSEs in check points, when the GCPs are evenly distributed. Moreover, in Case 7, the two results from DLT and bundle adjustment were much the same.

However, Case 7 could not be a practical case because that needs too much GCPs. Comparing to Case 4 to Case 6, we could get more good result in Case 4 with less GCPs. It's rather strange. We inferred that two more points of Case 6 compared to Case 4 would not be significant in geometric accuracy. So, we guess that Case 4 would be a practical case that needs not much GCPs in the field.

4. Conclusion and Future Works

In this paper, we analyzed the conversion processes from DLT parameters to camera parameters according to the number and arrangement of GCPs. As the result, we found that we can get suitable result when GCPs are evenly arrange in 4 corners and center.

In the future, we will apply this result to determine the 3 dimensional coordinates from stereo imagery using non-metric cameras such as on-the-shelf digital camera, CCD camera, video camera, etc.

Acknowledgement

This work was supported by the grant from Korea Ministry of Information and Communication (Project title: Development of Integrated Processing Technology for Multi-Sensor Spatial Imagery Information)

References

[1] Abdel-Aziz, Y.I. and H.M. Karara, 1971, *Direct linear transform from comparator coordinates into object-space coordinates*, In ASP Symposium on Close-Range Photogrammetry, Falls Church, VA:pp.1-18, American Society for Photogrammetry.

[2] Faugeras, O., 1999, Three-Dimensional Computer Vision-A Geometric Viewpoint, third prining, The MIT Press.

[3] Marzan, G.T. and H.M. Karara, 1975, *A computer program for direct linear transform of the collinearity condition and some applications of it*, In ASP Symposium on Close-Range Photogrammetry, pp.420-475, American Society for Photogrammetry.

[4] Mikhail, E.M., J.S. Bethel. and J.C. McGlone., 2001, *Introduction to Modern Photogrammetry*, John Wiley & Sons.