Super Resolution Image Reconstruction using the Maximum A-Posteriori Method

Hyuk-Jong Kwon*
Department of Geoinformatic Engineering
Inha University, Incheon 401-751, Korea
(*Tel: 82-32-865-5110; Fax: 82-32-863-1506; E-mail: xg94@korea.com)

Byung-Guk Kim**
Department of Geoinformatic Engineering
Inha University, Incheon 401-751, Korea
(**Tel: 82-32-860-7603; Fax: 82-32-863-1506; E-mail: byungkim@inha.ac.kr)

Abstract: Images with high resolution are desired and often required in many visual applications. When resolution can not be improved by replacing sensors, either because of cost or hardware physical limits, super resolution image reconstruction method is what can be resorted to.

Super resolution image reconstruction method refers to image processing algorithms that produce high quality and high resolution images from a set of low quality and low resolution images. The method is proved to be useful in many practical cases where multiple frames of the same scene can be obtained, including satellite imaging, video surveillance, video enhancement and restoration, digital mosaicking, and medical imaging.

The method can be either the frequency domain approach or the spatial domain approach. Much of the earlier works concentrated on the frequency domain formulation, but as more general degradation models were considered, later researches had been almost exclusively on spatial domain formulations.

The method in spatial domains has three stages: i) motion estimate or image registration, ii) interpolation onto high resolution grid and iii) deblurring process.

The super resolution grid construction in the second stage was discussed in this paper. We applied the Maximum A-Posteriori(MAP) reconstruction method that is one of the major methods in the super resolution grid construction.

Based on this method, we reconstructed high resolution images from a set of low resolution images and compared the results with those from other known interpolation methods.

Keywords: Super Resolution, image enhancement.

1. Introduction

Recently, resolution enhancement approach has been one of the most active research areas, and it is called super resolution(SR) (or HR) image reconstruction or simply resolution enhancement.

The Super Resolution technique is also useful in medical imaging such as computed tomography(CT) and magnetic resonance imaging (MRI) since the acquisition of multiple images is possible while the resolution quality is limited.

In satellite imaging applications such as remote sensing and LANDSAT, several images of the same area are usually provided, and the SR technique to improve the resolution of target can be considered.

We applied the Maximum A-Posteriori(MAP) reconstruction method that is one of the major methods in the super resolution construction.

2. The MAP Estimation

Given the general observation model,

$$Y = Hf + N \tag{1}$$

This is not unique solution for image expansion or enhancement. MAP method is proposed to compute an estimate of the high resolution image. MAP approach to estimating f seeks the estimate $\hat{f}_{MAP}$ for which the *a-posteriori* probability, $\Pr\{f|Y\}$ is a maximum. Formally, we seek $\hat{f}_{MAP}$ as,

$$\hat{f}_{MAP} = \arg\max_f [\Pr\{f \mid Y\}] \tag{2}$$

Where $\Pr\{f|Y\}$ is the *log-likelihood function*. This function can be computed using Bayes' rule.

$$\begin{aligned}\Pr\{f \mid Y\} &= \log\Pr(f \mid Y) \\ &= \log\Pr(Y \mid f) + \log\Pr(f) - \log\Pr(Y)\end{aligned} \tag{3}$$

Applying Bayes' rule yields,

$$\hat{f}_{MAP} = \arg\max_f \left[\frac{\Pr\{f \mid Y\}\Pr\{f\}}{\Pr\{Y\}}\right] \tag{4}$$

And since the maximum $\hat{f}_{MAP}$ is independent of Y we have,

$$\hat{f}_{MAP} = \arg\max_{f}[\Pr\{Y \mid f\}\Pr\{f\}] \qquad (5)$$

Since the logarithm is a monotonic increasing function, this is equivalent to finding,

$$\hat{f}_{MAP} = \arg\max_{f}[\log\Pr\{Y \mid f\} + \log\Pr\{f\}] \qquad (6)$$

Where logPr{Y|f} is the *log-likelihood function* and logPr{f} is the log of the *a-priori density* of f. since Y = Hf + N, it is easy to see that the likelihood function is determined by the probability density of the noise $f_N(\cdot)$ as,

$$\Pr\{Y \mid f\} = f_N(Y - Hf) \qquad (7)$$

Typically since the noise is assumed to be Gaussian, then the use of the natural logarithm in the above derivation removers the exponential term from the density $f_N(\cdot)$. Additionally, it is common to utilize a Markov Random Field prior which has a Gibbs probability density of the form,

$$\Pr\{f\} = \frac{1}{Z}\exp\left(-\frac{1}{\beta}u(f)\right) \qquad (8)$$

Where Z is a normalizing constant, β is the "temperature" parameter of the density and $u(f)$ is the "energy" of f. The use of the logarithm in the formulation for the MAP solution thus greatly simplifies manipulations in these cases.

If the noise is assumed to be Gaussian and the prior is chosen to to be a convex function of f, then it is optimization of (2) is convex, so that the solution $\hat{f}_{MAP}$ assured to exist and is unique. This is a very significant advantage of the MAP formulation.

Schultz and Stevenson extend their earlier work [1] on Bayesian (MAP) image interpolation for improved definition using a Huber Markov Random Field (HMRF) to the problem of super-resolution image. They propose a motion compensated subsampling matrix based observation model which accounts for both scene and camera motion which occurs between images acquisitions. Here we summarize the observation model.

Assume that p (odd) low-resolution frames, $y[m_1, m_2, k]$ with $k \in \{c-\frac{p-1}{2},\ldots,c,\ldots,c+\frac{p-1}{2}\}$ and $m_1 \in \{1,2,\ldots,M_1\}$, $m_2 \in \{1,2,\ldots,M_2\}$ are observed. The objective is to reconstruct a super resolution image $f[n_1,n_2,c]$, the center frame in the observed image sequence. A subsampling model, which models the spatial integration of sensors in the detector array, is proposed for the c^{th} observed frame:

$$y[m_1,m_2,c] = \sum_{n_1=qm_1-q+1}^{qm_1}\sum_{n_2=qm_2-q+1}^{qm_2} f[n_1,n_2,c] \qquad (9)$$

This relationship for the c^{th} (center) frame may be more succinctly written using lexicographic ordering of the LR and SR images as,

$$y_c = H_c f_c \qquad (10)$$

Where $H_c \in R^{M_1M_2 \times q^2M_1M_2}$ is the subsampling matrix relating the SR image f_c with the observed frame y_c. The remaining observed images y_k are related to f_c via motion-compensated subsampling matrices which compensate for the effects of motion occurring between frames as,

$$y_k = H_k f_c + u_k,$$
$$\text{for } k \in \{c-\tfrac{p-1}{2},\ldots,c-1,c+1,\ldots,c+\tfrac{p-1}{2}\} \qquad (11)$$

$H_k \in R^{M_1M_2 \times q^2M_1M_2}$ is the motion-compensated subsampling matrix which relates the k^{th} LR observation to the SR image f_c which is temporally coincident with the center frame in the LR sequence. The vector u_k contains pixels which are unobservable from f_c, but present in f_k. The elements of u_k are not known since f_c is unknown. Notice that rows of H_k which contain useful information are those for which elements of y_k are observed entirely from motion compensated elements of f_c. To improve robustness, rows for which this is not true are removed, yielding a reduced set of equations,

$$y'_k = H'_k f_c \qquad (12)$$

In practice H'_k is unknown and must be estimated from the observed LR frames y_k and y_c. This result in,

$$y'_k = \hat{H}'_k f_c + n_k \qquad (13)$$

Where n_k contain the errors resulting from the use of the estimate $\hat{H}'_k$. The elements of n_k are assumed to be independent identically distributed Gaussian random variables. With p observed frames we have the system of equations:

$$
\begin{array}{lclcl}
y'_{c-\left(\frac{p-1}{2}\right)} & = & \hat{H}'_{c-\left(\frac{p-1}{2}\right)} & f_c & + & n_{c-\left(\frac{p-1}{2}\right)} \\
\vdots & & \vdots & \vdots & & \vdots \\
y'_{c-1} & = & \hat{H}'_{c-1} & f_c & + & n_{c-1} \\
y_c & = & \hat{H}_c & f_c & + & 0 \\
y'_{c+1} & = & \hat{H}'_{c+1} & f_c & + & n_{c+1} \\
\vdots & & \vdots & \vdots & & \vdots \\
y'_{c+\left(\frac{p-1}{2}\right)} & = & \hat{H}'_{c+\left(\frac{p-1}{2}\right)} & f_c & + & n_{c+\left(\frac{p-1}{2}\right)}
\end{array} \qquad (14)
$$

This may be written as a stacked set of equations,

$$Y = Hf_c + N \qquad (15)$$

The SR image f_c is estimated using the MAP criterion as,

$$\hat{f}_c = \arg\max_{f_c}[\log \Pr\{f_c \mid \{y_k\}\}] \qquad (16)$$

which after applying Bayes' rule may be written,

$$\hat{f}_c = \arg\max_{f_c}[\log \Pr\{f_c\} + \log \Pr\{\{y_k\} \mid f_c\}] \qquad (17)$$

Schultz and Stevenson use the Huber Markov random field (HMRF) for the prior term $\log \Pr\{f_c\}$, which is a discontinuity preserving image model, which allows edge reconstruction while imposing smoothness constraints on reconstruction. It is assumed that motion estimation errors between frames are independent thus, the likelihood term may be written in the form $\Pr\{\{y_k\} \mid f_c\} = \prod_k \Pr\{y_k \mid f_c\}$. Taking into account that H_c is known exactly, finding $\hat{f}_c$ requires the solution of the constrained optimization,

Find

$$\hat{f}_c = \mathrm{argmax}_{f_c \in F}[\log \Pr\{f_c\} + \log \Pr\{\{y_k, k \neq c\} \mid f_c\}] \qquad (18)$$

Subject to

$$F = \{f : H_c f = y_c\} \qquad (19)$$

3. Examples

Camera: UNIQ UC 900
Frame grabber: imagination pxd-1000

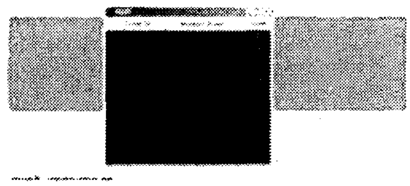

Fig. 1. CCD image capture

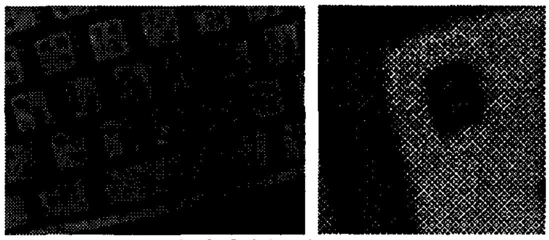

Fig. 2. Original image

Fig. 3. Blurring image(85×85, σ=2)

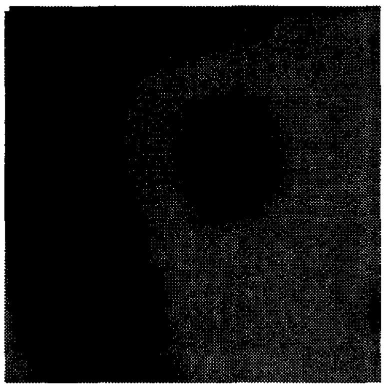

Fig. 4. Nearest neighbor 4× zoom(340×340)

Fig. 5. bilinear 4× zoom(340×340)

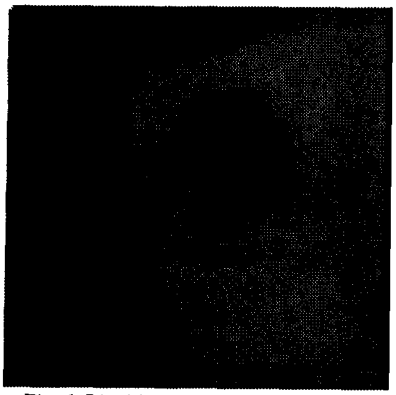

Fig. 6. Bicubic 4× zoom(340×340)

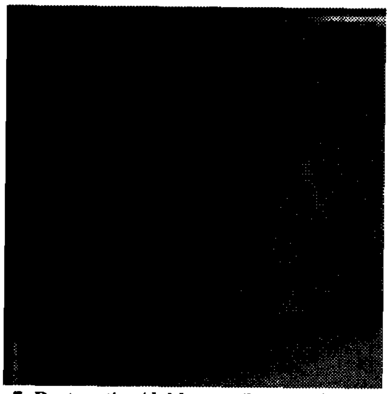

Fig. 7. Restoration/deblurrer (barash, iter=12)

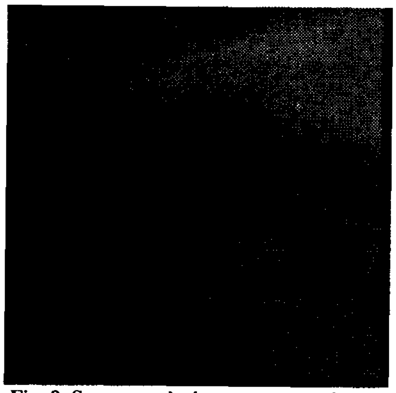

Fig. 8. Super resolution reconstruction

4. Conclusions

The MAP estimation that provides improved definition image expansion and restoration, when compare to various methods such as nearest neighbor interpolation, bilinear interpolation, bicubic interpolation.[Fig.4~8]

The MAP framework allows direct incorporation of a-priori constraints on the solution essential for finding high quality solution to the ill-posed super resolution inverse problem.

The MAP formulation is thus one of the most promising and flexible approaches to super resolution image reconstruction.

References

[1] R. R. Schultz and R. L. Stevenson, "A Bayesian approach to image expansion for improved definition," IEEE Transactions on Image Processing, vol. 3, no. 3, pp. 233–242, 1994.

[2] S. Borman and R.L. Stevenson, "Spatial resolution enhancement of low-resolution image sequences. A comprehensive review with directions for future research," Lab. Image and Signal Analysis, University of Notre Dame, Tech. Rep., 1998.

[3] Sung Cheol Park, Min Kyu Park and Moon Gi Kang, "Superresolution Image Reconstruction - A Technical Overview", IEEE Signal Processing Magazine, Vol. 20, No. 3, pp. 21~36, May. 2003

[4] N. Nguyen, 2000, Numerical Techniques for Image Superresolution, Ph.D.dissertation, Stanford Univ., Stanford CA, Apr. 2000

Thursday
28 October 2004

TA1 Data/Image Processing 3

TA4 KOMPSAT 1

TC1 EMSEA - National Program & Asian Land Monitoring

TD1 EMSEA - Asian Atmosphere & Ocean Monitoring

Interactive Presentations – I (POSTER)

Interactive Presentations – II (POSTER)

TC3 Remote sensing over land 3

TD3 Microwave remote sensing 2

TC4 KOMPSAT 2

TD4 KOMPSAT 3