

Optimization of Intentional Mistuning for Bladed Disk : Damping and Coupling Effect

ByeongKeun Choi*, HyunSeob Lee**, HakEun Kim***, SuJong Keun****

ABSTRACT

In turbomachinery rotor, there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Mistuning effects of the forced response of bladed disks can be extremely large as often reported in many studies. In this paper, the pattern optimization of intentional mistuning for bladed disks considering with damping and coupling effect is the focus of the present investigation. More specifically, the class of intentionally mistuned disks considered here is limited, for cost reasons, to arrangements of two types of blades (A and B, say) and Genetic Algorithm is used to optimize the arrangement of these blades around the disk to reduce the forced response of blade with different damping and coupling stiffness.

1. Introduction

In a dynamic analysis of a turbomachinery rotor, one traditionally has assumed that the blades are identical. But, in practice there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Much of the vast literature on this topic⁽¹⁻⁵⁾ has assumed these differences to be small and to arise either during the manufacturing process and/or as a consequence of in service wear. The motivation for considering such small variations is that their effects of the forced response of bladed disks can be extremely large as often reported in the above studies. Interestingly, the large sensitivity of the tuned system to these small variations has been linked⁽³⁾ to its high level of symmetry.

In this light, it would appear beneficial to design bladed disk not to be tuned, namely to exhibit intentional mistuning, to reduce the sensitivity of the forced response to unintentional mistuning. Certainly, the consideration of intentional mistuning is not new^(1,2) however, in the context of forced response, some papers^(6,7) have only recently investigated the use of harmonic patterns of mistuning. Recently, the authors have investigated and identified the effect of intentional mistuning which can significantly reduce the magnification of the forced response due to unintentional random mistuning.^(8,9)

In this paper, the pattern optimization of intentional mistuning for bladed disks considering with damping and

coupling effect is the focus of the present investigation using the two sets of blades A and B. Genetic Algorithm is used to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks.

2. Optimization Approach

In view of the complexity and cost of intentional mistuning, one should not look simply at set patterns, for example the harmonic patterns⁽⁷⁾ but rather one should optimize the pattern to reduce as much as possible the amplification of the forced response to a given excitation or set thereof. Accordingly, it was suggested by authors that the use of intentional mistuning is probably not a standard design tool but would be very valuable if: (a) it yields a large decrease in sensitivity to unintentional mistuning, and (b) it involves a minimum number of types of blades, ideally 2.^(8,9)

Therefore, in this paper, the disk will first be assumed to support only two different types of blades (blades A and B, say) and their arrangement that yields the smallest amplification of the forced response will be sought with different coupling stiffness and damping. The two sets of blades A and B were selected to have natural frequencies 5% lower and 5% higher, respectively, than the tuned ones (type C) in this paper.

The process described above has some rather dramatic computational implications. Indeed, for each mistuning pattern considered, it requires the determination of the largest amplitude of blade response that can be observed on a disk exhibiting both intentional and unintentional mistuning. At this point in time, however, reliable estimates of this largest amplitude can only be obtained by Monte Carlo simulations that are so computationally

* 경상대학교 기계항공공학부
E-mail : bgchoi@gsnu.ac.kr
Tel : (055) 640-3059, Fax : (055) 640-3188

** 경상대학교 대학원

*** 한국가스기술공업(주)

**** 하이드로텍(주)

expensive that they can only be performed after a problem has been detected and on very simple dynamic models of the bladed disk. Accordingly, a straightforward application of the proposed optimization strategy could only be done for very simple bladed disk models. It was thus proposed to proceed slightly differently by; (1) performing the optimization in the absence of unintentional mistuning, and (2) obtaining a qualitative/quantitative estimate of the sensitivity of a given intentionally mistuned disk to additional unintentional random mistuning.^(8,9) In this manner, the optimization effort as step (1) requires only one forced response evaluation per intentionally mistuned disk considered, as opposed to an entire population.

Mistuning also produces a very nonlinear effect on the forced response. That is, by switching the order of the blades around the disk, dramatic differences can be obtained in the variability of the blade-to-blade amplitudes of vibration as exemplified in particular by the harmonic mistuning analysis of Mignolet et al.⁽⁴⁾ It might thus be suspected that there exists a series of local optima in the complex, high dimensional space over which the optimization must take place. In this light, the present optimization effort has relied on the use of Genetic Algorithm (GA)⁽¹⁰⁾.

3. Simple Genetic Algorithm

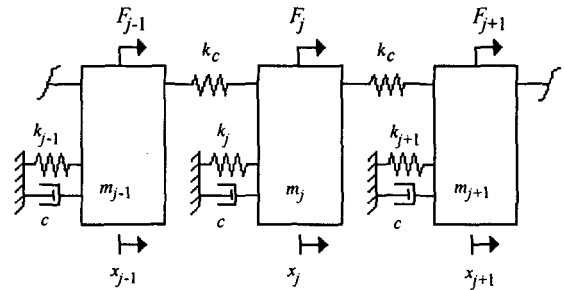
GA are particularly well suited for the present effort because the design variables only admit discrete values (i.e. a specific blade is only of type A or B), see References (10) for further details. The simple genetic algorithm (SGA) used here relies on a population of n_{pop} bladed disks each of which is a random arrangement of N genes (the type A or B of the different blades). Accordingly, each bladed disk can be characterized by a sequence of N A and B letters, for example AABBBAA..., which evolves from one generation to the next according to the rules of selection, crossover, and mutation until all the chromosomes yield essentially similar values of the fitness or objective function (the maximum amplitude of blade response).

The fitness proportionate selection, the single point crossover technique and an exponentially decreasing mutation function was used in the present investigation. Also, the one elite reservation strategy was used in this paper according to which the best disk is retained unchanged from one generation to the next.

4. Simple Bladed Disk Model

To identify the effect of coupling stiffness and damping to optimum pattern of intentional mistuning, the SDOF per blade model shown in Fig. 1 was first considered

with four values of the coupling stiffness ($k_c = 5000, 8606, 20000$ and 45430 N/m) and three values of damping coefficient ($c = 0.143, 0.4, 0.9$ N · s/m) with only 12 blades was considered in this paper. Specifically, each of the N blades is represented as a single mass (m) which is connected to the ground (i.e. the disk) and the aerodynamic and structural coupling between blades are modeled by springs (k_c) and dashpots (c). In the sequel, it is assumed that the coefficient vanishes in this paper. The values of mass ($m = 0.0114$ kg), and stiffness ($k_t = 430,300$ N/m) have already been used in a previous investigations to model a high-pressure turbine stage is used in this paper.



$$F_j = F_0 \cos\left(\omega t + \frac{2\pi r(j-1)}{N}\right)$$

$$m_t = 0.0114 \text{ kg}, k_t = 430,300 \text{ N/m}, c = 0.143 \text{ N}\cdot\text{s/m},$$

$$N = 12 \text{ blades}, F_0 = 1\text{N}, r = 4, k_j: \text{normal}$$

distribution of mean k_t and standard deviation σ

Fig. 1 Single-degree-of-freedom per blade disk model

The computations proceeded as follows. The bladed disk model and engine order of the excitation were first selected. The SGA described above was then used to obtain the intentionally mistuned disk formed of blades A and B such that the maximum of its response over the entire frequency range was the smallest possible.

First, the coupling stiffness effect to optimum pattern of intentional mistuning is identified. The value of the damping coefficient c was set to 0.143 N · s/m (approximately, 0.1% of the critical value) and the two sets of blades A and B were selected to have natural frequencies 5% lower and 5% higher, respectively, than the tuned ones (type C). In analysis, the stiffness of A and B type blade is selected to have 10% lower and 10% higher than the tuned ones. The optimum pattern of intentional mistuning is searched with $k_c = 5000, 8606, 20,000$ N/m (weakly coupling system) and $45,430$ N/m (an average to strong blade-to-blade coupling level) by Genetic Algorithm. Table 1 show the comparison of optimization result by Genetic Algorithm for each coupling stiffness level with the tune system (all C) and

Table 1 The comparison of optimization result by Genetic Algorithm for each coupling stiffness level with the tune and other mistuning pattern

COUPLING STIFFNESS (N/M)	TUNE SYSTEM (ALL C)	OPTIMUM SOLUTION BY GA	OTHER PATTERN
5,000	1.1181e-3 m	2A2B : 1.0541e-3 m (5.7%)	All A : 1.1787e-3m, All B : 1.0684e-3m AB : 1.1307e-3m, 3A3B : 1.081e-3m 4A2B : 1.2041e-3m
8,606	1.10434e-3 m	2A2B : 1.0012e-3 m (9.34%)	All B : 1.0561e-3m, 2BA : 1.0249e-3m 3A3B : 1.061e-3m, CDCBAB : 1.11e-3m
20,000	1.063e-3 m	2A2B : 8.388e-4 m (21.1%)	All A : 1.1139e-3m, All B : 1.0196e-3m AB : 1.2047e-3m, 3A3B : 1.0698e-3m 4A2B : 8.4922e-4m
45430	9.9167e-4 m	7A5B : 7.9776e-4 m (19.56%)	All A : 1.0289e-3m, All B : 9.5615e-4 m 2A2B : 8.5173e-4 m, A2B : 8.42271e-4 m 5A7B : 9.1139e-4 m

other possible mistuning pattern in 4th engine order case.

In Table 1, the optimum pattern that yields small/the smallest value of the largest amplitude of response to a given excitation by GA was changed according to the coupling stiffness level. The genetic optimization algorithm yielded the configuration 7A5B the highest responding blade of which experiences an amplitude of vibration of 0.79 times (reduced amplitude about 19.56%) the tuned value in strong coupling level while 2A2B is searched as optimum pattern in weakly coupling level. The largest amplitude on the disk can be reduced by up to 21.1% by using a disk with an A/B blade pattern as opposed to a tuned one.

Fig. 2 and 3 shows the forced response of optimum, tuned and other A/B patterns with $k_c = 8606$ (weakly coupling system) and 45,430 N/m (strong coupling system). The notation 3A3B refers to disks formed of 2 groupings AAABBB or (AAABBB)2 and similarly 2B1A represents (BBA)4. It is identified that 2A2B(weakly coupling system) and 7A5B(strong coupling system) are better than tuned and harmonic patterns in an amplitude of vibration.

Now, the damping coefficient effect to optimum pattern of intentional mistuning is identified. The value of the coupling stiffness k_c was set to 45,430 N/m and the two sets of blades A and B were also used. The optimum pattern of intentional mistuning is searched with $c = 0.143, 0.4$ and $0.9 \text{ N} \cdot \text{s/m}$ by Genetic Algorithm. Table 2 show the comparison of optimization result by Genetic Algorithm for each damping coefficient with the tune system (all C) and other possible mistuning pattern in 4th engine order case.

The optimum pattern by GA was changed according to the damping coefficient in Table 2. The GA yielded the configuration 7A5B in damping coefficient $c = 0.143$ and 0.4 while 2A2B is founded as optimum pattern in damping coefficient $c = 0.9 \text{ N} \cdot \text{s/m}$. But in case of

$c = 0.9 \text{ N} \cdot \text{s/m}$, the largest amplitude of forced response for optimum pattern by GA, 2A2B have a little bit smaller than that of 7A5B pattern.

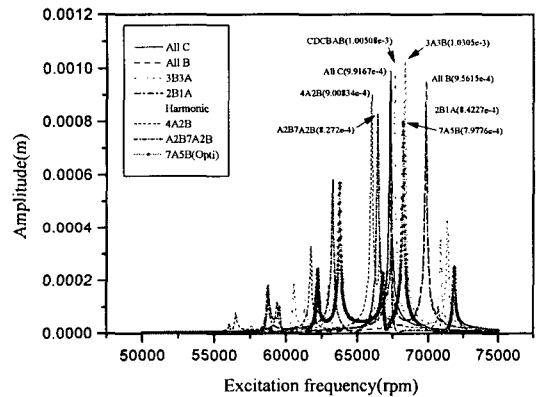


Fig. 2 Comparison with forced response of optimum, tuned and other A/B pattern ($k_c = 45,430 \text{ N/m}$)

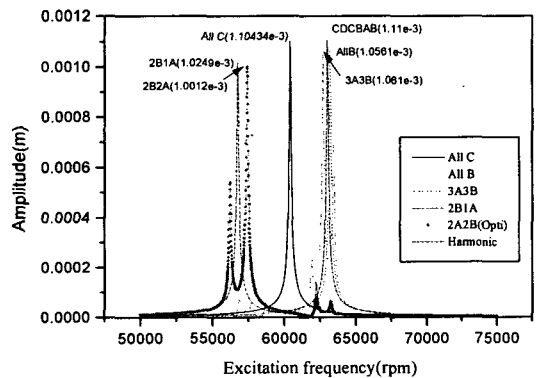


Fig. 3 Comparison with forced response of optimum, tuned and other A/B pattern ($k_c = 45,430 \text{ N/m}$)

tuned and other A/B pattern($k_c = 8,606$ N/m)

Table 2 The comparison of optimization result by Genetic Algorithm for each damping coefficient with the tune and other mistuning pattern

DAMPING COEFFICIENT (N · S/M)	TUNE SYSTEM (ALL C)	OPTIMUM SOLUTION BY GA	OTHER PATTERN
0.143	9.9167e-4 m	7A5B : 7.9776e-4 m (19.56%)	All A : 1.0289e-3m, All B : 9.5615e-4 m 2A2B : 8.5173e-4 m, A2B : 8.42271e-4 m 5A7B : 9.1139e-4 m
0.4	3.5461e-4m	7A5B : 2.9127e-4 m (17.86%)	All A : 3.6879e-4m, All B : 3.4187e-4m 2A2B : 3.1084e-4m, 1A2B : 3.1023e-4m 5A7B : 3.5063e-4m
0.9	1.5761e-4 m	2A2B : 1.432e-4 m (9.14%)	All A : 1.6396e-4m, All B : 1.5194e-4m A2B : 1.45567e-4m, 5A7B : 1.7515e-4m 7A5B : 1.4399e-4m

5. Summary

The investigation of this paper focused on the pattern optimization of intentional mistuning for bladed disks considering with damping and coupling effect using the two sets of blades A and B. Genetic Algorithm is used to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks. Through the optimization, it is found that the optimized pattern considering with damping and coupling effect may or may not appear as variations of simple harmonic patterns, the patterns 7A5B, 2A2B on the 12-blade disk may appear as distorted 1, 2, 3, and 4 harmonics of mistuning but the distortion plays an important role in reducing the amplitude magnification. Also it is identified that the optimum pattern of intentional mistuning by GA was changed according to the coupling stiffness level and damping coefficient in Table 1 and 2. Therefore, the effect of coupling stiffness and damping coefficient should be considered to optimize the intentional mistuning pattern which can reduce the forced response in blade.

Acknowledgements

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