한국소음진동공학회 2004년도 추계학술대회논문집, pp. 295~299.

주기 비안정 연속계의 파라메터공진에 관한 주파수 해석

Frequency Analysis on Parametric Resonance of Periodically Non-stationary Systems with Distributed Parameters

이 용 관*· 체추린**

Lee, Yong Kwan*, S.L. Chechurin**

Key Words : Parametric Resonance(파라메터공진), Distributed Parameter Systems(연속계), Periodically Non-stationary Systems(주기 비안정 스템), Frequency Analysis(주파수해석).

ABSTRACT

본 논문에서는 주기 비안정 연속계의 해석을 위한 주파수 방법이 제안된다. 비안정시스템의 안정화를 위한 기존의 주파수 해석법을 일부 수정하여 연속계를 포함한 비안정 시스템에 적합하도록 수정하였으며, 직류모터와 동기발전기로 구성되어 있는 전기-기계 시스템에 적용하여 유용성을 보였다. 복잡한 비안정 연속계의 문제를 각 요소별 주파수 응답을 분리하고 조합하는 작업들을 통하여 쉽게 풀 수 있음을 보였다. 모터-발전기로 구성되어있는 전기-기계 시스템에서 발전기의 상호유도인덕턴스의 시간에 따른 주기적 변화와 장선(long electrical line)의 부하가 시스템의 불안정성을 야기 함을 보였다.

기호설명

J - moment of inertia for «motor-generator» rotors;
α - angle of rotational shaft;
C_M - electromechanical constant of motor;
C_e - electrical constant of motor;
M - mutual inductance of stator windings and generator rotor;
Z_n - complex resistance of loading (impedance);
i - current of generator stator;
U and I- voltage and current in circuit of constant current motor;
R and L – active resistance and inductance of constant current motor;
$M_{\theta p}$ and M_c – rotational moment of motor and resist moment of generator to the shaft;
ω - angular rotational speed of motor and generator;
I_θ – excitational current of generator rotor;
u – voltage in loading circuit.
$W_n(p) = 1/Z_n$ - transfer function from voltage to the current of the loading circuit.
C_U – maximum value of the mutual inductance
p- Laplace variable
ξ and γ – some parameters depending on operator p,

1. Introduction

.Periodically non-stationary dynamic systems with distributed parameters are widely used in control engineering, electro-mechanics, mechanics, thermo-, hydro-, gas-dynamics and the like. Coordinate control system of robot-manipulator can be considered as a similar example. In this case periodically non-stationary element is MDM (modulator-demodulator) amplifier and resilient shaft of driving gripper serves as an element with distributed parameters. Another example of periodically non-stationary system with distributed parameters is an electro-mechanical transformer which consists of constant current motor and synchronous generator. In this system mutual inductance between stator windings and rotor of the synchronous generator serves as a periodically changing parameter by time, and long electrical line plays a role of element with distributed parameters. Methodology and procedures for solving time-varying systems were presented in many literatures(1~3). Most of stability analysis problems in those literatures are generally based on classical matrix and time manipulation.

This thesis presents methodology to evaluate the stability of such periodically non-stationary dynamic systems with distributed parameters in frequency domain and gives usability of the proposed method with an applied example.

2. Parametric resonance in periodically non-stationary systems with distributed parameters

As the systems with distributed parameters have theoretically infinite number of natural frequencies, parametric resonances in such systems occur in many regions of frequencies as well. Fig. (1) shows the characteristic of parametric resonances by combining parametric circle(4) with frequency responses of systems with distributed parameters.

In case of the systems with distributed parameters it is more convenient to use central parametric circumference than current non-central parametric circumference owing to many cross points of frequency contours of the systems(5). When the periodically non-

• (주) 삼성전기
E-mail : lurpy@hanmail.net

** St. Petersburg State Polytechnic University

stationary element $a(t)$ consists of mean value a_0 and periodically non-stationary signal $a_1(t)$,

$$a(t) = a_0 + a_1(t), \tag{1}$$

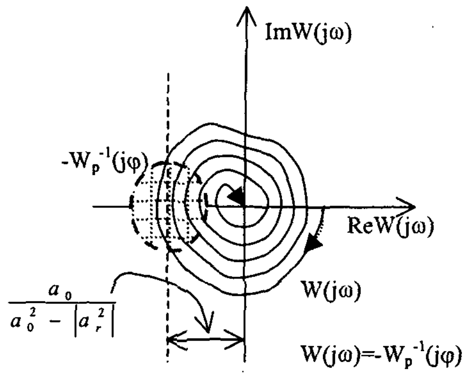

Fig. 1. Condition of parametric resonances for periodically non-stationary systems with distributed parameters in complex plane.

Parametric circle can be positioned on the center of the complex plane by moving mean value of the signal to the non-periodical element. Then, the regions of instability can be easily found and experimentally obtained frequency responses can be also included.

In this case modified non-periodical transfer function becomes

$$W_M(p) = \frac{W(p)}{1 + a_0 W(p)}, \tag{2}$$

Therefore, the first parametric resonance of the system can be found by simple relations;

$$\left|W_M^{-1}\left(j\frac{r\Omega}{2}\right)\right| = |a_r| \quad \text{or} \quad \left|W_M\left(j\frac{r\Omega}{2}\right)\right| = |a_r^{-1}|. \tag{3}$$

Equation (3) can be graphically expressed in both complex and frequency-magnitude planes as shown in fig. 2. The advantage of the modified method is the possibility to use frequency-magnitude graph which is widely used by engineers and scientists in theory and practice. In many cases engineers can hardly find theoretical solutions of complex structures, but experimental results. The proposed method can easily include such experimental data for the analysis of non-stationary systems with distributed parameters.

On the point of transient response, general view of periodically non-stationary dynamic systems with distributed parameters can be depicted as shown in Fig. 3. For the computer evaluation of the non-stationary systems with distributed parameters it is necessary to model distributed parameters by well-known methods such as finite element method. Another way to evaluate time response of the system is to choose specific interesting mode from whole frequency responses and approximate the mode as the second order system

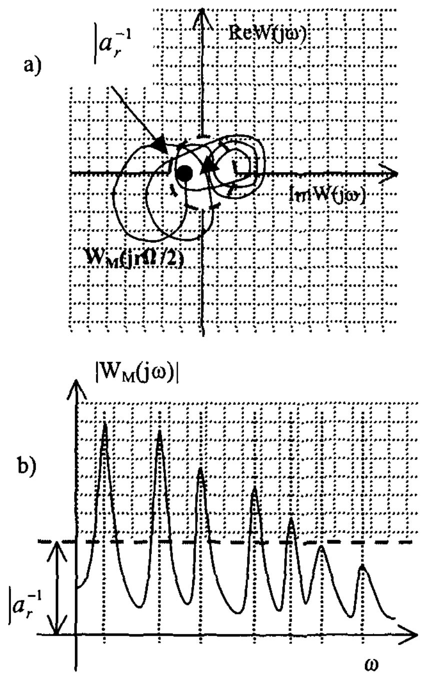

Fig. 2 Central parametric circumference with frequency response of systems with distributed parametrs in complex plane (a) and frequency-magnitude plane (b).

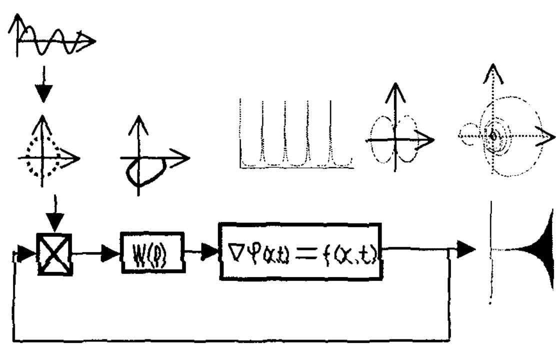

Fig. 3 Block diagram with frequency and time responses of the non-stationary systems with distributed parameters

3. Numerical example

M.L. Levinshtein[6] performed an experimental research of parametric resonance on a synchronous generator with capacitance loading, but he considered a generator without motor. However, "motor-generator" system with long electrical line shall be considered in this thesis. Look at the «motor-generator» system as shown in Fig. 4. In the frame of single-phased synchronous generator, it is not difficult to write the equation of motion with general variables;

$$J\ddot{\alpha} = C_M I - M_c, U = RI + L\dot{I} + C_e\omega,$$
$$M_c = \frac{\partial M}{\partial \alpha} iI_e, \frac{\partial M}{\partial t} I_e = Z_n i \quad , \qquad (4)$$

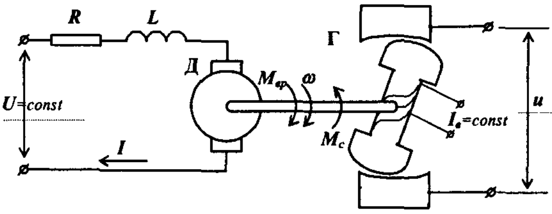

Fig. 4 Motor-Generator electrical transformer system

Alteration of the mutual position between rotor and stator of the generator reflects that the inductance of the generator M can be considered to change by time. Ideal law of alteration M and $\partial M/\partial t$ during constant shaft rotation is generally adapted(7) as shown in Fig. 5.

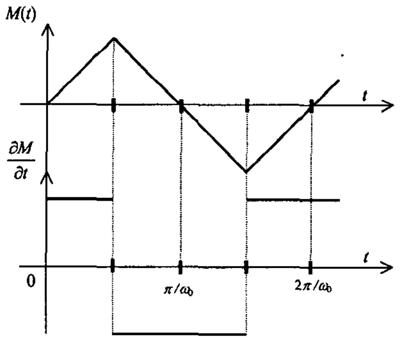

Fig. 5. Alteration character of the mutual inductance according to time during shaft roation

Smooth variation of this dependence is applied in this work.

$$M(t) = C_U \sin\alpha = C_U \sin\omega_0 t,$$

Applying Laplace transform to the system and arranging the equation by the relative angle of rotational shaft α, following nonlinear equation can be obtained;

$$T_a T_M p^3\alpha + T_M p^2\alpha + p\alpha + k_1 W_n(p)\left(\frac{\partial M}{\partial \alpha}\right)^2 p\alpha \qquad , (5)$$
$$+ T_a k_1 W_n(p)\left\{2\frac{\partial M}{\partial \alpha}\left(\frac{\partial^2 M}{\partial \alpha^2}\right)p^2\alpha^2 + \left(\frac{\partial M}{\partial \alpha}\right)^2 p^2\alpha\right\} = \frac{U}{C_e}$$

where $k_1 = T_M I_e^2 / J$, $T_a = L/R$, $T_M = RJ/C_e C_M$.

As the mutual inductance M is the function of rotation angle α, equation (5) is nonlinear. Let us consider transient motion – rotation of the rotor with constant speed ω_0. For the purpose of stability research of the rotation equation in variational form is used. Introducing small variation of angle $\Delta\alpha$ from equal rotation $\alpha=\omega_0 t$, equation (5) can be written in view of

$$T_a T_M p^3\Delta\alpha + T_M p^2\Delta\alpha + p\Delta\alpha + \left\{2k_1 W_n(p)\left(\frac{\partial M}{\partial \alpha}\right)\left(\frac{\partial^2 M}{\partial \alpha^2}\right)\omega_0\right\}\Delta\alpha + \left\{k_1 W_n(p)\left(\frac{\partial M}{\partial \alpha}\right)^2 p\right\}\Delta\alpha +$$
$$+ T_a k_1 W_n(p)\left[2\left\{\left(\frac{\partial^2 M}{\partial \alpha^2}\right)^2\omega_0^2 + \left(\frac{\partial M}{\partial \alpha}\right)\left(\frac{\partial^3 M}{\partial \alpha^3}\right)\omega_0^2 + \left(\frac{\partial M}{\partial \alpha}\right)\left(\frac{\partial^2 M}{\partial \alpha^2}\right)(2\omega_0)p\right\} + \left(\frac{\partial M}{\partial \alpha}\right)^2 p^2\right]\Delta\alpha = 0. \qquad (6)$$

Induced harmonic stationarization(4) of equation (6) gives following excitation condition of first parametric resonance.

$$T_a T_M p^3 + \left(T_M + \frac{1}{2}T_a q\right)p^2 + \left(1 + \frac{1}{2}q\right)p \qquad (7)$$
$$+ \left\{\frac{1}{2}qT_a p^2 + \left(\frac{1}{2}q + p(2T_a)q\right)p + pq\right\}\left(-\frac{1}{2}e^{-j\varphi}\right) = 0 \quad ,$$

where $q = k_1 W_n(p) C_u^{\ 2}$, and $p=j\omega_0=j\Omega/2$, where Ω - alternating frequency of parameter (in this research single pole machine $\Omega = 2\omega_0$).

Equation (7) can be considered as characteristic polynomial with exponential multiplier of some linear stationary systems. Equality condition of left part is analogous to the searching condition on the boundary of system stability with feedback connection which gives following transfer function.

$$W(j\Omega) = \frac{2.5T_a q(j\Omega/2) + 1.5q}{T_a T_M (j\Omega/2)^2 + (T_M + T_a q/2)(j\Omega/2) + (1 + q/2)} \qquad (8)$$

$$W_1(j\varphi) = \left(-0.5e^{-j\varphi}\right). \qquad (9)$$

Hence, in accordance with Nyquist criterion, uniform rotation of the system (motor-generator) rotor with speed ω_0 can be unstable in condition of loading circuit $W_n(p)$ and alternating mutual inductance $M(t) = C_U \sin\omega_0 t$ if Nyquist diagram of the closed-loop transfer function $W(p)W_1(j\varphi)$ encloses point $(-1;j0)$ on imaginary plane.

$$W(j\Omega/2)W_1(j\varphi) = -1 \quad \text{or}$$
$$W_1(j\varphi) = -W^{-1}(j\Omega/2) \qquad (10)$$

Therefore, critical frequencies (and amplitudes) of changing parameter which causes first parametric resonance excitation can be found.

Let us consider electromechanical transformer on long (250m) line with parameters: r_L=1e-6 Ohm/m, l_L=1e-5 H/m, c_L=1e-8 F/m. Lumped loading inductance l_n=0.032 H is assumed to be attached at the end of line. Transfer function of the long line with reactive loading inductance can be written(5) as

$$W_n(p) = \frac{1}{(\sinh\xi/\gamma)\sinh\xi N + \cosh\xi N \cdot R(p)} \qquad (11)$$

where $R(p) = l_n p$ and N indicates the number of finite elements.

Nyquist diagram and amplitude-frequency characteristics of the given system with distributed long electrical line loading is shown in Fig. (6).

Calculation shows that the first parametric resonance can occur at following frequency outskirts: till 5.91; from 151.06 to 151.49; from 302.99 to 303.19; from 455.20 to 455.30 rad/sec. The most interesting resonance is the third one as it is closely located near the nominal frequency of rotation 300 rad/sec.

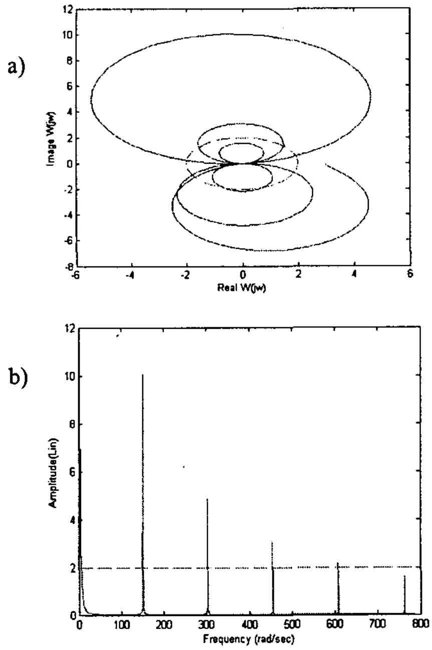

Fig. 6 Conditions of first parametric resonance excitement on electromechanical transformer with long line (250m) loading;

Standard commercial software Simulink (see Fig. 7) was used for the numerical experiment with equivalent transfer functions $W_1(p)$ and $W_3(p)$ as

$$W_1(p) = \frac{\frac{1}{2} k_1 C_U^2 (T_a p + 1)}{T_a T_M p^2 + T_M p + 1},$$

$$W_3(p) = \frac{\frac{1}{2} k_1 C_U^2 (5 T_a p + 3)}{T_a T_M p^2 + T_M p + 1},$$

and $W_2(p) = W_n(p)$.

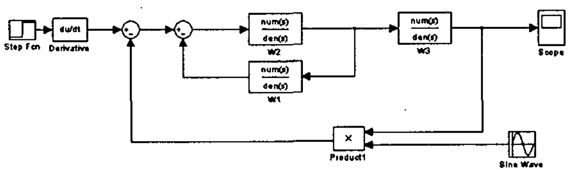

Fig. 7. Block scheme of Simulink for numerical experiment

It is easy to prove that in this case whole transfer function

$$W(p) = \frac{W_2(p)}{1 + W_1(p) W_2(p)} W_3(p)$$

coincides with above equation (8).

In case that the transfer function of loading is hyperbolic it is not possible to use standard simulation tool of Simulink. However for the observation of first parametric resonance it is not always necessary to use exact transfer function of equation (11), but enough to use its approximation around interesting frequency range.

Experiment shows the presence of the first parametric resonance (see Fig. 8).

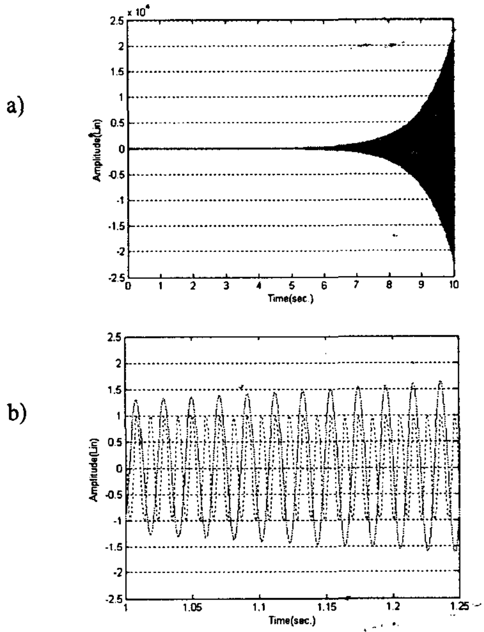

Fig. 8. Oscillogram of the vibration growth for the electromechanical transformer by numerical experiment. (a) general view, (b) fragment of (a) (dotted line – vibration of parameter, solid line – vibration in the system).

4. Conclusion

Frequency analysis of periodically non-stationary systems with distributed parameters was studied and found following basic results.

1) Famous method of «stationarization» of periodically non-stationary systems, which simplifies the search of parametric resonance excitement condition in frequency domain, is modified.
2) Essence of the modification comes from the separation of the stationary elements from the complex non-stationary parts and such procedure gives the central parametric circumference for the non-stationary element.
3) Calculation of electromechanical transformer system, which consists of constant current motor and

synchronous generator with distributed loading, was studied as an example of non-stationary system with distributed parameters.

4) Condition of loss stability of transformer rotation owing to the parametric resonance excitement was obtained.

References

(1) Yakubovich V. A. and Strazhinskii V. M., 1987, "Parametric resonance in linear systems. (in Russian)", Nauka.

(2) Vesnitskii A. I., Krysov S. V. and Patapov A. I., "Experimental research of parametric vibration excitation in one-dimensional mechanical systems. (in Russian)", Applied mechanics, 1980. V. 16, № 12. PP. 122-125.

(3) Rapoport L. B. 1997, "About stability of linear mechanical systems in conditions of parametric excitations. (in Russian)" MTT. 1997. № 5. C. 41-45.

(4) Chechurin S. L., 1983, "Parametric oscillations and stability of periodical motion. (in Russian)", LGU publication.

(5) Lee Y. K., 2004, "Frequency models of periodically non-stationary systems with distributed parameters. (in Russian)", St.Petersburg State Polytechnic Univ. Ph.D dissertation

(6) Levinshtein M. L., "Phenomenon of parametric resonance in works of synchronous machines on capacitance loading. (in Russian)", Transactions of LPI named after Kalinina, 1948, № 3, pp 13–41.

(7) Vazhnov A. I. 1969, "Electrical machines. (in Russian)", Leningrad, Energiya.