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비동위치화된 센서와 액추에이터를 이용한 외팔보의 끝단 진동에 대한 직접속도 피드백제어

Direct Velocity Feedback for Tip Vibration Control of a Cantilever Beam with a Non-collocated Sensor and Actuator Pair

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Key Words : Direct Velocity Feedback Control (직접속도 피드백제어), Non-collocation of Sensor and Actuator (센서 및 액추에이터 비동위치화), Active Control (능동제어), Smart Structures (지능구조).

ABSTRACT

This paper presents a theoretical and experimental study of a non-collocated pair of piezopolymer PVDF sensor and piezoceramic PZT actuator, which are bonded on a cantilever beam, in order to suppress unwanted vibration at the tip of the beam. The PZT actuator patch was bonded near the clamped part and the PVDF sensor, which was triangularly shaped, was bonded on the other part of the beam. This is because the triangular PVDF sensor is known that it can detect the tip velocity of a cantilever beam. Because the arrangement of the sensor and actuator pair is not collocated and overlapped each other, the pair can avoid so called "the in-plane coupling". The test beam is made of aluminum with the dimension of 200 × 20 × 2 mm, and the two PZT5H actuators are both 20 × 20 × 1 mm and bonded on the beam out-of-phase, and the PVDF sensor is 178 mm × 6 mm × 52μm. Before control, the sensor-actuator frequency response function is confirmed to have a nice phase response without accumulation in a reasonable frequency range of up to 5000 Hz. Both the DVFB and displacement feedback strategies made the error signal from the tip velocity (or displacement) sensor is transmitted to a power amplifier to operate the PZT actuator (secondary source). Both the control methods attenuate the magnitude of the first two resonances in the error spectrum of about 6 -7 dB.

1. Introduction

Distributed transducers such as piezoelectric transducers have been widely used in structural control because they are commercially available, compact, light weight, embeddable, not expensive, and disposable transducers, which can be permanently bonded to structures [1]. The distributed property of those transducers like piezoelectric vinylidene fluoride (PVDF) and piezoelectric zirconate titanate (PZT) are known as they can potentially minimise control spillover. They have been used as actuators, sensor, or sensoriactuators (or self-sensing actuators) especially in active vibration control and active structural acoustical control [1].

This research targets ultimately to implement a simpler active control system. The simpler analogue control system can be consisted of sensors, actuators and power amplifiers without digital signal processors and low pass filters. The control system adopts the direct velocity feedback (DVFB) strategy as suggested by Balas using a collocated sensor and actuator system [2].

Conventional point collocated sensor and actuator pairs offer an extremely robust active feedback control system, particularly when the DVFB strategy is implemented. [3,4,5] This strategy is unconditionally stable for any type of primary disturbance acting on a structure, in spite of having a very simple controller.

This point collocation strategy has inspired to a new arrangement with a distributed and collocated sensor and actuator with the same shape on either side of a structure. When a distributed sensor and actuator of the same but arbitrary shape are positioned on the either side of a structure, the transducers pairs are said to be *matched* [6] or *dual* [7]. The dynamics of smart structures with distributed transducers suffer from so-called the "in-plane" coupling, which cannot guarantee unconditional stability with DVFB [8].

In the case of non-collocated systems the "trade-off" between stability and performance has been essential because a better stability for those systems could cause the degradation of performance. H_2 and H_∞ are methods of designing control systems satisfying robust stability and robust performance [9].

However, in this study, a non-collocated piezoelectric sensor-actuator pair is considered to actively control of unwanted vibration with analogue feedback startegies.

Therefore, section 2 of this paper discusses the control property of a point collocated sensor and actuator pair arrangement first with the triangular shping of a PVDF film for the detection of tip motion of a cantilever beam. Also the details of the DVFB control strategy for application to the active control of vibration will be described theoretically in the section. Section 3 shows the design of a non-collocated sensor and actuator pair

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arrangement for feedback control. Section 4 describes the expermental results of the sensor and actuator pair arrangement with two different analogue feedback strategies.

The non-collocated sensor-actuator pair could be useful for the tip position control of a beam-like structure with a simple analogue controller [10].

2. DVFB Control with Collocated sensor-actuator

A transfer function for a vibrational system consists of the infinite number of complex conjugate pairs of poles, indicating the resonances due to the vibration modes, which are a certain boundary condition and properties of the material. The numerator, which is defineded by the location of actuator and sensor, determines the system zeros and influences the phase relation between the system input (actuator) and output (sensor). Thus whether the system is minimum phase or not is also decided by their locations.

As a special case of a minimum phase system [11], a system which is *strictly positive real* (*SPR*) can be defined as "a system for which all the complex conjugate pairs of poles and zeros are alternating each other as well as located in the left hand side in the s-plane". Thus its phase response exists between ±90° as shown in Fig. 1. The real part of the frequency response of such a system of $G(j\omega)$ is always greater than zero for all frequencies, and can be written as [11]

$$\mathrm{Re}[G(j\omega)] > 0, \qquad \omega \in (-\infty, +\infty). \tag{1}$$

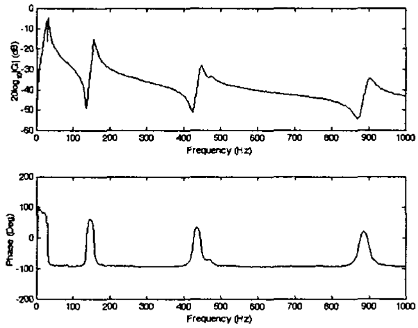


Fig. 1 Measured sensor-actuator FRF of a point collocated system: Shaker input and integrated accelerometer (velocity) output.

Thus the SPR condition provides a very important requirement in the design of unconditionally stable control system with constant gain or DVFB control [2] because it allows stable inversion.

If a single channel feedback control system for disturbance rejection is considered as shown in Fig. 2, the sensor-actuator system is composed by a beam characterized by its plant response $G(s)$, a feedback controller $H(s)$ and a primary source (disturbance).

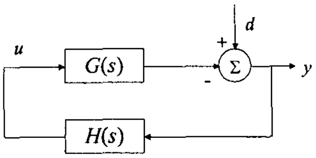


Fig. 2 A single-channel feedback control system for disturbance rejection.

The plant $G(s)$ includes the beam, a secondary source (point force actuator) and an error sensor (point velocity sensor). If the feedback control system is stable, the spectrum of the error sensor output $y(j\omega)$ is related to that of the sensor output before control, $d(j\omega)$, by the expression [11]

$$y(j\omega) = [1 + G(j\omega)H(j\omega)]^{-1} d(j\omega) \tag{2}$$

For this control scheme, the control input to the secondary source (actuator), $u(j\omega)$, is given by [11]

$$u(j\omega) = H(j\omega)[1 + G(j\omega)H(j\omega)]^{-1} d(j\omega) \tag{3}$$

For the DVFB control strategy, the controller is assumed to be a constant gain, so that $H(j\omega) = h$, where h is the feedback gain.

If the actuator and sensor pair is point-collocated, then the plant frequency response $G(j\omega)$ is SPR, since the total power supplied to the uncontrolled system by the actuator must be positive [12]. If a point force actuator and a velocity sensor are used for the control, then $u(j\omega) = f(j\omega)$, where $f(j\omega)$ is the applied force and $y(j\omega) = Q(j\omega)$, where $\dot{w}(j\omega)$ is the measured velocity, and time-averaged power transfer to a mechanical system driven by a point force excitation with a simple harmonic time dependence to the system at a frequency ω can be expressed [12] as

$$\Pi(\omega) = \frac{1}{2}\mathrm{Re}[f^{*}(j\omega)\dot{w}(j\omega)] \tag{4}$$

where f^{*} is the complex conjugate of the applied force. Since $\dot{w}(j\omega) = G(j\omega)f(j\omega)$, equation (4) can be written as

$$\Pi(\omega) = \frac{1}{2}|f(j\omega)|^{2}\,\mathrm{Re}[G(j\omega)] \tag{5}$$

where $G(j\omega)$ is the *mechanical mobility* of the system. The controller is designed to have a positive definite real part at all frequencies since $H(j\omega) = h$ and $h > 0$. Thus the plant of the sensor-actuator and the controller FRF are both SPR so that the velocity feedback control system in Fig. 2 is *unconditionally stable*. In practice, the system under study only approximates a volume velocity sensor and uniform force actuator so that its open-loop FRF $G(j\omega)H(j\omega)$ has to be analyzed with reference to the Nyquist stability condition [11].

It has been known that a triangularly shaped piezoelectric sensor bonded on a cantilever beam as shown in Fig. 3 can detect the tip flexural displacement. The details are summarised as the following. [13]

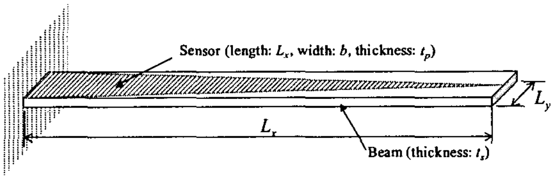


Fig. 3 A triangularly shaped piezoelectric tip position sensor for a cantilever beam.

When an Euler-Bernoulli cantilever beam (length × width × thickness $= L_x \times L_y \times 2h_s$, $L_x >> L_y$) is subject only to flexural motion, the charge output $q(t)$ of a PVDF film sensor bonded on the beam can be written, based on equation (1), as [14]

$$q(t) = e_{31} h_{sen} L_y \int_0^{L_x} S(x,y) \frac{\partial^2 w(x,t)}{\partial x^2} dx \tag{6}$$

where e_{31} is the piezoelectric stress constants, h_{sen} is the distance between the neutral axis of the beam and the PVDF sensor, and $w(x,t)$ is the displacement of the beam.

If the PVDF film sensor is triangular shaped with the dimension of $L_x \times L_y \times h_{pe}$ and bonded on one side of the beam, the spatial sensitivity function $S(x,y)$ of the sensor can be defined for $x = 0$ to L_x by

$$S(x,y) = -k(x - L_x) \tag{7}$$

where $k = L_y / L_x$ is the slope of the triangular shaped sensor. Using integration by parts, equation (6) can be expressed as

$$q(t) = e_{31} h_{sen} L_y \left\{ \left[S(x,y) \frac{\partial w(x,t)}{\partial x} \right]_0^{L_x} - \frac{\partial S(x,y)}{\partial x} \int_0^{L_x} \frac{\partial w(x,t)}{\partial x} dx \right\}, \tag{8}$$

where $\partial S(x,y)/\partial x$ is a constant. Since the spatial sensitivity function $S(x,y)$ with the triangular shaped sensor has the following conditions:

$$S(0,y) = kL_x,\ S(L_x,y) = 0,\ \text{and}\ \partial S(x,y)/\partial x = -k \tag{9}$$

and the boundary conditions of the cantilever beam are given by $w(0,t) = 0$ and $\partial w(0,t)/\partial x = 0$, equation (8) can be rewritten by

$$q(t) = e_{31} h_{sen} L_y k w(L_x, t) \tag{10}$$

where $w(L_x,t)$ is the flexural tip position of the cantilever beam. This triangular PVDF position sensor is predicted to have charge output $q(t)$ proportional to the tip deflection when mounted a cantilever beam. Also the current output $i(t)$ of the PVDF sensor can provide the information of the flexural tip velocity $\partial w(L_x,t)/\partial t$.

As described above, a point collocated sensor and actuator system offers an extremely robust active feedback control system, particularly when the DVFB strategy is applied. A matched pair of triangular PVDF sensor and actuator bonded either side of a cantilever beam was made to take advantage of the attractive SPR property as a point collocated sensor-actuator pair. However it is found that a matched piezoelectric sensor and actuator pair suffers from so-called the "inplane-coupling" problem, which cannot offers unconditional stability in DVFB feedback control. The measured sensor-actuator FRF is plotted in Fig. 4.

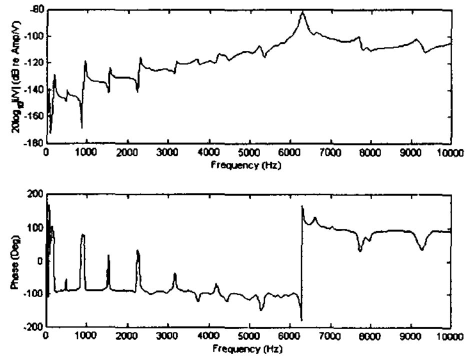


Fig. 4 Measured sensor-actuator FRF with matched triangular piezoelectric sensor and actuator.

In spite of the phase response shows ±90° below about 3500 Hz, a sudden phase change at about 6300 Hz destroys the SPR property with the increasing magnitude with frequency. Therefore, if a higher feedback gain with DVFB control is applied to a matched sensor-actuator pair, the system goes unstable definately.

3. Non-collocated sensor-actuator pairs

Because of the "inplane-coupling" problem of a matched piezoelectric sensor-actuator pair, three different types of non-collocated actuator/sensor pairs for a cabtilever beam are suggested as shown in Fig. 5.

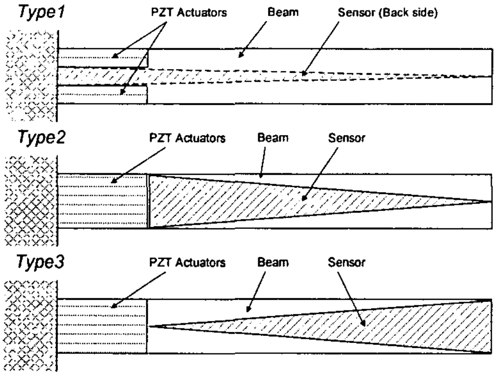


Fig. 5 Three suggested types of non-collocated actuator and sensor pairs.

It is assumed that the actuators are PZT patches and the sensors are triangularly shaped PVDF film to detect the beam tip motion. A steel beam is considered with the dimensions of $L_x \times L_y \times 2h_s = 200\text{mm} \times 20\text{mm} \times 2\text{mm}$ to calculate the suggested actuator/sensor types. The bending moments due to PZT actuation are assumed applied at 20mm away from the clamped ends.

As shown in Fig. 6, since the actuators and sensors are non-collocated, all the calculated responses are all non-minimum phase. The frequency responses of $q(j\omega)/M(j\omega)$ which is proportional to displacement/moment.

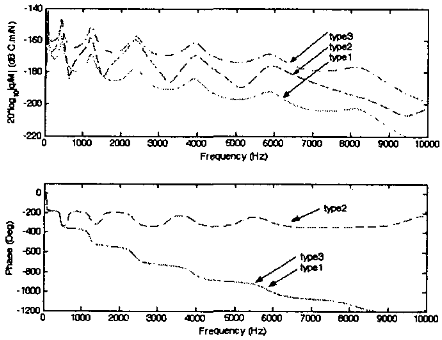


Fig. 6 Frequency responses ($q(j\omega)/M(j\omega)$) of the three suggested triangular sensors, which is proportional to displacement/moment.

Type 1 may be exposed to in-plane motion due to PZT actuators/ motion to in-plane directions, however, because the sensor is not overlapped with the PZT actuators directly, the in-plane coupling might not be so serious. In spite of this problem, type 1 could provide more exact tip displacement of the beam, the magnitude of type 1 response is the smallest compared with other two types. Although type 2 showed an increased charge output compared with type 1, it has a less accumulated phase response which could provide a better stability. Type 3 showed an increased charge output like type 2 and its phase response is accumulated like type 1.

Among the above three types, thus type 2 sensor-actuator pair can be considered with DVFB control or a simple proportional feedback control. But the first mode which shows a sudden phase change must be handled carefully. In this section, type 2 sensor-actuator pair in Fig. 5 will be examined in detail. A pair of PZT patches and a triangular PVDF film sensor are bonded on a cantilever beam. The dimensions are shown in Table 1.

The sensor and actuators are bonded on the beam with a polyester adhesive and a gap of 2 mm is given between the PZT actuator and PVDF sensor as shown in Fig. 7(a).

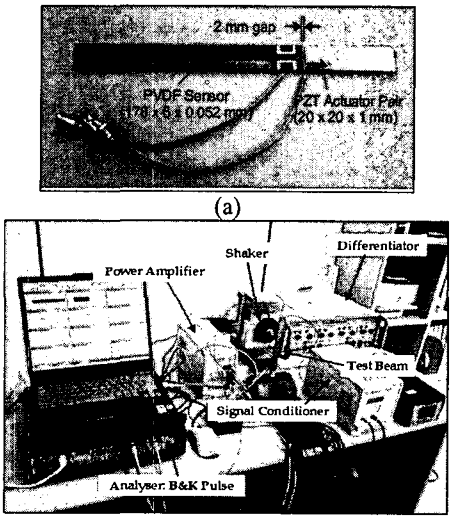
***Table 1** Physical properties of the beam and piezoceramic actuator and piezopolymer sensor.*

	Beam (aluminium)	Piezo actuator (PZT*)	Piezo sensor (PVDF**)
Dimension ($L \times B \times t$)	200 × 20 mm	20 × 20 mm	178 × 6 mm
Thickness	$2h_b = 2$ mm	$h_p = 1.0$ mm	$h_p = 0.052$ mm
Mass density	$\rho_b = 2700\ \text{kgm}^{-3}$	$\rho_p = 7600\ \text{kgm}^{-3}$	$\rho_p = 1780\ \text{kgm}^{-3}$
Young's modulus	$Y_b = 7.1 \times 10^{10}\ \text{Nm}^{-2}$	$Y_p = 6.1 \times 10^{10}\ \text{Nm}^{-2}$	$Y_p = 2 \times 10^{9}\ \text{Nm}^{-2}$

* PZT5H (Morgan Matroc) ** PVDF (Kynar film)

4. Experiment and Discussion

After the completion of the test beam with the non-collocated piezo sensor and actuator in Fig. 7(a), the test beam was installed vertically by a clamping device as shown in Fig. 7(b). The PVDF sensor is connected to a charge amplifier (PCB Dual Mode Vibration Amplifier 441A101) and then the output of the charge amplifier is connected to a differentiator.



(a)

(b)

Fig. 7 (a) A pair of PZT actuators and a PVDF film sensor bonded on the test beam. (b) DVFB control experiment set-up.

The differentiator is applied to obtain the velocity signal of beam tip motion from the PVDF sensor, and its measured frequency response is plotted in Fig. 8.

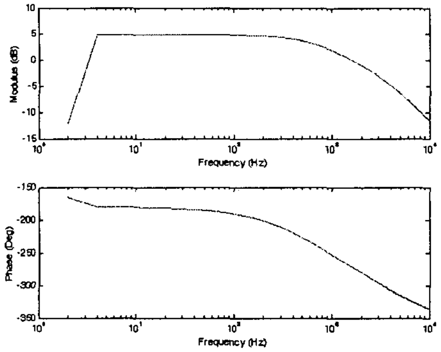


Fig. 8 Measured frequency response of the differentiator used for control.

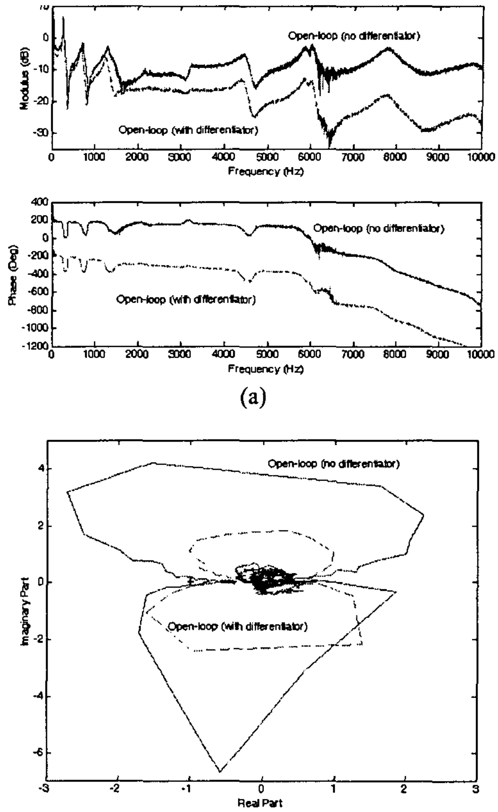
In active control experiment, two feedback strategies have been applied: one is DVFB with the differentiator and the other is a simple displacement feedback without differntiation. That is, the plant in the DVFB control includes the differentiator but the plant in displacement feedback does not.

A shaker (B&K 4810) is attached to the test beam as a primary source at the location of about 80 mm away from the clamping part. The feedback gains in both controls were implemented with a power amplifier (PCB Power Amplifier 790 Series), which was connected from differentiator or directly from the charge amplifier.

As shown in Fig. 9, the measured open-loop responses of the two plants below 10000 Hz with proper feedback gains are plotted. The open-loop FRFs in Fig. 9(a) show that the response without the differentiator (solid line) has a nice phase response without accumulation below about 5000 Hz except at the very first resonance. The response with the differentiator (dashed line) indicates less magnitude especially at higher frequencies due to the differentiator and has steeper phase change.

The open-loop FRFs in Fig. 9(a) also show that the response without the differentiator has more control power to suppress unwanted vibration compared to the other case. The open-loop Nyquist plot in Fig. 9(b) also represents the plant without differentiation can have more feedback gain than that with differentiation.

Fig. 10 show the measured error spectra before and after control, and attenuations with the two different plants at the frequency range of 0 - 2500 Hz. Fig. 10(a) shows active control results of displacement feedback without differentiation, where attenuation is achieved about 7 dB at about 100 - 200 Hz and 2000 Hz, and some reduction were shown at other frequencies. But 4 dB of enhancement was also observed at about 1300 Hz.



(a)

(b)

Fig. 9 Measured open-loop responses of the non-collocated sensor-actuator pair with and without the differentiator.

However, the DVFB control with differentiation as plotted in Fig. 10(b) shows the achieved attenuation of about 6 dB at about 100 - 200 Hz and 2000 Hz, and some reduction were shown at other frequencies. But a strong enhancement was observed was at about 1300 Hz (4th resonance).

This is because the 4th resonant frequency was slightly shifted to a higher frequency, which seemed that as if even higher enhancement was measured.

It is noted that the non-collocated piezoelectric sensor and actuator arrangement can be used for the vibration control of a cantilever beam with a very simple control strategies: DVFB and displacement. Especially the displacement control with the arrangement gaves a better stability and performance than those of DVFB control.

The approach of non-collocated piezoelectric sensor-actuator pair in this paper could be applicable to a tip position tracking control of beam-like structures, although the controller is very simple.

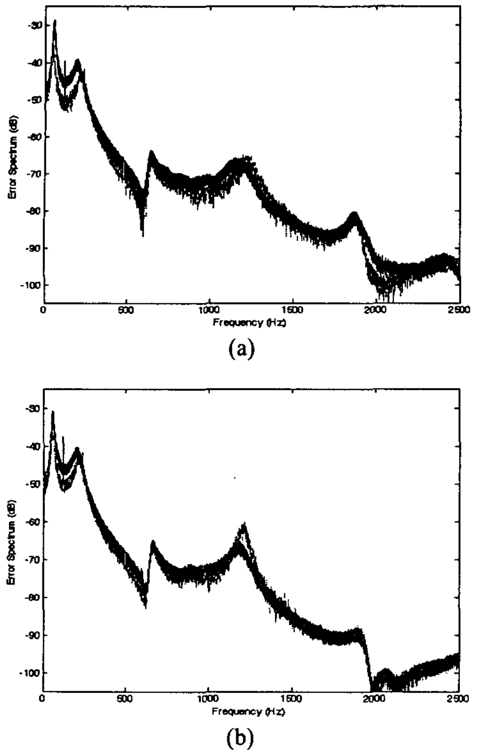


Fig. 10 Non-collocated control results: comparison of measured error spectra. Before control (solid line) and after control (dashed line) (a) Wthout a differentiator. (b) Wth a differentiator.

5. Conclusions

This paper presents a theoretical and experimental study of a non-collocated pair of piezopolymer PVDF sensor and piezoceramic PZT actuator, which are bonded on a cantilever beam, in order to suppress unwanted vibration at the tip of the beam. The PZT actuator patches were bonded out-of-phase at the clamped part and the PVDF sensor, which was triangularly shaped, was bonded on the other part of the beam. This is because the triangular PVDF sensor is known that it can detect the tip velocity of a cantilever beam. At the DVFB control, the non-collocated sensor-actuator pair showed a good active suppression of the beam tip vibration. The displacement feedback control gave a better result then DVFB control with the test beam.

The suggested non-collocated piezoelectric sensor-actuator arrangement could also be applicable to the tip position control of a cantilever beam.

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