

# Reflective Liquid Crystal Cell Parameters Measurement

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## Abstract

We report a method for simultaneous measurements of cell gap, twist angle, birefringence dispersion of liquid crystal (LC) material and alignment directions in filled reflective LC cells. The method is based on spectral measurements of the LC cell reflectivity for three orientations ( $0^\circ, 45^\circ, 90^\circ$ ) of the exit polarizer and may be applied for LC cells with low and high gaps.

## 1. Introduction

Recently reflective liquid crystal displays (LCDs) have found wide applications in various mobile devices, because of their low power consumption and light weight. The twist angle and the retardation are two important parameters that determine the optical performance of conventional twisted nematic (TN) and super twisted nematic (STN) LCDs. In order to get an image of preferred quality, the precise control of these parameters is necessary. Therefore a simple, fast and accurate method to measure the twist angle and the retardation of filled reflective TN (or STN) cells is highly desirable.

Various methods for measuring a reflective twist nematic liquid crystal cell properties have been proposed [1-7]. Some of them use a single wavelength as an optical probe, the other are based on analysis of spectral distribution of light. A major disadvantage of monochromatic methods is the impossibility to find the wavelength dispersion of the retardation (or birefringence). To derive the cell gap, it is necessary to know the birefringence for the wavelength of light used in the measuring. Since the most materials are only supplied with refractive indices at one wavelength (typically 589,3 nm) and if this wavelength differs from the exploitable one, the additional measurement of the birefringence is needed. The same problem exists for the spectral methods, which use certain special points in the spectrum of observed light, for example, the wavelength of null reflectance [3,4].

In this paper we propose a method for measuring retardation of the LC cell, its wavelength dispersion and twisted angle. Alignment directions can be found too. We use almost the same geometry as in the method

introduced by H. L. Ong in his monochromatic measurements [5] and in our analogous spectroscopic method for transparent LC cells [7]. In the paper [7], retardation, its wavelength dispersion and twisted angle are derived from the three spectra of light passed through the LC cell in the configuration, where the entrance polarizer is always oriented at  $45^\circ$  to the entrance LC director, and the exit polarizer is oriented at the angles  $0^\circ$  (parallel),  $45^\circ$  (diagonal) and  $90^\circ$  (perpendicular) to the entrance polarizer direction. In comparison with Ong's monochromatic method, spectral measurements provide a large array of experimental data that makes the spectroscopic method more accurate. This allows us to determine the wavelength dispersion of the birefringence and can be used for finding twist angle and orientation of alignment direction. In addition, our method is not very sensitive to noise or to contribution caused by multi-reflections between layers of the sandwich structure of the LC cell as the monochromatic method is, when coherent light source is used.

The main idea of the proposed method is to find the unknown parameters of the LC cell from quantitative comparison between theoretical calculations and experimental data.

## 2. Theoretical background

From optical point of view twisted nematic liquid crystal is a non-uniform anisotropic medium. In the general case a light wave passed through a layer of this medium changes its state of polarization that depends on optical retardation of the layer of liquid crystal, twist angle, alignment direction and wavelength. Polarization of light reflected from a reflective LC cell can be calculated by using Jones matrix technique. The Jones matrix of a reflected LC cell is:

$$\hat{G} = \hat{M}^T \hat{M}, \quad (1)$$

where  $\hat{M}$  is the Jones matrix of the LC layer, the notation  $T$  indicate a transpose operation.  $\hat{M}$  is given [8]

$$\hat{M} = \hat{R}(\varphi) \begin{pmatrix} \cos X - i \frac{\Gamma \sin X}{2X} & \varphi \frac{\sin X}{X} \\ -\varphi \frac{\sin X}{X} & \cos X + i \frac{\Gamma \sin X}{2X} \end{pmatrix}, \quad (2)$$

where  $\hat{R}(\varphi)$  is a coordinate rotated matrix,  $\varphi$  is the twisted angle,  $X = \sqrt{\varphi^2 + \left(\frac{\Gamma}{2}\right)^2}$ ,  $\Gamma = \frac{2\pi(n_e - n_o)d}{\lambda}$ ;  $n_e, n_o$  are principal refractive indexes of LC,  $d$  is the cell gap,  $\lambda$  is the wavelength. After multiplication (1), we obtain

$$\hat{G} = \begin{pmatrix} A+iB & -iC \\ -iC & A-iB \end{pmatrix}, \quad (3)$$

Where  $A = \cos^2 X - \frac{\sin^2 X}{X^2} \left( \varphi^2 - \frac{\Gamma^2}{4} \right)$ ,

$$B = \frac{\sin 2X}{2X}, \quad C = \frac{\Gamma \varphi \sin^2 X}{2X^2}$$

If the Jones vector of input light is  $\vec{J}_{in} = \begin{pmatrix} g \\ f \end{pmatrix}$ ,

then the Jones vector of output is

$$\vec{J}_{out} = \begin{pmatrix} Ag + i(Bg - Cf) \\ Af - i(Cg + Bf) \end{pmatrix} \quad (4)$$

From Eq. (4) it is easy to see that the polarization state of reflected light depends on  $\frac{d\Delta n}{\lambda}$ ,  $\varphi$  and cell orientation that, as a rule, is described by orientation of alignment direction ( $\alpha$ ). In other words the complex number  $\chi = e^{i\delta} \tan \psi = \frac{J_{out,y}}{J_{out,x}}$  that represents the state of polarization [9] is a function of these parameters. Studying this function we concluded that  $\chi$  uniquely depends on  $\frac{d\Delta n}{\lambda}$  for fixed  $\varphi$  and  $\alpha$ . Typical dependencies of  $\psi$  and  $\delta$  versus  $\frac{d\Delta n}{\lambda}$  and  $\varphi$  for fixed cell orientation are shown in Fig.1 and Fig.2.

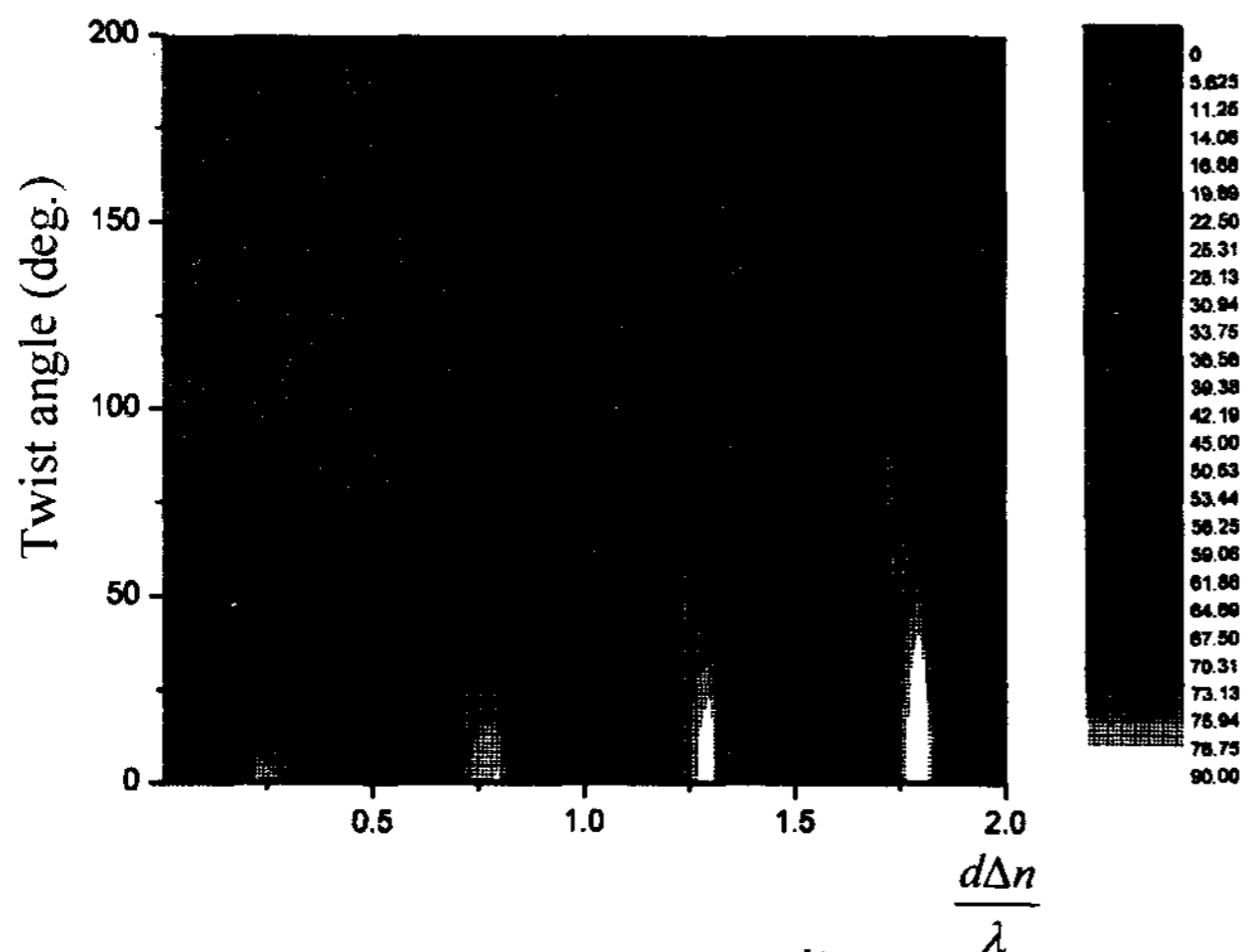


Fig.1 Dependence  $\psi$  versus  $\frac{d\Delta n}{\lambda}$

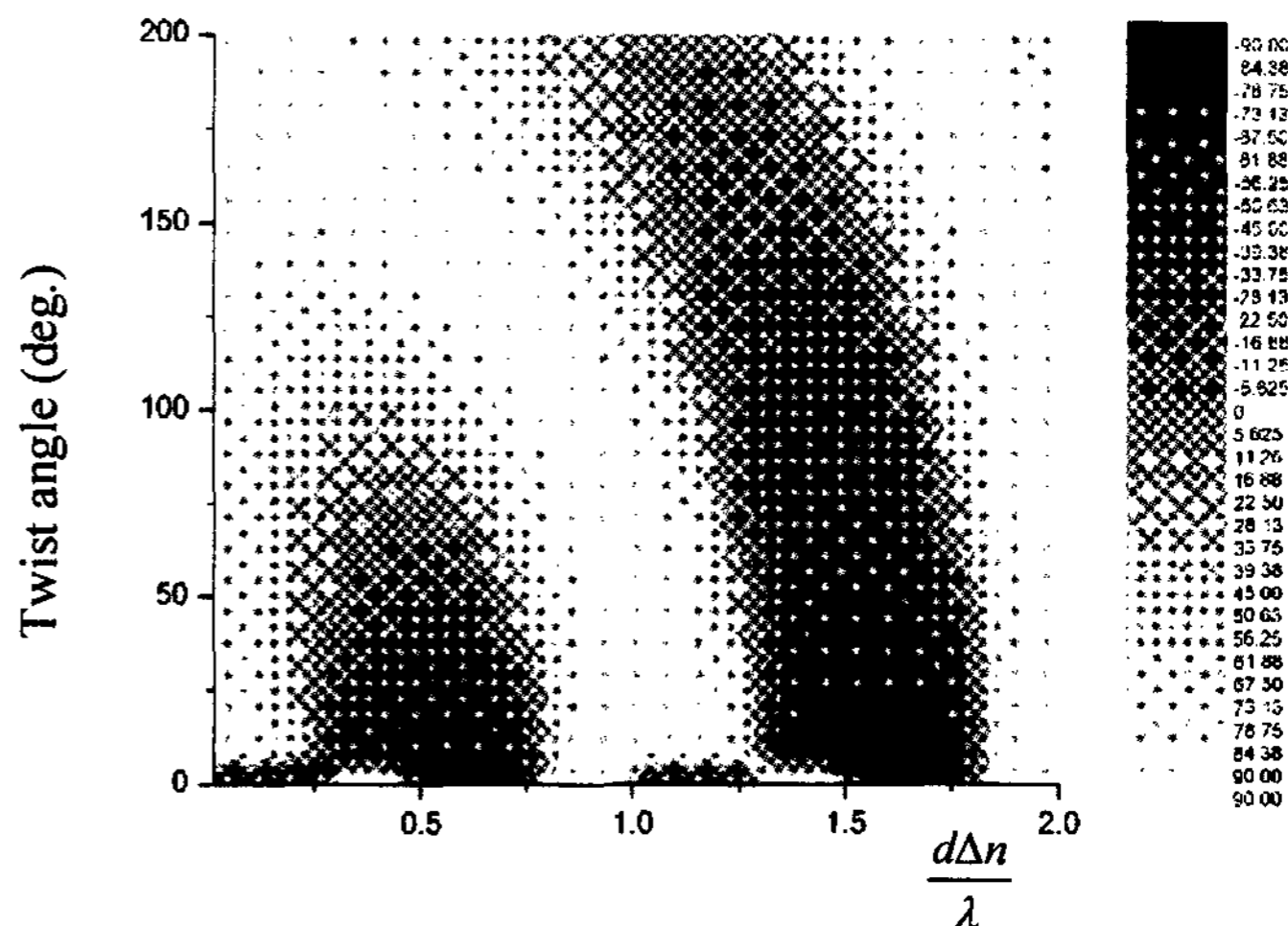


Fig.2 Dependence  $\delta$  versus  $\frac{d\Delta n}{\lambda}$

In these calculations the LC cell orientated at  $45^\circ$  to the transmittance axis of the entrance polarizer.

In fact, spectral dependence of polarization of reflected light from the LC cell is a horizontal trace in the presented above diagrams. The length of this trace depends on the retardation of the LC cell and the measuring spectral range.

Let us show that only three measurements of intensity of reflected light when analyzer parallel ( $I_0$ ), diagonal ( $I_{45}$ ) and perpendicular ( $I_{90}$ ) to entrance polarizer are enough for finding a polarization state of polarized light. Really, having  $I_0, I_{45}, I_{90}$ , coordinates of the Stokes vector of completely polarized light can be found as:

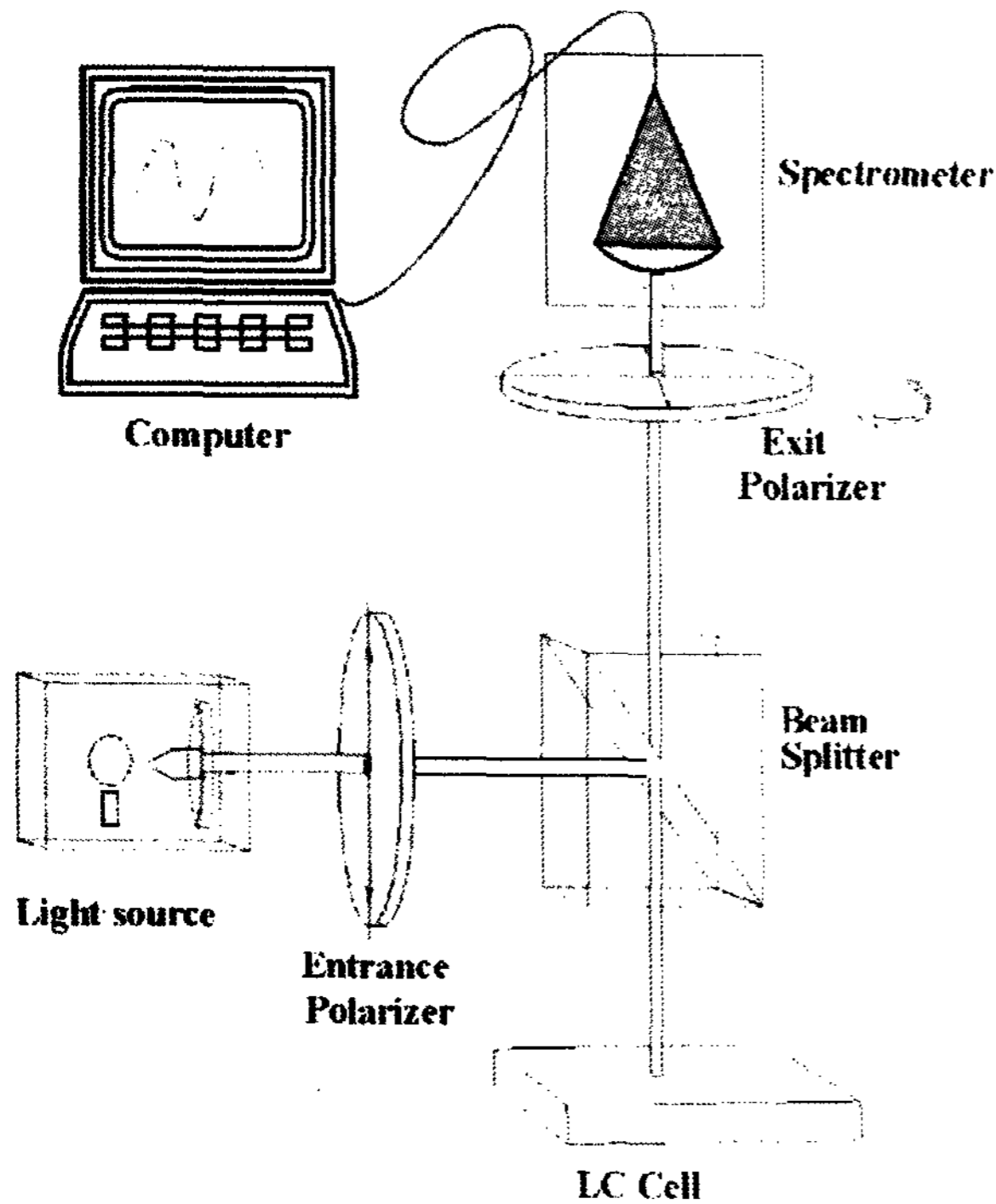
$$S_0 = I_0 + I_{90}, \quad S_1 = I_0 - I_{90}, \quad S_2 = 2I_{45} - I_0 - I_{90},$$

$S_3 = \sqrt{2I_0 I_{90} - 4I_{45}^2 + 4I_{45}(I_0 + I_{90}) - I_0^2 - I_{90}^2}$ . This means that  $I_0, I_{45}, I_{90}$  are unique functions of  $\frac{d\Delta n}{\lambda}$ ,  $\varphi$

and  $\alpha$ . So, having three dependencies  $I_0(\lambda), I_{45}(\lambda), I_{90}(\lambda)$ , parameters  $d\Delta n, \varphi$ , and  $\alpha$  can be found as roots of the system of three nonlinear equations, where measured reflectancies equal to theoretical calculations.

### 3. Experimental setup

The optical setup for measuring parameters of reflective LC cells is shown in Fig.1. The collimated light from an incandescent bulb passes through an entrance polarizer and then reflects to the LC cell in a beam splitter. The light is then reflected back by the reflector imbedded in the inner side of the LC cell. Then it is transmitted through the beam splitter and an exit polarizer. After the exit polarizer (analyzer) the light is registered by a computer-controlled spectrometer that measures the spectrum in the range of 380 to 800 nm. We used a spectrometer from "Ocean Optics".



**Fig.3 Experimental setup**

Due to the accumulated phase retardation, the outgoing light polarization is different from that of the incident beam. Information about the LC cell parameters is obtained from the measured spectra of the reflected light for three orientations of the exit polarizer ( $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ).

#### 4. Measuring procedure

The procedure of measurements contains the following steps:

1) In the beginning we must evaluate reflectivity caused by interfacial reflections. A piece of the covered plate (glass or plastic) treated with ITO is necessary to put on an absorption material (the side with ITO must be in bottom). After that reflectivity of the plate is measured for open and crossed polarizers ( $I_{p0}$ ,  $I_{p90}$ ).

2) Measuring intensities of reflected light for open, diagonal and crossed polarizers ( $I_0$ ,  $I_{45}$ ,  $I_{90}$ )

3) Recalculation of intensities as follow:

$$I_0^* = \frac{I_0 - I_{p0}}{I_0 + I_{90} - I_{p0} - I_{p90}},$$

$$I_{90}^* = \frac{I_{90} - I_{p90}}{I_0 + I_{90} - I_{p0} - I_{p90}},$$

$$I_{45}^* = \frac{I_{45} - \frac{1}{2}(I_{p0} + I_{p90})}{I_0 + I_{90} - I_{p0} - I_{p90}}.$$

4) Solving the system of non-linear equations:

$$\begin{cases} I_0^* \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) = R_0 \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) \\ I_{45}^* \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) = R_{45} \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) \\ I_{90}^* \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) = R_{90} \left( \frac{d\Delta n}{\lambda}, \varphi, \alpha \right) \end{cases}, \quad (5)$$

where  $R_0$ ,  $R_{45}$ , and  $R_{90}$  are calculated reflectivities of the reflected LC cell.

The wavelength dispersion of the birefringence ( $\Delta n(\lambda)$ ) can be approximated by the Cauchy formula [10]

$$\Delta n(\lambda) = A + \frac{B}{\lambda^2}, \quad (6)$$

where A, B are constants. If the birefringence is known for some wavelength  $\lambda_0$  (for example 589,3 nm), one of the coefficients can be expressed through the other:

$$A = \Delta n(\lambda_0) - \frac{B}{\lambda_0^2} \quad (7)$$

#### 5. Results and discussions

From mathematical point of view the problem under consideration is an inverse task. Solving of equations means to find such unknown parameters of a tested LC cell when theoretical and measuring results are sufficiently close one to another. In other words this is an optimisation procedure. To find reflectivities  $R_0$ ,  $R_{45}$ , and  $R_{90}$ , we used the Jones vector obtained in Eq. (4) and the Jones vector of analyser ( $\vec{P}_{an}$ ):

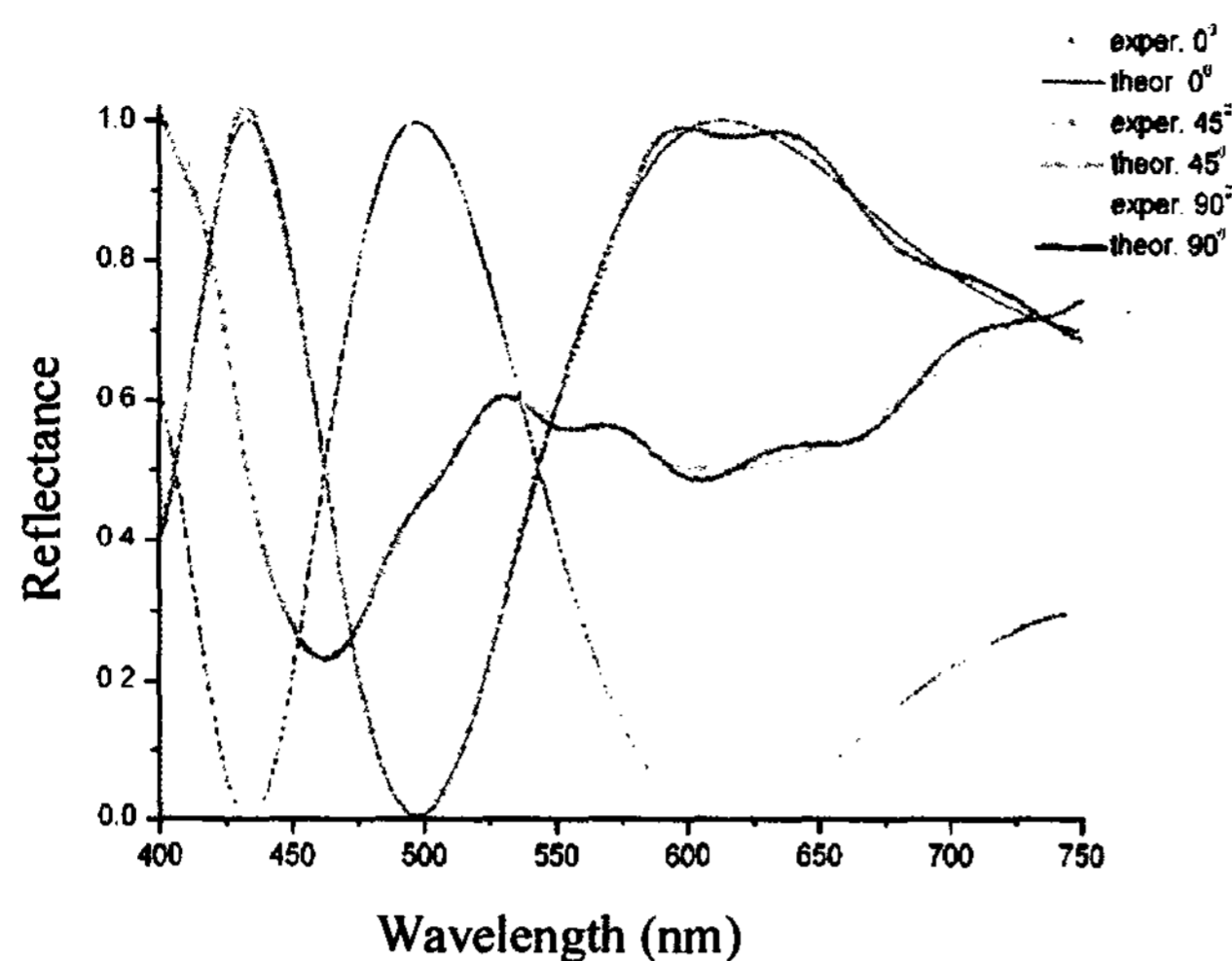
$$R = \left| \vec{P}_{an} \cdot \vec{J}_{out} \right|^2. \quad (8)$$

Several methods exist to solve this optimisation procedure [11]. We used the following: a new function describing the absolute difference between theoretical and experimental results is build:

$$s = \sum_{i=1}^N \left\{ \left( I_0^*(\lambda_i) - R_0(\lambda_i) \right)^2 + \left( I_{45}^*(\lambda_i) - R_{45}(\lambda_i) \right)^2 + \left( I_{90}^*(\lambda_i) - R_{90}(\lambda_i) \right)^2 \right\} \quad (9)$$

$s$  is a function of unknown parameters, that reaches its global minimum (zero in the ideal case) when experimental results equal theoretical ones. It is also understandable that accuracy of measurements and calculations depends on a number of unknown variables. As a rule, the less the number of unknown parameters, the higher accuracy and shorter calculation time for finding the global minimum.

In order to test the proposed method we measured parameters of a reflective LC cell with help the procedure described above. The experimental results and theoretical approximation are shown in Fig. 4. The theoretical curves are solid and smooth. The small oscillations of the experimental curves are explained by interference of multi-reflections from covered plates.



**Fig.4 Experimental and theoretical reflectencies**

The tested LC cell was filled with MLC-6809-000 ( $\Delta n=0,1295$  for  $589,3$  nm). The parameters obtained by this method are  $d=4.12$   $\mu\text{m}$ ,  $\varphi=91^\circ$ ,  $B=0,0089$   $\mu\text{m}^2$ , and  $\alpha=34^\circ$ . Parameters of this cell measured with other methods [1,6] are  $d=4.15$ ,  $\varphi=90^\circ$ ,  $B=0,0092$   $\mu\text{m}^2$ , and  $\alpha=35^\circ$ .

Accuracy of the proposed method depends on both precision in calibration of the optical setup and possibility of the computer program to find correct global minimum. As we mentioned above accuracy of finding of the global minimum is as higher as the less the number of unknown parameters. In the ideal case one or two unknown parameters must be. Moreover, calculation time and measured accuracy depend on the range in which the program must find the unknown parameters.

## 6. Conclusion

We proposed and demonstrated a method for measuring parameters of reflective LC cell such as the cell gap, wavelength dispersion of the birefringence, the twist angle, and entrance director orientation. The method requires three spectral measurements when orientation of the analyzer is  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  relatively to the entrance polarizer. Unknown parameters are found from computer comparison between experimental and theoretical results.

## 7. References

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