A Modified Soft Output Viterbi Algorithm for Quantized Channel Outputs

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Abstract: In this paper, a modified-SOVA (soft output viterbi algorithm) of turbo codes is proposed for quantized channel receiver filter outputs. We derive optimum branch values for the Viterbi algorithm. Here we assume that received filter outputs are quantized and the channel is additive white Gaussian noise. The optimum non-uniform quantizer is used to quantize channel receiver filter outputs. To compare the BER (bit error rate) performance we perform simulations for the modified SOVA algorithm and the general SOVA.

1. INTRODUCTION

Since turbo codes is introduced in [1], researchers have been interested in finding good encoders and good decoding algorithms which is efficient to the iterative decoding. The well known decoding algorithms are the maximum a posteriori (MAP) [2], the log-MAP, the max-log-MAP [3], and the soft output Viterbi algorithm (SOVA) [4].

Among all the above algorithms, SOVA algorithm has some advantages in the aspect of the complexity and the speed of decoding. SOVA is operated with additions and subtractions instead of multiplications. Memory requirement of SOVA is much less than that of MAP.

Due to these advantages, in the real-time decoder, it is likely that SOVA would be implemented using quantized data. Recently a research [5] is performed to evaluate the performance for the SOVA algorithm with the quantized receiver filter output. The optimal range of quantization has been studied with the given levels of quantization. In [6], we find some different kinds of quantizers, uniform, non-uniform and Logarithmic quantizer. With the quantized receiver filter output, the decoding algorithm with the approximated conditional probability derived from the pdf of continuous received filter output is normally used. In this case, we can expect that there exist some performance degradation since the maximum likely function is not derived from the PMF (probability mass function) of quantized receiver filter output. Paper [8] has proposed a modified MAP algorithm with the maximum likely function derived from PMF.

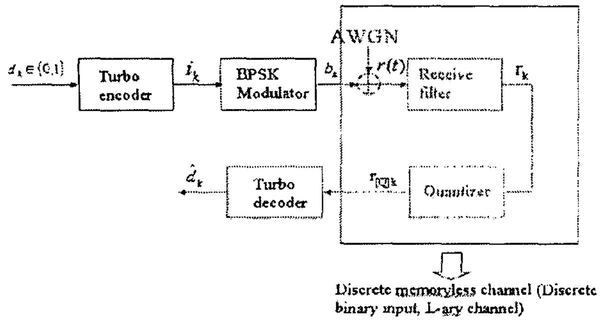
In this pâper, we consider the modified SOVA algorithm with the maximum likely function derived for the PMF of the quantized received filter output. Here we cosider the optimum nonuniform quantizer (Lloyd-Max quantizer [11]) instead of the uniform quantizer. The non-uniform quantization can reduce the quantization error. The modified SOVA algorithm for the quantized output achieves the same performance as the SOVA algorithm operating on continuous data as the number of quantized levels is going to the large value. Simulation results show that, with the quantizer which has more than four bits quantization, the modified SOVA achieves the performance which is almost equivalent to that of the general SOVA with the continuous channel output.

The remainder of the paper is organized as follows: The system model & channel is introduced in Section II. In Section III, the modified SOVA for the quantized channel with the optimum nonuniform quantizer is expained. In Section IV various simulation results are presented. Finally, Section V concludes.

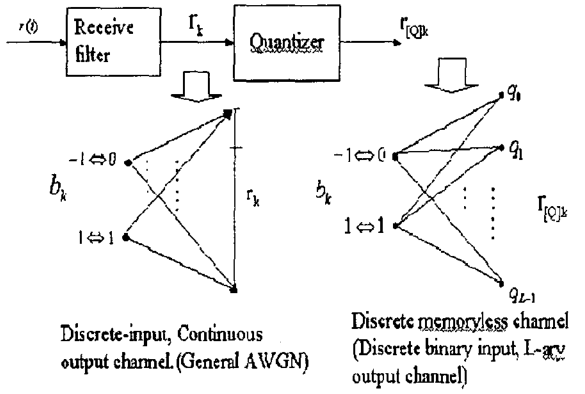
2. THE SYSTEM MODEL

A block diagram of turbo coded system considered in this paper is shown in Figure-1. Information bits $d_k = 0$ or $d_k = 1$ are produced with equal probabilities. We consider the system which consists of two identical parallel recursive systematic convolutional (RSC) encoders whose octal generators are $(7,5)_8$ with code rate 1/2, and constraint length 3. Source inputs d_k's are grouped into a frame of 512 bits. A pseudo random interleaver is implemented between two component encoders The encoded binary symbols are assigned to the signals of antipodal-BPSK.

The only additional block to the general system model of turbo code is the quantizer which changes the continuous receive filter output r_k into the corresponding discrete output $r_{[Q]k}$. The quantized data $r_{[Q]k}$ takes values from the discrete set $Q = \{q_0, q_1, \cdots\cdots, q_{L-1}\}$. There are $L = 2^n$ quantization levels for n bits quantizer. With this quantization the channel could be modeled as a discrete channel shown in Fig.2 described by a set of transition probabilities. For simplicity of the system model, information bits $i_k = \{0,1\}$ is modulated into the channel symbols $b_k = \{-1,1\}$. Quantizer converted receive filter output into quantized output.



[Figure-1 System model]



[Figure-2 AWGN and DMC]

After the quantization, the AWGN channel is changed to DMC (Discrete memoryless channel). DMC channel has two discrete-input levels and 2^n discrete-output levels.

3. DECODING ALGORITHM & OPTIMUM NONUNIFORM QUANTIZER

For a DMC channel, we consider a SOVA decoding algorithm based on channel transition probabilities. Let I be a set of path numbers in a trellis of a convolutional code, i be its element which is assigned to each path as $S^{(i)}_{k+l} = \{s^{(i)}_0, s^{(i)}_1, s^{(i)}_2, \cdots\cdots, s^{(i)}_{k+l}\}$,

where $l(0 \le l \le D)$ represent a position in the trace-back depth , D be a trace-back depth, The a posteriori probability is given by

$$P(S^{(i)}_{k+D}|R_{k+D}) = P(R^{(i)}_{k+D}|S_{k+D})\cdot\frac{S^{(i)}_{k+D}}{R_{k+D}} \quad (1)$$

Since the probability $P(R_{k+D})$ is independent of i, SOVA algorithm is then modified to

$$\max_{i\in I} P(S^{(i)}_{k+D},R_{k+D}) = \max_{i\in I} P(R^{(i)}_{k+D},S_{k+D})\cdot P(S^{(i)}_{k+D}) \quad (2)$$

After determining the Survivor path based on the received signal up to time $k+D$, an information symbol d_k at time k is decoded

The joint probability in Eq (2) at time j can be calculated from the product of branch transition probability by

$$P(S^{(i)}_{k+l},R_{k+l}) = P(S^{(i)}_{k+l-1},R_{k+l-1})\cdot\gamma(R^{(i)}_{k+l},s^{(i)}_{k+l-1},s^{(i)}_j) \quad (3)$$

where $P(S^{(i)}_{k+l-1},R_{k+l-1})$ is path transition probability at the time of $k-l-1$ to time $k+l$ branch transition probability is represented by (eq-4)

$$\gamma(R_{k+l},s^{(i)}_{k+l-1},s^{(i)}_j) = P(y^s_{k+l},y^p_{k+l}|d_{k+l},s^{(i)}_{k+l-1},s^{(i)}_{k+l-1}) \cdot P(d_{k+l}|s^{(i)}_{k+l-1},s^{(i)}_{k+l})\cdot P(s^{(i)}_{k+l}|s^{(i)}_{k+l-1}) \quad (4)$$

Here, a variable with a suffix $k+l$ shows a value at time $k+l$ is the received signal corresponding to a parity bit, $s^{(i)}_{k+l-1}$ corresponds to the originating state and $s^{(i)}_{k+l}$ corresponds to the terminating state of a code. $P(y^s_{k+l},y^p_{k+l}|d_{k+l},s^{(i)}_{k+l-1},s^{(i)}_{k+l-1})$ is the likelihood value of an combination of information bit and parity bit, $P(d_{k+l}|s^{(i)}_{k+l-1},s^{(i)}_{k+l})$ is either 1 or 0 depending upon the code, and $P(s^{(i)}_{k+l}|s^{(i)}_{k+l-1})$ is a priori state transition probability.

In a conventional VA for maximum likelihood decoding, the last factor $P(s^{(i)}_{k+l}|s^{(i)}_{k+l-1})$ is not included. On the other hand, since we are now discussing SOVA, this factor is taken into consideration.

The first fraction of the right side of Eq (4) is given by

$$P(y^s_{k+l},y^p_{k+l}|d_{k+l},s^{(i)}_{k+l-1},s^{(i)}_{k+l}) = P(y^s_{k+l}|d^s_{k+l})\cdot P(y^p_{k+l}|d^p_{k+l} \quad (5)$$

and assuming that channel is AWGN, each factor is represented by pdf.

$$p(y^s_k|i^s_k) = \frac{1}{\sqrt{2\pi\sigma}}e^{\frac{-(y^s_k-i^s_k)^2}{2\sigma^2}}$$

$$p(y^p_k|i^p_k) = \frac{1}{\sqrt{2\pi\sigma}}e^{\frac{-(y^p_k-i^p_k)^2}{2\sigma^2}} \quad (6)$$

Note that, due to the quantized channel, the input to the decoder is no longer continuous but instead is constrained to a set of discrete values as decided by the choice of the quantizer. Let us express these quantized signal of $y^{(s)}_k, y^{(p)}_k$ as $y^{(s)}_{[Q]k}, y^{(p)}_{[Q]k}$.

$$\gamma(R_{k+l[Q]},s^{(i)}_{k+l-1},s^{(i)}_j) = P(x_{j[Q]},y_{j[Q]} \setminus d_j,s^{(i)}_{j-1},s^{(i)}_j) \cdot P(d_j \setminus s^{(i)}_{j-1},s^{(i)}_j)\cdot P(s^{(i)}_j \setminus s^{(i)}_{j-1}) \quad (7)$$

The first term on the right-hand side in (eq) becomes

$$P(x_{j[Q]},y_{j[Q]}|d_j,s^{(i)}_{j-1},s^{(i)}_j) = P(x^s_{j[q]}|d^s_j)\cdot P(y^p_{j[q]}|d^p_j) \quad (8)$$

and we are taken into a logarithm on the every terms. each probability is changed to the branch metric $\Gamma_{[Q]}$, and the path metric $\Lambda_{[Q]}(S^{(i)}_{k+l})$.

In SOVA, branch metric in a path $S^{(i)}_{k+D}$ is at the time $k+l$ defined as $k+l$,

$$\Gamma_{[Q]}(s^{(i)}_{k+l-1}, s^{(i)}_{k+l}) = \ln \gamma(R_{k+l[Q]}, s^{(i)}_{k+l-1}, s^{(i)}_{k+l}) \quad (9)$$

path metric at the time $k+l$ is given recursively by

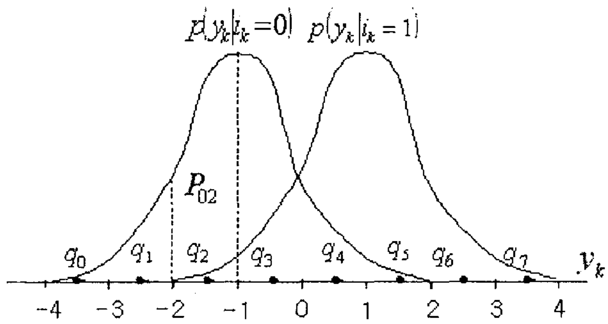
$$\Lambda_{[Q]}(S^{(i)}_{k+l}) = \Lambda_{[Q]}(S^{(i)}_{k+l-1}) + \Gamma_{[Q]}(s^{(i)}_{k+l-1}, s^{(i)}_{k+l}) \quad (10)$$

The difference of $\Delta^{(s,n)}_{k+l}$ between the metric of a survivor path and that of a non-survivor path is given by

$$\Delta^{(s,n)}_{k+l} = \Lambda(S^{(s)}_{k+l}) - \Lambda(S^{(n)}_{k+l}) \quad (11)$$

From now on, we calculate the pmf . As an example we explains the case of a 3-bit uniform quantizer shown in [Fig-3]. The pdf of received signal is a bimodal Gaussian distribution. We derive that channel transition probability is pmf in quantized channel (DMC). The pmfs of received signal are defined by the integral of the conditional pdf of the received signal lying over the region (bin) whose mid-point (for uniform quantization) corresponds to $y^{(s)}_{[Q]k}, y^{(p)}_{[Q]k}$

$$P_{ij} = \Pr(y_k \in T_j \setminus i_k) = \Pr(y_{[Q]k} \in q_j \setminus i_k)$$
$$= \int_{T_j} p(y_k \setminus i_k) dy_k \qquad i = 0,1 \quad j = 0,1,\cdots\cdots,L-1 \quad (12)$$

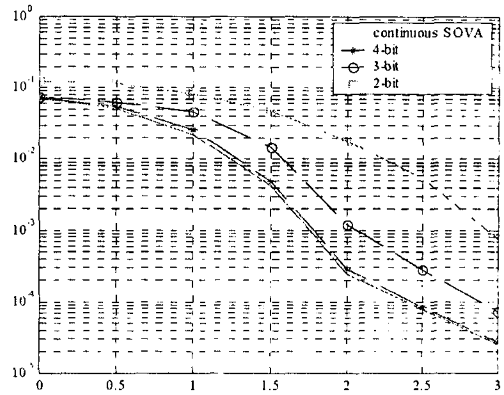


[Figure-3 Example of 3bit uniform quantizer(-4,4)]

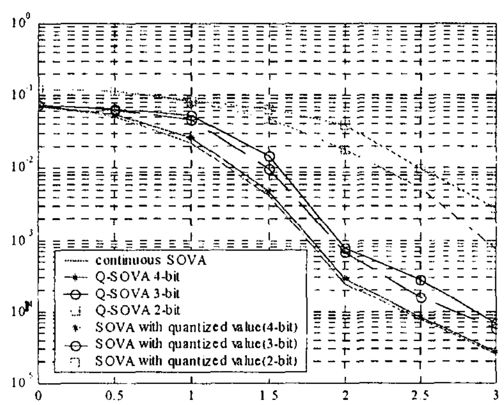
4. SIMULATION RESULTS

To illustrate the above derivation of the decoding algorithm and to compare the BER performance. We perform simulations for the cases of an uniform quantizer and the optimum nonuniform quantizer with the number of quantization bits, n = 2,3,4-bits. The length of interleaver is 512. Puncturing is not used. The overall code rate of the system is 1/3. The dotted lines of Fig. 4 shows simulation results of the modified SOVA (Q-SOVA) for the different numbers of quantization bits and the solid line of Fig. 4 shows the simulation results of general SOVA with continuous received filter outputs with eight iterations.

Fig. 4 shows that the BER performace of the modified SOVA (Q-SOVA) approaches to that of the general SOVA as the number of quantization bits goes to the large value. With 4 bits quantization, the modified SOVA has almost the same performance to the general SOVA.



[Figure-4 Performance for bits of the quantization]

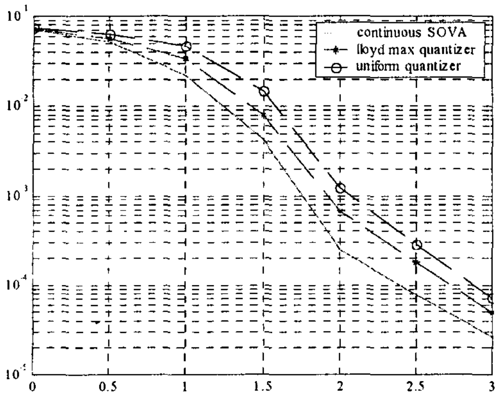


[Figure-5 Performance of the Modified SOVA and the SOVA for quantized data]

Fig. 5 shows simulation results for the modified SOVA and the general SOVA with quantized values. The SOVA with quantized value represents the general SOVA algorithm which use the quantized receiver filter outputs as branch metrics. This branch metric is corresponding to the approximation of the likelihood value of the quantized output. This indicates that the performance of modified SOVA (Q-SOVA) is better than that of SOVA with quantized values.

For the case that quantized receiver ouputs are given, with the modified SOVA algorithm has better BER performance than SOVA with quantized values.

In this paper we consider the Lloyd-Max quantizer [1] which is optimized for the incoming data having a Gaussian distribution for a given transmitting symbol. Fig 6 shows the simulation results for the general SOVA and modified SOVA's (Q-SOVA) with nonuniform and uniform quantizers with 3 bits quantization. Nonuniform quantizer has better BER performance than uniform quantizer.



[Figure-6 BER Performance of the Modified SOVA with nonuniorm (Lloid max) and uniform quantizer]

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5. CONCLUSION

We have proposed the modified SOVA for quantized channel filter outputs. We derived maxium likelihood values for received sequences in considering pmf values instead of Gaussian pdf's for discrete channel outputs. This gives us the concept of maximum likelihood functions of DMC and we derive the branch values of the modified SOVA. Simulation results show that the modified SOVA outperfoms the SOVA with quantized values. However we should note that the SOVA with quantized values is implemented with quantized branch values and the modified SOVA is performed with continuous branch values which are the derived from the likelihood function. The nonuniform quantizer (Lloid-Max quantizer) has better performace in the modified SOVA, but the uniform quantizer might be used in the SOVA with quantization values.

Acknowledgement

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References

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error correcting coding and decoding : turbo codes," Proc. of ICC '93, pp. 1064-1070, Geneva, Switzerland, May 1993.
[2] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," IEEE Trans. on Inform. Theory, vol. IT-20, pp 284-287, Mar. 1974.
[3] P. Robertson, E Villebrun, and P.Hoeher,"A comparison of sub-optimal and MAP decoding algorithm operating in the log domain," in Proc. ICC '95, June 1995, pp. 1009-1013.
[4] J. Hagenauer and P. Robertson, "Iterative ("TURBO") Decoding of Systematic Convolutional Codes with the MAP and SOVA Algorithms," ITG-Fachberichte, v.130, pp.21-29, 1995.
[5] Y. Wu and B. D. Woerner, "The influence of quantization and fixed point arithmetic upon the BER performance of turbo codes," in Proc. IEEE Int. Conf. on Vehicular Technology(VTC Spring '99), vol. 2, May 1999, pp. 1683-1687
[6] G. Jeong and D Hsia, "Optimal quantization for soft-decision turbo decoder," in IEEE VTC Fall'99, vol. 3, Sept. 1999, pp. 1620-1624.
[7] H. Michel, A. Worm, and N.Wehn, "Influence of quantization on the bit-error performance of turbo-decoders," in Proc. VTC "00 Spring, Tokyo, Japan, May 2000
[8] U. Dasgupta and Costas N. Georghiades, " Turbo Decoding of Quantized Data" in IEEE transactions on communications, vol. 50, NO. 1, January 2002.