BER of Rectangular QAM signals with MRC over Correlated Nakagami Fading Channels

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Abstract: The average bit error rate (BER) performance of a Gray coded arbitrary rectangular quadrature amplitude modulation (AR-QAM) signal with maximal ratio combining (MRC) diversity in an arbitrarily correlated Nakagami-*m* fading channel is derived and analyzed. The derived two types of general solutions are a simple closed-form and an integral form, depending on the types of the values (integer and non-integer) of the fading parameter. Using the derived equations in this paper, we analyzed the BER performances numerically based on the practical base station antenna configuration. The results show that MRC reception is a very effective scheme so far even though the combined signals are not independent each other because of the correlation values. The antenna height and separation of the MRC system relate to the correlation coefficient value between antennas, and go a long way with the diversity advantage. In particular, it is needed to be determined the antenna height that is carefully do for the diversity advantage because the correlation coefficient and the antenna height gain are contrary to each other from the aspect of the system performance. The expressions presented here can offer a convenient way to evaluate the exact BER performance of an arbitrary rectangular QAM signal with MRC diversity reception for various cases of practical interest on a correlated Nakagami fading channel.
Digital Communication, Fading Channels, QAM, BER

1. INTRODUCTION

Mobile radio systems require spectrally efficient modulation schemes due to the fact that available radio spectrum is limited. With the increasing demands of various mobile communication services, transmissions at higher rates will be required in band-limited mobile radio systems. A quadrature amplitude modulation (QAM) scheme is an attractive technique to achieve an improved high rate transmission over wireless links without increasing the bandwidth [1]. For this reason, QAM signaling is strongly recommended for future wireless communication systems. A great deal of recent attention has been devoted to the study of bit error probability for *M*-ary QAM [2]-[5], and an exact and general closed-form expression of the BER for QAM with arbitrary rectangular constellation size in AWGN has recently been developed [5]. When a received signal experiences fading during transmission, both its envelope and phase fluctuate over time. The fading governs the channel characteristics to decrease the receiving performance of mobile communication systems. Because the fading phenomena caused by multipath propagation characteristics depend on geographical features, there are different models describing the statistical behavior of the multipath fading envelopes named Rayleigh, Rican, Nakagami and others. The Nakagami *m*-distribution among these is a versatile statistical model, which can accurately fit experimental data for many physical propagation channels [6]. To deal successfully with the detrimental effects of channel fading, diversity receptions have been recognized as effective techniques and have been discussed extensively in recent years to improve the communication system performance in multi-path fading channels. The diversity techniques for improving the system performance without incurring a substantial penalty in terms of implementation complexity or cost are of practical interest. For coherent communication systems, one of the most prevalent space diversity techniques is MRC. If the diversity channels are sufficiently separated, the assumption of statistical independence between the diversity branches is valid. However, in real system, if the diversity branches are very closely spaced, the signals on different branches are no longer independent. When the diversity antennas are placed in a small area with insufficient antenna spacing between diversity antennas, the received and combined signals are arbitrarily correlated with each other to some degree [7],[8]. The error rate performances for MRC diversity in correlated Nakagami fading channels have been presented in many places in the literature [9]-[15]. Aalo [13] provided MRC reception performance of PSK and FSK with several approximated correlation fading model. Especially, Zhang [9] provided two types of exact solutions of error probabilities of BPSK and BFSK to MRC's over Nakagami fading channels with an arbitrary branch covariance matrix for the two types of fading parameters *m*, integer and non-integer. However, general BER expression of AR-QAM with MRC diversity receptions in correlated Nakagami fading channels has not yet been evaluated.

In this Paper, using the our previous results of [5], the exact expressions of BER for AR-QAM, with *L*-branch MRC diversity reception affected by a correlated

Nakagami fading and embedded in AWGN, is derived and analyzed with the excellent results of [9],[10].

2. SYSTEM MODEL

The modulated rectangular QAM signal is assumed to be transmitted over a frequency non-selective slow Nakagami fading channel where the channel transfer function is given by

$$c(t) = \alpha \cdot e^{j\theta}, \quad (1)$$

where α represents the Nakagami fading envelope and θ the phase of the channel which is uniformly distributed over $[-\pi, \pi]$ for the duration of one signaling interval.

Considering a MRC reception system using L antenna branch, the received signal at the k-th branch antenna over the symbol duration can be written as

$$r_k(t) = \alpha \cdot e^{j\theta} \cdot s(t) + n_k(t), \quad (2)$$

where $s(t)$ is the transmitted modulated signal with its symbol energy E_0, $n(t)$ is an additive white Gaussian noise (AWGN) with its uniform power spectral density $N_0/2$, and k=1, 2, ⋯, L. The instantaneous signal to noise ratio (SNR) γ_k from the k-th branch antenna is given by $\gamma_k = \alpha^2 \cdot E_0/N_0$. For Nakagami fading, γ_k is a gamma-distributed variate. Thus, the maximal ratio combiner output SNR γ is the sum of the jointly gamma distributed L-branch SNRs, $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_L$.

From [9], the joint CF $\Phi_\gamma(s)$ of the instantaneous output SNR can be obtained as

$$\Phi_\gamma(s) = E\left[\exp\left(j\sum_{k=1}^{L} t_k \gamma_k\right)\right] = \det(\mathbf{I} - j\mathbf{T}\boldsymbol{\Gamma})^{-m} = \det(\mathbf{I} - s\boldsymbol{\Gamma})^{-m}, \quad (3)$$

where m is the real-valued fading parameter and $\boldsymbol{\Gamma}$ is symmetric and positive definite matrix determined by the branch covariance matrix $\mathbf{R_r}$ of the instantaneous SNR vector $\mathbf{r} = [\gamma_1, \gamma_2, \cdots, \gamma_L]$; $\mathbf{T}$=diag$\{t_1, t_2, \cdots, t_L\}$; $s = jt$. Through the derivative operations of the CF involving matrices, we can show the relationship between $\mathbf{R_r}$ and $\boldsymbol{\Gamma}$ matrices as [9]

$$\mathbf{R}_r(k,l) = m\boldsymbol{\Gamma}^2(k,l) \quad (4)$$

with $\boldsymbol{\Gamma}(k,l)$ denoting the (k,l)th entry of $\boldsymbol{\Gamma}$.

The conditional probability of a bit error for a rectangular QAM signal in the AWGN environment is given by [5]

$$P_b(e\,|\,\gamma) = \frac{1}{\log_2(I\cdot J)}\left(\sum_{k=1}^{\log_2 I} P_I(k) + \sum_{l=1}^{\log_2 J} P_J(l)\right), \quad (5a)$$

where $P_I(k)$and $P_J(l)$ are the probabilities that the k-th bit of the in-phase components and the l-th bit of the quadrature components are in error in terms of SNR, respectively

$$P_I(k) = \frac{1}{I}\sum_{i=0}^{(1-2^{-k})I-1}\left[\Theta(i,k,I)\cdot erfc\left(\sqrt{\gamma\cdot\Omega(i,I,J)}\right)\right] \quad (5b)$$

$$P_J(l) = \frac{1}{J}\sum_{j=0}^{(1-2^{-l})J-1}\left[\Theta(j,l,J)\cdot erfc\left(\sqrt{\gamma\cdot\kappa^2\cdot\Omega(j,I,J)}\right)\right], \quad (5c)$$

where

$$\Theta(a,b,c) = (-1)^{F\left(\frac{a\cdot 2^{b-1}}{c}\right)}\cdot\left[2^{b-1} - F\left\{\left(\frac{a\cdot 2^{b-1}}{c}\right)+\frac{1}{2}\right\}\right] \quad (5d)$$

$$\Omega(a,b,c) = \frac{3(2a+1)^2\cdot\log_2(bc)}{(b^2-1)+\kappa^2(c^2-1)}. \quad (5e)$$

In (5e), κ is the ratio of the minimum distance between the in-phase and quadrature axes; F(•)is the floor function; $k \in \{1,2,\cdots\log_2 I\}$; $l \in \{1,2,\cdots\log_2 J\}$. When I = J = $\sqrt{M}$ and κ = 1, (5) reduces to the BER of M-ary square QAM.

The average bit error probability, $\overline{P_b}$, for diversity reception in a fading channel is obtained by

$$\overline{P_b} = \int_0^\infty P_b(e\,|\,x) f_\gamma(x)dx, \quad (6)$$

where $P_b(e\,|\,x)$ is the bit error probability in AWGN and $f_\gamma(x)$ is the probability density function (PDF) of the instantaneous SNR γ at the diversity combiner output.

By substituting (5) into (6), the average BER $\overline{P_b}$ can now be expressed as the sum of two integrals, which corresponds to the bit error probabilities of in-phase and quadrature components, respectively

$$\overline{P_b} = \frac{1}{\log_2(I\cdot J)}\left(\frac{1}{I}\sum_{k=0}^{\log_2 I}\sum_{i=0}^{(1-2^{-k})I-1}[\Theta(i,k,I)\cdot\Psi_I(i,I,J)] + \frac{1}{J}\sum_{l=0}^{\log_2 J}\sum_{j=0}^{(1-2^{-l})J-1}[\Theta(j,l,J)\cdot\Psi_J(j,I,J)]\right), \quad (7a)$$

where

$$\Psi_I(i,I,J) = \int_0^\infty erfc\left(\sqrt{\gamma\cdot\Omega(i,I,J)}\right) f_D(\gamma)d\gamma, \quad (7b)$$

$$\Psi_J(i,I,J) = \int_0^\infty erfc\left(\sqrt{\gamma\cdot\kappa^2\cdot\Omega(i,I,J)}\right) f_D(\gamma)d\gamma. \quad (7c)$$

Eq. (7) shows that the $\overline{P_b}$ is solely characterized by $\Psi_I(i,I,J)$ and $\Psi_J(i,I,J)$. Here, we investigate $\Psi_I(i,I,J)$ for the in-phase axis only because $\Psi_J(i,I,J)$ for the quadrature axis can be obtained in a similar manner by substituting $\kappa^2\bar{\gamma}$ for $\bar{\gamma}$.

3. BIT ERROR PROBABILITY (BER)

In this chapter, we calculate the BER of AR-QAM signals with MRC system over Nakagami fading channel. In the case of the fading parameter m of Nakagami PDF is an integer; it is possible to write the BER as closed-form expression. But, if the fading parameter is not an integer, the BER shall have an integral-form expression. Thus, we will drive the BER expressions with integer and non-integer fading parameter, individually.

3.1 BER with an Integer Fading Parameter *m*

From (3), because the $\boldsymbol{\Gamma}$ matrix is a positive definite matrix, it can be factorized into $\boldsymbol{\Gamma}=\mathbf{Q\Lambda Q}^T$ (spectral theorem) with the orthonormal eigenvectors in $\mathbf{Q}$ and the eigenvalues in $\boldsymbol{\Lambda}$ (diagonal matrix) when all eigenvalues of $\boldsymbol{\Gamma}$ matrix are distinct [16]. Also, the $\boldsymbol{\Gamma}$ matrix is a symmetric, then, $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ ($\mathbf{Q}^T = \mathbf{Q}^{-1}$). Thus, the determinant term of (3) can be rewrite as

$$\begin{aligned}\det(\mathbf{I}-s\mathbf{\Gamma}) &= \det[\mathbf{Q}(\mathbf{I}-s\mathbf{\Lambda})\mathbf{Q}^{\mathrm{T}}] \\ &= \det(\mathbf{Q}\mathbf{Q}^{\mathrm{T}})\cdot\det(\mathbf{I}-s\mathbf{\Lambda}) = \det(\mathbf{I}-s\mathbf{\Lambda})\end{aligned} \quad . (8)$$

Hence $\det(\mathbf{I} - s\mathbf{\Lambda})$ is an $L\times L$ diagonal matrix, it can be expressed as

$$\det(\mathbf{I}-s\mathbf{\Lambda}) = \det\begin{pmatrix} 1-s\lambda_1 & 0 & \cdots & 0 \\ 0 & 1-s\lambda_2 & \cdots & \cdots \\ \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & 1-s\lambda_L \end{pmatrix} = \prod_{k=1}^{L}(1-s\lambda_k) \quad (9)$$

where λ_k stand for the eigenvalues of $\mathbf{\Gamma}$ matrix.

Take as a whole, in order to derive the associate pdf, the product must be converted in to a sum, using partial fraction expansion method, considering the fading parameter m is an integer. Thus we can rewrite the new form of CF (3) with (8) and (9) to partial product and sum expressions after some manipulations as

$$\begin{aligned}\Phi_\gamma(s) &= \det(\mathbf{I}-s\mathbf{\Gamma})^{-1}\cdot\det(\mathbf{I}-s\mathbf{\Gamma})^{-1}\cdots \ (m \text{ times}) \\ &= \prod_{k=1}^{L}(1-s\lambda_l)^{-1}\cdot\prod_{k=1}^{L}(1-s\lambda_l)^{-1}\cdots \ (m \text{ times}) \\ &= \prod_{k=1}^{L}(1-s\lambda_l)^{-m} = \sum_{l=1}^{L}\sum_{r=1}^{m}\frac{\beta_{lr}}{(1-s\lambda_l)^r}\end{aligned} \quad . (10)$$

where the each term corresponds to a Gamma distribution, and the coefficient β_{lr}, for $0 < r < m$, can be written by

$$\beta_{lr} = \frac{1}{(m-r)!(-\lambda_l)^{m-r}}\frac{d^{m-r}}{ds^{m-r}}\left(\prod_{k=1,k\neq l}^{L}(1-s\lambda_k)^{-m}\right)\Bigg|_{s=(1/\lambda_l)} \quad ,(11)$$

and for $r = m$

$$\beta_{lr} = \left(\prod_{k=1,k\neq l}^{L}(1-s\lambda_k)^{-m}\right)\Bigg|_{s=(1/\lambda_l)} \quad . \quad (12)$$

Because the coefficient term β_{lr} is involved with the calculation of higher order differentiation of a product function, it is not easy to calculate manually. To evaluate that function, we use a general algorithm proposed in [9]. In (10), because γ is the linear combination of the distributions of mL independent gamma variables, the PDF corresponding to the CF $(1-s\lambda_l)^{-r}$ is

$$p_\gamma(x;l,r) = \frac{1}{\lambda_l\Gamma(r)}\left(\frac{x}{\lambda_l}\right)^{r-1}\exp\left(-\frac{x}{\lambda_l}\right) \quad ,x>0\,, \quad (13)$$

and, thus, the PDF of the instantaneous SNR γ can be expressed as

$$f_\gamma(x) = \sum_{l=1}^{L}\sum_{r=1}^{m}\beta_{lr}p_\gamma(x;l,r)\,. \quad (14)$$

By applying (14) to (6), $\Psi_I(i,I,J)$ can be acquired as

$$\begin{aligned}\Psi_I(i,I,J) &= \int_0^\infty erfc\left(\sqrt{x\,\Omega(i,I,J)}\right)\sum_{l=1}^{L}\sum_{r=1}^{m}\beta_{lr}p_\gamma(x;l,r)dx \\ &= 2\sum_{l=1}^{L}\sum_{r=1}^{m}\beta_{lr}\left(\frac{1-\mu_l}{2}\right)^r\sum_{k=0}^{r-1}\binom{r-1+k}{k}\left(\frac{1+\mu_l}{2}\right)^k\end{aligned} \quad (15)$$

where $\mu_l = \sqrt{\dfrac{\lambda_l\,\Omega(i,I,J)}{1+\lambda_l\,\Omega(i,I,J)}}$.

3.2 BER with an Non-integer Fading Parameter *m*

The integral-form PDF of the instantaneous output SNR γ can be acquired by solving the inverse Fourier transformation of the CF (3) for non-integer fading parameter m as

$$f_\gamma(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_\gamma(jt)\exp(-jxt)dt \quad (16)$$

Thus, $\Psi_I(i,I,J)$ is obtained as (17) by (6) and (16)

$$\begin{aligned}\Psi_I(i,I,J) &= \frac{1}{2\pi}\int_0^\infty erfc\left(\sqrt{\gamma\,\Omega(i,I,J)}\right)f_\gamma(x)d\gamma \\ &= \frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{j\pi t}\left(1-\sqrt{\frac{\Omega(i,I,J)}{jt+\Omega(i,I,J)}}\right)\prod_{k=1}^{L}(1-s\lambda_k)^{-m}dt \\ &= \int_0^\infty \mathrm{Re}\{F(jt)\}dt\end{aligned} \quad . (17)$$

Using the result in [9], $\mathrm{Re}\{F(jt)\}$ in (17) can rewrite as

$$\mathrm{Re}\{F(jt)\} = \frac{\eta(t)\sin\left(\varphi(t)+m\sum_{k=1}^{L}\tan^{-1}(\lambda_k t)\right)}{\pi\prod_{k=1}^{L}\left(1+(\lambda_k t)^2\right)^{m/2}} \quad (18)$$

where,

$$\eta(t) = \sqrt{1+\left(\frac{\Omega(i,I,J)^2}{t^2+\Omega(i,I,J)^2}\right)^{1/2} - 2\left(\frac{\Omega(i,I,J)^2}{t^2+\Omega(i,I,J)^2}\right)^{1/4}\cos\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{\Omega(i,I,J)}\right)\right]},$$

$$\varphi(t) = \tan^{-1}\left(\frac{\sin\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{\Omega(i,I,J)}\right)\right]}{\left(\frac{\Omega(i,I,J)^2}{t^2+\Omega(i,I,J)^2}\right)^{-1/4} - \cos\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{\Omega(i,I,J)}\right)\right]}\right)$$

When the fading parameter m is an integer, (15) and (17) have the same result.

4. NUMERICAL RESULTS

The correlation value between antennas is affected by many factors; orientation of antennas relative to mobile unit, antenna height (H), antenna separation (S), carrier frequency and *etc* [7]. In this paper, we consider a base station has linear array antenna configurations, the signals are arrived at the array with an angle of incidence of 45 degrees for the linear array axis, and the signal frequency is located at 2GHz. To simplify the numerical analysis, we assume that all branches have same average SNR. Thus, scaling factor can be used for the relation between $\mathbf{R_r}$ and $\mathbf{\Gamma}$ in (4) resulting $\mathbf{\Gamma} = (\mathrm{SNR}/m)\sqrt{\mathbf{R_r}}$.

With the above assumptions, we can get the correlation matrix for the given parameters using results of the empirical curve in [7]. For example, if the base station antenna height is 20m, antenna spacing is 6 wavelengths (λ) and diversity order L is 5, the correlation matrix can be acquired by

$$\mathbf{R_r} = \begin{pmatrix} 1 & 0.720 & 0.451 & 0.310 & 0.217 \\ 0.720 & 1 & 0.720 & 0.451 & 0.466 \\ 0.451 & 0.720 & 1 & 0.720 & 0.451 \\ 0.310 & 0.451 & 0.720 & 1 & 0.720 \\ 0.217 & 0.310 & 0.451 & 0.720 & 1 \end{pmatrix} \quad . (19)$$

Note that the correlation matrices for the diversity order

L=2, 3, and 4 are subsets of the correlation matrix (19).

Fig. 1 and 2 show the effect of the diversity orders L and the fading parameters m on the error performance, respectively. From the results, as expected, we observe that the more increase of the diversity orders and the fading parameters, the more improvement of the BER performance. From Fig. 1, a close inspection of these numerical results reveals that a significant improvement in BER performance is achieved as the order of L increases from 1 to 2 and the improvement in BER performance is some extent retained as the order of L increases from 2 to higher orders. For example, at a BER of 10^{-3}, there is a 3.1dB diversity gain by using L =3 over L =2, and as much as a 6.6dB diversity gain by using L =2 over L =1 for 32-QAM. Also, we see that with the same diversity orders for different orders of QAM signals, the diversity gain is same regardless of the orders of QAM signals at the same parameter environment.

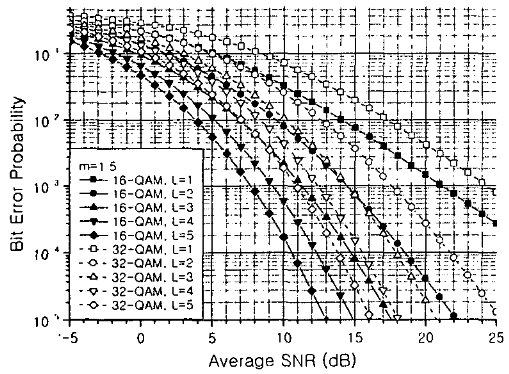

Fig.1 The effect of the diversity orders (L) on the error performance (m=1.5, H = 20m, S = 6 λ)

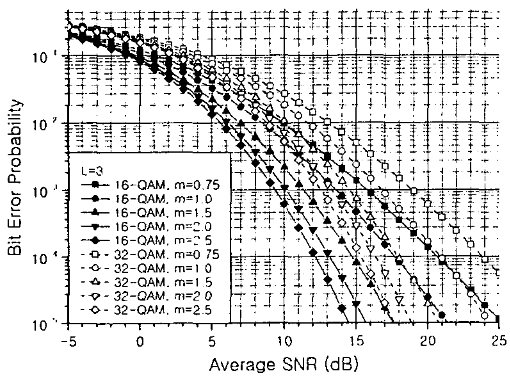

Fig.2 The effect of the fading parameters (m) on the error performance (L = 3, H = 20m, S = 6 λ)

Fig.3 shows the antenna separation dependency of the BER performance. From the curves, we observe that the wider antenna separation (smaller correlation coefficient), the more improvement of the BER performance. Fig.4 shows the antenna height dependency of the BER performance. The curves in the figure show that lower antenna height give more advantage of diversity than higher one. It is because lowering the antenna height results in smaller correlation coefficient. But, lowering antenna height, to get the lower correlation coefficient to enhance the diversity gain, should be considered with aspect to the total system performance because determining the antenna height affects antenna height gain and diversity advantage each other reversely.

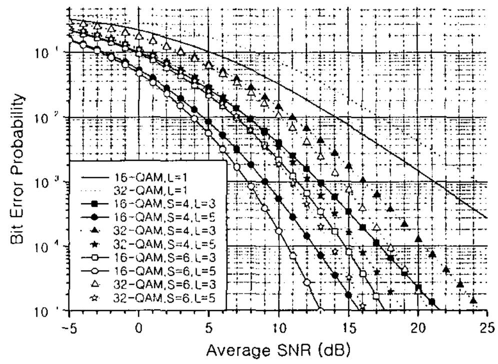

Fig.3 The effect of the antenna separations (S) on the error performance (H=20m, m=1.5, L=3)

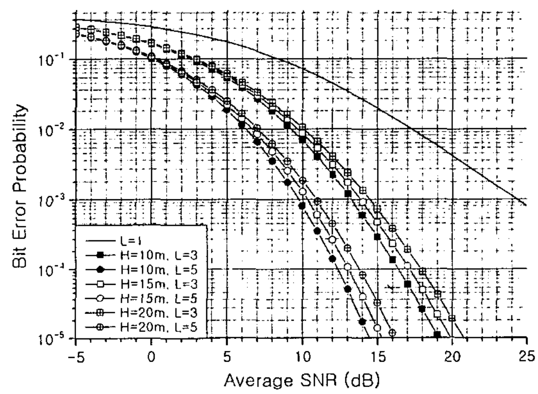

Fig.4 The effect of the antenna Height (H) on the error performance (32-QAM, S= 6λ, m=1.5, L=3)

5. CONCLUSIONS

In this Paper, the two types of BER expressions of Gray coded arbitrary rectangular QAM, a closed-form and an integral form dependent on integer and non-integer fading parameters, have been presented and analyzed for MRC diversity system in arbitrarily correlated Nakagami-m fading channels. The BER performances have been evaluated for the fading parameters and the diversity orders with covariance matrices considering the antenna spacing and height. Also, using the derived equations in this paper, we analyzed the BER performances numerically based on the practical base station antenna configurations. From the results, we observed that the antenna height and separation determine the correlation coefficient value between antennas and affect diversity advantage. In particular, determining the antenna height is carefully do because the correlation coefficient effect for the diversity advantage and the antenna height gain are contrary to each other from the aspect of the total system performance.

Since derived expressions are exact, they offer a convenient way to evaluate the performance of arbitrary rectangular QAM in conjunction with MRC diversity for various cases of practical interest in correlated fading channels.

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