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# Bayesian Test for the Difference of Exponential Guarantee Time Parameters

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## Abstract

When $X$ and $Y$ have independent two parameter exponential distributions, we develop a Bayesian testing procedures for the equality of two location parameters. Under the noninformative prior, we propose a Bayesian test procedures for the equality of two location parameters using fractional Bayes factor and intrinsic Bayes factor. Simulation study and some real data examples are provided.

*Keywords* : Fractional Bayes Factor, Intrinsic Bayes Factor, Reference Prior, Matching Prior, Exponential Location Parameters

## 1. Introduction

The two parameter exponential distribution plays an important role in the field of life testing and reliability theory since it is the only continuous distribution with a constant hazard function. The reciprocal of the scale parameter is the hazard rate. The location (threshold) parameter can translate the distribution along the time axis, so it is also known as the minimum life or guarantee time parameter. The guarantee time parameter can be used to model warranty periods for some products.

The present paper focuses on Bayesian testing procedure for the equality of two location parameters. In Bayesian testing problem, the Bayes factor under proper priors or informative priors have been very successful. However, limited information and time constraints often require the use of noninformative priors. Since noninformative priors such as Jeffreys' priors or reference priors (Berger and Bernardo, 1989, 1992) are typically improper so that such priors are only defined up to arbitrary constants which affects the values of Bayes factors. Spiegalhalter and Smith (1982), O'Hagan (1995) and Berger and Pericchi (1996) have made

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efforts to compensate for that arbitrariness.

Berger and Pericchi (1996) introduced the intrinsic Bayes factor using a data-splitting idea, which would eliminate the arbitrariness of improper priors. O'Hagan (1995) proposed the fractional Bayes factor. For removing the arbitrariness he used to a portion of the likelihood with a so-called the fraction $b$. These approaches have shown to be quite useful in many statistical areas.

For the statistical inference of the exponential distribution, Epstein and Tsao (1953) studied some statistical tests for two exponential distributions. Epstein and Sobel (1954) obtained the minimum variance unbiased estimator for the scale and location parameters. The shrinkage estimators for the scale parameter have been proposed by Bhattacharya and Srivastava (1974). Chiou and Han (1989) proposed a pre-test estimator and a pre-test shrinkage estimator for the location parameter. Chiou and Miao (2004) studied the shrinkage estimator for the difference between location parameters.

Almost all the work mentioned above is the analysis based on the classical point of view, there is a little work on this problem from the viewpoint of the objective Bayesian framework. Because the two parameter exponential distribution is the non-regular case, so the noninformative priors such as reference prior or probability matching prior were hard to derive. Almost all theory related to these priors were developed based on the assumption of regular distribution. Recently, Ghosal (1997, 1999) developed the procedures to derive the reference and matching priors for non-regular cases. Using his results, we feel a strong necessity to develop objective Bayesian testing procedure for the difference between location parameters. For dealing this problem, we use the fractional Bayes factor (O'Hagan, 1995) and the intrinsic Bayes factor (Berger and Pericchi, 1996).

The outline of the remaining sections is as follows. In Section 2, using the noninformative priors, we provide the Bayesian testing procedure based on the fractional Bayes factor and intrinsic Bayes factor for the testing equality of two location parameters. In Section 3, simulation study and some real examples are given.

## 2. Bayesian Test Procedures

### 2.1 Preliminaries

Models (or Hypotheses) $H_1$, $H_2, \cdots$, $H_q$ are under consideration, with the data $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ having probability density function $f_i(\boldsymbol{x} \mid \boldsymbol{\theta}_i)$ under model $H_i, i=1,2,\cdots,q$. The parameter vectors $\boldsymbol{\theta}_i$ are unknown. Let $\pi_i(\boldsymbol{\theta}_i)$ be the prior distribution of model $H_i$, and let $p_i$ be the prior probabilities of model $H_i, i=1,2,\cdots,q$. Then the posterior probability that the model $H_i$ is true is

$$P(H_i \mid \boldsymbol{x}) = \left( \sum_{j=1}^{q} \frac{p_j}{p_i} \cdot B_{ji} \right)^{-1}, \tag{1}$$

where $B_{ji}$ is the Bayes factor of model $H_j$ to model $H_i$ defined by

$$B_{ji} = \frac{m_j(\boldsymbol{x})}{m_i(\boldsymbol{x})} = \frac{\int f_j(\boldsymbol{x} \mid \boldsymbol{\theta}_j)\pi_j(\boldsymbol{\theta}_j)d\boldsymbol{\theta}_j}{\int f_i(\boldsymbol{x} \mid \boldsymbol{\theta}_i)\pi_i(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i}. \tag{2}$$

The $B_{ji}$ interpreted as the comparative support of the data for the model $j$ to $i$. The computation of $B_{ji}$ needs specification of the prior distribution $\pi_i(\boldsymbol{\theta}_i)$ and $\pi_j(\boldsymbol{\theta})$. Usually, one can use the noninformative prior, often improper, such as uniform prior, Jeffreys prior, reference prior or probability matching prior. Denote it as $\pi_i^N$. The use of improper priors $\pi_i^N(\cdot)$ in (2) causes the $B_{ji}$ to contain unspecified constants. To solve this problem, O'Hagan (1995) proposed the fractional Bayes factor for Bayesian testing and model selection problem as follow.

When the $\pi_i^N(\boldsymbol{\theta}_i)$ is noninformative prior under $H_i$, equation (2) becomes

$$B_{ji}^N = \frac{\int f_j(\boldsymbol{x} \mid \boldsymbol{\theta}_j)\pi_j^N(\boldsymbol{\theta}_j)d\boldsymbol{\theta}_j}{\int f_i(\boldsymbol{x} \mid \boldsymbol{\theta}_i)\pi_i^N(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i}.$$

Then the fractional Bayes factor of model $H_j$ versus model $H_i$ is

$$B_{ji}^F = B_{ji}^N \cdot \frac{\int f_i^b(\boldsymbol{x} \mid \boldsymbol{\theta}_i)\pi_i^N(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i}{\int f_j^b(\boldsymbol{x} \mid \boldsymbol{\theta}_j)\pi_j^N(\boldsymbol{\theta}_j)d\boldsymbol{\theta}_j},$$

and $f_i(\boldsymbol{x} \mid \boldsymbol{\theta}_i)$ is the likelihood function and $b$ specifies a fraction of the likelihood which is to be used as a prior density. He proposed three ways for the choice of the fraction $b$. One frequently suggested choice is $b = m/n$, where $m$ is the size of the minimal training sample, assuming this is well defined. (see O'Hagan, 1995, 1997 and the discussion by Berger and Mortera of O'Hagan, 1995).

Berger and Pericchi (1996) proposed the intrinsic Bayes factor (IBF) for Bayesian testing and model selection. The arithmetic intrinsic Bayes factor is given by

$$B^I_{ji} = B^N_{ji} \cdot \frac{1}{L} \sum_{l=1}^{L} B^N_{ij}(\boldsymbol{x}(l)),$$

where

$$B^N_{ij}(\boldsymbol{x}(l)) = \frac{m_i(\boldsymbol{x}(l))}{m_j(\boldsymbol{x}(l))} = \frac{\int f_i(\boldsymbol{x}(l) \mid \boldsymbol{\theta}_i)\pi^N_i(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i}{\int f_j(\boldsymbol{x}(l) \mid \boldsymbol{\theta}_j)\pi^N_j(\boldsymbol{\theta}_j)d\boldsymbol{\theta}_j}.$$

Here $\boldsymbol{x}(l)$ is minimal training sample and $L$ is the number of all possible minimal training samples.

## 2.2 Bayesian Test

Let $X$ be a two parameter exponential distribution with density function

$$f(x \mid \eta, \theta) = \frac{1}{\theta}\exp\{-\frac{x-\eta}{\theta}\}, \ x > \eta, \ \theta > 0, \tag{3}$$

where $\eta$ is the location parameter (guarantee parameter) and $\theta$ is the scale parameter. Suppose that $X = (X_1, \cdots, X_{n_1})$ is a random sample of size $n_1$ from a two parameter exponential distribution with location parameter $\eta_1$ and scale parameter $\theta_1$ and $Y = (Y_1, \cdots, Y_{n_2})$ is a random sample of size $n_2$ from a two parameter exponential distribution with location parameter $\eta_2$ and scale parameter $\theta_2$. Then the joint probability density function is

$$f(\boldsymbol{x}, \boldsymbol{y} \mid \eta_1, \eta_2, \theta_1, \theta_2) = \theta_1^{-n_1}\theta_2^{-n_2}\exp\{-\frac{n_1(\bar{x}-\eta_1)}{\theta_1} - \frac{n_2(\bar{y}-\eta_2)}{\theta_2}\},$$

where $\theta_1 > 0$, $\theta_2 > 0$, $x_i > \eta_1, i = 1, \cdots, n_1$ and $y_j > \eta_2, j = 1, \cdots, n_2$, and $\bar{x}$ and $\bar{y}$ are the sample mean for each population.

We want to test the hypotheses $H_1: \eta_1 = \eta_2$ vs. $H_2: \eta_1 \neq \eta_2$. Our interest is to develop a Bayesian test based on the fractional and intrinsic Bayes factors for $H_1$ vs. $H_2$ under the noninformative priors. The two parameter exponential distribution is belong to non-regular cases. However for non-regular cases, the reference and probability matching prior is developed by Ghosal (1997,1999). In our model (3), the reference prior is given by $\pi(\eta, \theta) \propto 1/\theta$.

## 2.2 Bayesian Test using the Fractional Bayes Factor

Under the hypothesis $H_1$, the reference prior for $\eta (\equiv \eta_1 = \eta_2)$, $\theta_1$ and $\theta_2$ is

$$\pi_1(\eta, \theta_1, \theta_2) = \pi(\eta, \theta_1)\pi(\eta, \theta_2) = \theta_1^{-1}\theta_2^{-1}.$$

The likelihood function under $H_1$ is

$$L(\eta, \theta_1, \theta_2 \mid \boldsymbol{x}, \boldsymbol{y}) = \theta_1^{-n_1}\theta_2^{-n_2}\exp\{-\frac{n_1(\bar{x}-\eta)}{\theta_1} - \frac{n_2(\bar{y}-\eta)}{\theta_2}\}.$$

Then the element of fractinal Bayes factor under $H_1$ is given by

$$\begin{aligned} m_1^b(\boldsymbol{x},\boldsymbol{y}) &= \int_0^{m_{x,y}}\int_0^{\infty}\int_0^{\infty} L^b(\eta, \theta_1, \theta_2 \mid \boldsymbol{x}, \boldsymbol{y})\pi_1(\eta, \theta_1, \theta_2)d\theta_1 d\theta_2 d\eta \\ &= (n_1 b)^{-n_1 b}(n_2 b)^{-n_2 b}\Gamma(n_1 b)\Gamma(n_2 b)\int_0^{m_{x,y}}(\bar{x}-\eta)^{-n_1 b}(\bar{y}-\eta)^{-n_2 b}d\eta, \end{aligned}$$

where $m_{x,y} = \min_{1\le i\le n_1, 1\le j\le n_2}\{x_i, y_j\}$. For the $H_2$, the reference prior for $\eta_1$, $\eta_2$, $\theta_1$ and $\theta_2$ is

$$\pi_2(\eta_1, \eta_2, \theta_1, \theta_2) = \pi(\eta_1, \theta_1)\pi(\eta_2, \theta_2) = \theta_1^{-1}\theta_2^{-1}.$$

The likelihood function under $H_2$ is

$$L(\eta_1, \eta_2, \theta_1, \theta_2 \mid \boldsymbol{x}, \boldsymbol{y}) = \theta_1^{-n_1}\theta_2^{-n_2}\exp\{-\frac{n_1(\bar{x}-\eta_1)}{\theta_1} - \frac{n_2(\bar{y}-\eta_2)}{\theta_2}\}.$$

Thus the element of fractional Bayes factor under $H_2$ gives as follows.

$$\begin{aligned} m_2^b(\boldsymbol{x}, \boldsymbol{y}) &= \int_0^{m_x}\int_0^{m_y}\int_0^{\infty}\int_0^{\infty} L^b(\eta_1, \eta_2, \theta_1, \theta_2 \mid \boldsymbol{x}, \boldsymbol{y})\pi_2(\eta_1, \eta_2, \theta_1, \theta_2)d\theta_2 d\theta_1 d\eta_2 d\eta_1 \\ &= \frac{1}{(n_1 b-1)(n_2 b-1)}(n_1 b)^{-n_1 b}(n_2 b)^{-n_2 b}\Gamma(n_1 b)\Gamma(n_2 b) \\ &\times [(\bar{x}-m_x)^{-n_1 b+1}-(\bar{x})^{-n_1 b+1}][(\bar{y}-m_y)^{-n_2 b+1}-(\bar{y})^{-n_2 b+1}], \end{aligned}$$

where $m_x = \min_{1\le i\le n_1}\{x_i\}$ and $m_y = \min_{1\le j\le n_2}\{y_j\}$. Therefore the $B_{21}^N$ is given by

$$B_{21}^N = \frac{[(\bar{x}-m_x)^{-n_1+1}-(\bar{x})^{-n_1+1}][(\bar{y}-m_y)^{-n_2+1}-(\bar{y})^{-n_2+1}]}{(n_1-1)(n_2-1)S(\boldsymbol{x},\boldsymbol{y})}, \tag{4}$$

where

$$S(\boldsymbol{x}, \boldsymbol{y}) = \int_0^{m_{x,y}}(\bar{x}-\eta)^{-n_1}(\bar{y}-\eta)^{-n_2}d\eta.$$

And

$$\frac{m_1^b(\boldsymbol{x}, \boldsymbol{y})}{m_2^b(\boldsymbol{x}, \boldsymbol{y})} = \frac{(n_1 b-1)(n_2 b-1)S^b(\boldsymbol{x}, \boldsymbol{y})}{[(\bar{x}-m_x)^{-n_1 b+1}-\bar{x}^{-n_1 b+1}][(\bar{y}-m_y)^{-n_2 b+1}-\bar{y}^{-n_2 b+1}]},$$

where

$$S^b(\mathbf{x},\mathbf{y}) = \int_0^{m_{x,y}} (\bar{x}-\eta)^{-n_1 b}(\bar{y}-\eta)^{-n_2 b} d\eta.$$

Thus the fractional Bayes factor of $H_2$ versus $H_1$ is given by

$$\begin{aligned} B_{21}^F &= \frac{(n_1 b-1)(n_2 b-1)S^b(\mathbf{x},\mathbf{y})}{(n_1-1)(n_2-1)S(\mathbf{x},\mathbf{y})} \\ &\times \frac{[(\bar{x}-m_x)^{-n_1+1}-(\bar{x})^{-n_1+1}][(\bar{y}-m_y)^{-n_2+1}-(\bar{y})^{-n_2+1}]}{[(\bar{x}-m_x)^{-n_1 b+1}-\bar{x}^{-n_1 b+1}][(\bar{y}-m_y)^{-n_2 b+1}-\bar{y}^{-n_2 b+1}]}. \end{aligned}$$

Note that the calculation of the fractional Bayes factor of $H_2$ versus $H_1$ is requires a one dimensional integration.

Remark. In the calculation of $m_1^b(\mathbf{x},\mathbf{y})$, if $n_1 b$ is 1, then $m_1^b(\mathbf{x},\mathbf{y})$ is $\log(\bar{x}/(\bar{x}-m_x))$. In like manner, if $n_2 b$ is 1, then $m_2^b(\mathbf{x},\mathbf{y})$ is $\log(\bar{y}/(\bar{y}-m_y))$.

2.3 Bayesian Test using the Intrinsic Bayes Factor

The element $B_{21}^N$, (4), of the intrinsic Bayes factor is computed in the fractional Bayes factor. So using minimal training sample, we only calculate the marginal densities $m^N(x_i,x_j,y_k,y_l)$ under $H_1$ and $H_2$, respectively.

The marginal densities $m_1^N(x_i,x_j,y_k,y_l)$ under $H_1$ is given by

$$\begin{aligned} m_1^N(x_i,x_j,y_k,y_l) &= \int_0^{m_x}\int_0^{\infty}\int_0^{\infty} f(x_i,x_j,y_k,y_l \mid \eta,\theta_1,\theta_2)\pi_1(\eta,\theta_1,\theta_2)d\theta_1 d\theta_2 d\eta \\ &= \int_0^{m_x}(x_i+x_j-2\eta)^{-2}(y_k+y_l-2\eta)^{-2}d\eta \\ &\equiv T(x_i,x_j,y_k,y_l), \end{aligned}$$

where $1 \le i < j \le n_1, 1 \le k < l \le n_2$. And the marginal density $m_2^N(x_i,x_j,y_k,y_l)$ under $H_2$ is given by

$$\begin{aligned} & m_2^N(x_i,x_j,y_k,y_l) \\ &= \int_0^{m_x}\int_0^{m_y}\int_0^{\infty}\int_0^{\infty} f(x_i,x_j,y_k,y_l \mid \eta_1,\eta_2,\theta_1,\theta_2)\pi_2(\eta_1,\eta_2,\theta_1,\theta_2)d\theta_2 d\theta_1 d\eta_2 d\eta_1 \\ &= \frac{m_x m_y}{(x_i+x_j)(y_k+y_l)(x_i+x_j-2m_x)(y_k+y_l-2m_y)}. \end{aligned}$$

Therefore the IBF of $H_2$ versus $H_1$ is given by

$$\begin{aligned} B_{21}^I &= \frac{1}{L}\sum_{i,j}\sum_{k,l}\frac{(x_i+x_j)(y_k+y_l)(x_i+x_j-2m_x)(y_k+y_l-2m_y)T(x_i,x_j,y_k,y_l)}{(n_1-1)(n_2-1)S(\mathbf{x},\mathbf{y})} \\ &\times \frac{[(\bar{x}-m_x)^{-n_1+1}-(\bar{x})^{-n_1+1}][(\bar{y}-m_y)^{-n_2+1}-(\bar{y})^{-n_2+1}]}{m_x m_y}, \end{aligned}$$

where $L = n_1(n_1-1)n_2(n_2-1)/4$. Note that the calculation of the IBF of $H_2$ versus $H_1$ is requires a one dimensional integration. In Section 3, we investigate our testing procedures using some real examples.

## 3. Numerical Examples

Example 1. To investigate the Bayesian test procedures, we examine the cases when $(\theta_1, \theta_2) = (1,3), (3,1)$, $(\eta_1, \eta_2) = (1,1), (1,2), (1,3)$ and $(n_1, n_2) = (5,5), (5,10), (10,10)$ $(10,20)$. The posterior probabilities of $H_1$ being true are computed assuming equal prior probabilities. The Table 1 shows that the results of the averages and the standard deviations in parentheses of posterior probabilities for each case based on 1,000 replications.

From the Table 1, when $(\eta_1, \eta_2) = (1,1)$, the fractional Bayes factor does not select $H_1$ for some small sample size cases. However the intrinsic Bayes factor gives fairly reasonable answers. Also for moderate sample sizes, the fractional and intrinsic Bayes factors give fairly reasonable answers.

[Table 1]. The averages ( the standard deviations) of posterior probabilities

| $(\theta_1, \theta_2)$ | $(\eta_1, \eta_2)$ | $(n_1, n_2)$ | $P^F(H_1 \mid \boldsymbol{x}, \boldsymbol{y})$ | $P^I(H_1 \mid \boldsymbol{x}, \boldsymbol{y})$ |
|---|---|---|---|---|
| 1,3 | 1,1 | 5,5 | 0.4699(0.1280) | 0.5870(0.1598) |
| | | 5,10 | 0.5122(0.1491) | 0.6552(0.1752) |
| | | 10,10 | 0.5491(0.1572) | 0.6984(0.1749) |
| | | 10,20 | 0.6245(0.1696) | 0.7703(0.1713) |
| | 1,2 | 5,5 | 0.3269(0.1171) | 0.4187(0.1519) |
| | | 5,10 | 0.2034(0.1273) | 0.2866(0.1732) |
| | | 10,10 | 0.1786(0.1038) | 0.2721(0.1541) |
| | | 10,20 | 0.0376(0.0448) | 0.0657(0.0713) |
| | 1,3 | 5,5 | 0.2133(0.1011) | 0.2997(0.1427) |
| | | 5,10 | 0.0591(0.0522) | 0.0915(0.0883) |
| | | 10,10 | 0.0482(0.0432) | 0.0865(0.0768) |
| | | 10,20 | 0.0014(0.0026) | 0.0026(0.0051) |
| 3,1 | 1,1 | 5,5 | 0.4682(0.1257) | 0.5839(0.1594) |
| | | 5,10 | 0.4861(0.1409) | 0.6154(0.1650) |
| | | 10,10 | 0.5449(0.1616) | 0.6900(0.1799) |
| | | 10,20 | 0.5755(0.1649) | 0.7210(0.1755) |
| | 1,2 | 5,5 | 0.2669(0.1816) | 0.3113(0.2228) |
| | | 5,10 | 0.1673(0.2285) | 0.1969(0.2644) |
| | | 10,10 | 0.0826(0.1671) | 0.1021(0.1981) |
| | | 10,20 | 0.0237(0.1042) | 0.0290(0.1227) |
| | 1,3 | 5,5 | 0.1181(0.1367) | 0.1270(0.1600) |
| | | 5,10 | 0.0415(0.1384) | 0.0446(0.1524) |
| | | 10,10 | 0.0037(0.0277) | 0.0042(0.0315) |
| | | 10,20 | 0.0005(0.0121) | 0.0006(0.0139) |

Example 2. The data in Table 2 is taken from Bain and Engelhardt (1991). Suppose a certain additive is proposed for increasing the length of time of tread wear of a tire. Suppose 40 of the present and 40 tires made under the new process are placed in service and the experiment is continued until the 20 smallest observations are obtained for each sample.

The value of fractional Bayes factor and arithmetic intrinsic Bayes factor of $H_2$ versus $H_1$ is $B^F_{21}=0.1314$ and $B^I_{21}=0.0563$, respectively. We assume that the prior probabilities are equal. Then the posterior probability for $H_1$ is 0.8839 and 0.9467, respectively. Thus there are strong evidence for $H_1$ in terms of the posterior probability.

[Table 2]. Length of time of tread wear

| | |
|---|---|
| Present | 10.03 10.47 10.58 11.48 11.60 12.41 13.03 13.51<br>14.48 16.96 17.08 17.27 17.90 18.21 19.30 20.10<br>20.51 21.78 21.79 25.34 |
| Additive | 10.10 11.01 11.20 12.95 13.19 14.81 16.03 17.01<br>18.96 24.10 24.15 24.52 26.05 26.44 28.59 30.24<br>31.03 33.51 33.61 40.68 |

Example 3. Proschan (1963) gives the time of successive failures of the air conditioning system of each member of a fleet of Boeing 720 jet airplanes. The hours of flying time between failures are listed Table 3 for two of the planes.

The value of fractional Bayes factor and arithmetic intrinsic Bayes factor of $H_2$ versus $H_1$ is $B^F_{21}=4.0977$ and $B^I_{21}=1.3836$, respectively. We assume that the prior probabilities are equal. Then the posterior probability for $H_1$ is 0.1962 and 0.4195, respectively. Thus there are evidence for $H_1$ in terms of the posterior probability

Therefore both of the fractional and intrinsic Bayes factors factor give reasonable answers in our examples.

[Table 3]. Time of successive failures

| | |
|---|---|
| Plane 7911 | 55 320 56 104 220 239 47 246 176 182 33 |
| Plane 7912 | 23 261 87 7 120 14 62 47 225 71 246 21<br>42 20 5 12 120 11 3 14 71 11 14 11 16<br>90 1 16 52 95 |

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