T-S Fuzzy Model Based Indirect Adaptive Fuzzy Observer Design

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Abstract

This paper proposes an alternative observation scheme, T-S fuzzy model based indirect adaptive fuzzy observer. Nonlinear systems are represented by fuzzy models since fuzzy logic systems are universal approximators. In order to estimate the unmeasurable states of a given nonlinear system, T-S fuzzy modeling method is applied to get the dynamics of an observation system. T-S fuzzy system uses the linear combination of the input state variables and the modeling applications of them to various kinds of nonlinear systems can be found. The adaptive fuzzy scheme estimates the parameters comprising the fuzzy model representing the observation system. The proposed indirect adaptive fuzzy observer based on T-S fuzzy model can cope with not only unknown states but also unknown parameters. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used. To show the performance of the proposed observation method, it is applied to an inverted pendulum on a cart.

Keywords : T-S fuzzy model, indirect, adaptive observer, lipchitz condition, an inverted pendulum

I. INTRODUCTION

There often conflicts the problems that states are partially or fully unavailable in many practical control problems because the state variables are not accessible for direct connection or, sensing devices or transducers are not available or very expensive. In such cases, observer based control schemes should be designed to generate estimates of the states . Therefore, Observer design has been a very active field during the last decade and has turned out to be much more difficult than the control problem. The design of state observers and the design of controllers can be carried out independently [8].

Based on a consideration of works to represent and identify practical systems, a number of studies on fuzzy modeling and identification have been developed over last decades since fuzzy logic systems are universal approximators [10]. As the structure of fuzzy models, Takagi-Sugeno(T-S) fuzzy system is widely accepted as a powerful tool for design and analysis of fuzzy control systems [9][10]. T-S fuzzy system uses the linear combination of the input state variables and the modeling applications of them to various kinds of nonlinear systems can be found [9][10]. To design fuzzy observers, nonlinear systems are represented by T-S fuzzy models.

Fuzzy systems are supposed to work in situation where there is a large uncertainty or unknown variation in plant parameters and structures. Generally, the basic objective of adaptive control is to maintain consistent performance of a system in the presence of these uncertainties. Therefore, advanced fuzzy systems should be adaptive. If a controller is conducted from adaptive fuzzy systems (an adaptive fuzzy system is a fuzzy logic system equipped with a training algorithm), it is called an adaptive fuzzy controller. The most important advantage of adaptive fuzzy control over conventional adaptive control is that adaptive fuzzy controllers are capable of incorporating linguistic fuzzy information from human operators, whereas conventional adaptive controllers are not [11]. Adaptive schemes for nonlinear systems that incorporate fuzzy systems have been enormously popular [12-18].

Our goal is to design T-S fuzzy model based indirect adaptive fuzzy observer. In order to estimate the unmeasurable states of a given nonlinear system, T-S fuzzy modeling method is applied to get the dynamics of an observation system and based on the T-S fuzzy model, an adaptive fuzzy scheme is proposed. The adaptive fuzzy scheme estimates the parameters comprising the fuzzy model representing the observation system. The proposed indirect adaptive fuzzy observer based on T-S fuzzy model can cope with not only unknown states but also unknown parameters. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used. In addition, the stability condition is given by them.

The rest of this paper is organized as follows. In the Section II, the fuzzy system is briefly reviewed and the problem to be considered is formulated. The proposed observer is designed in Section III and the derivation of adaptive law is derived in Section IV.. In Section V, some example and its computer simulations are given to demonstrate the effectiveness and applicability of the proposed observer. Finally, some conclusions are drawn in Section VI.

II. OVERVIEW AND PROBLEM STATEMENT

A. Takagi-Sugeno Fuzzy Model

T-S fuzzy model can express a highly nonlinear functional relation in spite of a small number of implications of rules [9][19]. T-S fuzzy model can be briefly presented below by the following IF-THEN form or In-Out form.

1) IF-THEN form

Plant rule i:

$$IF\ x\ is\ M_{i1}\ \text{and}\ \dot{x}\ is\ M_{i2}\ \text{and}\ldots\ \text{and}\ x^{(n-1)}\ is\ M_{in}$$

$$THEN\ x^{(n)} = a_i^T x + b_i u, \quad i=1,\ 2,\ ,\cdots, r$$

where

$$x = [x\ \dot{x} \cdots x^{(n-1)}]^T \in R^n, \quad a_i \in R^n, \quad b_i \in R$$

M_{ij} is the fuzzy set and r is the number of rules.

2) Input-Output form

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(x)\{\ a_i^T x + b_i u\ \}}{\sum_{i=1}^{r} w_i(x)} = \sum_{i=1}^{r} h_i(x)\{\ a_i^T x + b_i u\ \}$$

where $w_i(x) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)})$, $h_i(x) = \dfrac{w_i(x)}{\sum_{i=1}^{r} w_i(x)}$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij}.

It is assumed that $w_i(x) \geq 0$, $\sum_{i=1}^{r} w_i(x) > 0$

$i=1,\ 2,\ 3,\ \cdots,\ r$

Hence, $h_i(x) \geq 0$, $\sum_{i=1}^{r} h_i(x) = 1$

B. Problem Specification

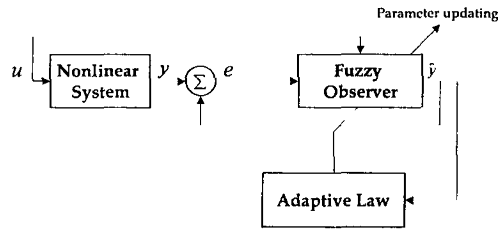
Consider the regulation problem of the following n-th order nonlinear SISO system.

$$\dot{x} = Ax + B[f(x) + g(x)u]$$
$$y = Cx \tag{2-1}$$

where,

$$A=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},\ C=[\ 1\ 0\ \cdots\ 0\ 0]\ .$$

and $f(x)$, $g(x)$ are unknown but bounded continuous nonlinear functions. u is a control input. Let $x=[x\ \dot{x} \cdots x^{(n-1)}]^T = [x_1\ x_2 \cdots x_n]^T \in R^n$ be the state vector of the system which is assumed to be unmeasurable.



<Fig. 1> Configuration of the proposed observer

The goal of this paper is to estimate unknown states and unknown parameters. T-S fuzzy model and indirect adaptive scheme are used to solve this problem. Using T-S fuzzy model, the structure of the proposed observer is designed and the unknown parameters of that is estimated by derived adaptive law. Fig.1 shows the block diagram of the proposed observation system. The nonlinear function can be expressed using TS fuzzy model as follows.

$$f(x) = \sum_{i=1}^{r} h_i(x)\ a_i^T x \tag{2-2}$$

$$g(x) = \sum_{i=1}^{r} h_i(x) b_i \tag{2-3}$$

Therefore, the original system and the observer system can be described based on T-S fuzzy model as follows.

<The original system>

$$\dot{x} = Ax + B\Big[\sum_{i=1}^{r} h_i(x)\ a_i^T x + \sum_{i=1}^{r} h_i(x)\ b_i u\Big]$$
$$y = Cx \tag{2-4}$$

III. INDIRECT ADAPTIVE FUZZY OBSERVER

In this section, a fuzzy observer and adaptive law are developed. The developed observer guarantees to estimate states well. The design process starts with the TS fuzzy model based presentation of the proposed observer system.

Let $\hat{x}$ and $\hat{y}$ be an estimated state vector and the outpu of the proposed observer system.

Where, $\hat{x}=[\hat{x}\ \dot{\hat{x}}\ \cdots\ \hat{x}^{(n-1)}]^T=[\hat{x}_1\ \hat{x}_2\ \cdots\ \hat{x}_n]^T\in R^n$

<The proposed observer system>

$$\dot{\hat{x}}=A\hat{x}+B\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\hat{a}_i^T\hat{x}+\sum_{i=1}^{r}h_i(\hat{x})\hat{b}_iu\Big]+L(y-\hat{y})$$

$$\hat{y}=C\hat{x} \tag{3-1}$$

where, $\hat{a}_i$ and $\hat{b}_i$ are adaptive parameters.

The estimation error, which is the error between the real states and the estimated states is defined as

$$e:=x-\hat{x} \tag{3-2}$$

Therefore, After differentiating e and substituting the original system and the observer system into it, an error dynamic equation can be obtained as follows.

$$\dot{e}=(A-LC)e+B\Big[\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx-\hat{a}_i^T\hat{x})+\sum_{i=1}^{r}h_i(\hat{x})(b_i-\hat{b}_i)u\Big] \tag{3-3}$$

Note that $h_i(\hat{x})$ should be used as the membership function for both x and $\hat{x}$ since only $\hat{x}$ is measurable.

It is very obvious that all entries of e will approach zero if all eigenvalues of $(A-LC)$ have negative real parts ,and $\hat{a}$ and $\hat{b}$ estimate the original parameters a and b well. Hence, The adaptive law should be derived in next process.

IV. DERIVATION OF ADAPTIVE LAW

A Lyapunov function is chosen as follows

$$V=e^TPe+\frac{1}{\alpha_1}\sum_{i=1}^{r}\tilde{a}_i\tilde{a}_i^T+\frac{1}{\alpha_2}\sum_{i=1}^{r}\tilde{b}_i\tilde{b}_i^T \tag{4-1-a}$$

$$\tilde{a}_i=a_i-\hat{a}_i,\ \tilde{b}_i=b_i-\hat{b}_i \tag{4-1-b}$$

where,

V : a positive definite and radially unbounded function

P : a symmetric positive definite matrix

α_1, α_2 : positive adaptation constant gains

After differentiating V, an adaptive law to make $\dot{V}\le 0$ (negative semi-definite) can be constructed. The derivation of V is as follows.

$$\dot{V}=\dot{e}^TPe+e^TP\dot{e}+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \tag{4-2}$$

By substituting (3-3) into (4-2), $\dot{V}$ is expressed as follows

$$\dot{V}=e^T(A-LC)^TPe+e^TP(A-LC)e$$
$$+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx-\hat{a}_i^T\hat{x})+\sum_{i=1}^{r}h_i(\hat{x})(b_i-\hat{b}_i)u\Big]$$
$$+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T$$
$$=-e^TQe+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx+a_i^T\hat{x}-\hat{a}_i^T\hat{x}-a_i^T\hat{x})$$
$$+\sum_{i=1}^{r}h_i(\hat{x})(b_i-\hat{b}_i)u\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T$$
$$=-e^TQe+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\,a_i^Te+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\tilde{a}_i^T\hat{x}$$
$$+\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_iu\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \tag{4-3}$$

Since the error state e has to approach zero as time goes to infinite, an adaptive law that makes $\dot{V}$ negative definite should be derived from the equation, (4-3). In order to obtain the adaptive law, we need some definition and condition as follows.

Definition 1 :

let $\lambda_{min}(M)$ denote the smallest eigenvalue of M and $\lambda_{max}(M)$ the largest. Then, it follows from $M=U^T\Lambda U$ that

$$\lambda_{min}(M)\|x\|^2\le x^TMx\le\lambda_{max}(M)\|x\|^2 \tag{4-4}$$

where, M is a positive definite matrix, $U^TU=I$ and Λ is a diagonal matrix containing the eigenvalues of the matrix M.

Using *Lipchitz condition* [7] ,

$$\sum_{i=1}^{r}h_i(\hat{x})(a_i^Tx-a_i^T\hat{x})\le k_l\|x-\hat{x}\|$$

In addition, according to *Definition 1*, the following inequality is accomplished.

$$\dot{V}=-e^TQe+2e^TPB\sum_{i=1}^{r}h_i(\hat{x})\,a_i^Te+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\tilde{a}_i^T\hat{x}$$
$$+\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_iu\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T$$
$$\le-e^TQe+2k_l\|B\|\lambda_{max}(P)\|e\|^2+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\tilde{a}_i^T\hat{x}$$
$$+\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_iu\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T$$
$$\le-\lambda_{min}(Q)\|e\|^2+2k_l\|B\|\lambda_{max}(P)\|e\|^2$$
$$+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\tilde{a}_i^T\hat{x}+\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_iu\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T$$
$$+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T$$
$$\le-[\lambda_{min}(Q)-2k_l\|B\|\lambda_{max}(P)]\|e\|^2+2e^TPB\Big[\sum_{i=1}^{r}h_i(\hat{x})\,\tilde{a}_i^T\hat{x}$$
$$+\sum_{i=1}^{r}h_i(\hat{x})\tilde{b}_iu\Big]+\frac{2}{\alpha_1}\sum_{i=1}^{r}\dot{\tilde{a}}_i\tilde{a}_i^T+\frac{2}{\alpha_2}\sum_{i=1}^{r}\dot{\tilde{b}}_i\tilde{b}_i^T \tag{4-5}$$

In (4-5), in order to make $\dot{V}$ negative definite, we have to not only find suitable P and Q but also derive the adaptive law of $\hat{a}_i$ and $\hat{b}_i$.

P and Q can be easily chosen, which satisfy (4-6).

$$k_i\|B\| < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{4-6-a}$$

$$B^T P = C \tag{4-6-b}$$

(4-6-a) means that $-[\lambda_{\min}(Q) - 2k_i\|B\|\lambda_{\max}(P)]\|e\|^2$ in (4-5) is negative definite. Therefore, What we only have to do is to converge the rest part of (4-5) to zero such as

$$2e^T PB\left[\sum_{i=1}^{r} h_i(\hat{x})\,\tilde{a}_i^{\,T} x + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u\right] + \frac{2}{\alpha_1}\sum_{i=1}^{r} \dot{\tilde{a}}_i\,\tilde{a}_i^{\,T} + \frac{2}{\alpha_2}\sum_{i=1}^{r} \dot{\tilde{b}}_i\,\tilde{b}_i^{\,T} = 0 \tag{4-7}$$

Now let assume that

$$S = 2e^T PB\left[\sum_{i=1}^{r} h_i(x)\,\tilde{a}_i^{\,T} x + \sum_{i=1}^{r} h_i(x)\,\tilde{b}_i u\right],$$

then $S = S^T$ since S has a scalar value. Hence,

$$2e^T PB\left[\sum_{i=1}^{r} h_i(x)\,\tilde{a}_i^{\,T} x + \sum_{i=1}^{r} h_i(\hat{x})\tilde{b}_i u\right] = 2\left[\sum_{i=1}^{r} h_i(x)\,\tilde{a}_i^{\,T} x + \sum_{i=1}^{r} h_i(x)\tilde{b}_i u\right]^T (y - \hat{y}) \tag{4-8}$$

Note that $B^T P^T e$ equals $(y - \hat{y})$ because of a symmetric positive definite matrix, P and (4-6-b).

Using (4-8), the equation, (4-7) can be expressed as follows.

$$2\left[\sum_{i=1}^{r} h_i(x)\,\tilde{a}_i^{\,T} x + \sum_{i=1}^{r} h_i(x)\tilde{b}_i u\right]^T (y - \hat{y}) + \frac{2}{\alpha_1}\sum_{i=1}^{r} \dot{\tilde{a}}_i\,\tilde{a}_i^{\,T} + \frac{2}{\alpha_2}\sum_{i=1}^{r} \dot{\tilde{b}}_i\,\tilde{b}_i^{\,T} = 0 \tag{4-9}$$

with respect to a,

$$2\sum_{i=1}^{r} h_i(x)\,x^T\,\tilde{a}_i (y - \hat{y}) + \frac{2}{\alpha_1}\sum_{i=1}^{r} \dot{\tilde{a}}_i\,\tilde{a}_i^{\,T} = 0$$

$$\sum_{i=1}^{r} \dot{\tilde{a}}_i\,\tilde{a}_i^{\,T} = -\alpha_1 \sum_{i=1}^{r} h_i(\hat{x})\,\hat{x}^T\,\tilde{a}_i (y - \hat{y})$$

$$= -\alpha_1 (y - \hat{y})\sum_{i=1}^{r} h_i(x)\,x^T\,\tilde{a}_i \tag{4-10}$$

with respect to b,

$$2\sum_{i=1}^{r} h_i(\hat{x}) u^T\,\tilde{b}_i^{\,T}(y - \hat{y}) + \frac{2}{\alpha_2}\sum_{i=1}^{r} \dot{\tilde{b}}_i\,\tilde{b}_i^{\,T} = 0$$

$$\sum_{i=1}^{r} \dot{\tilde{b}}_i\,\tilde{b}_i^{\,T} = -\alpha_2 \sum_{i=1}^{r} h_i(\hat{x}) u^T\,\tilde{b}_i^{\,T}(y - \hat{y})$$

$$= -\alpha_2 (y - \hat{y})\sum_{i=1}^{r} h_i(\hat{x}) u^T\,\tilde{b}_i^{\,T} \tag{4-11}$$

where, from (4-1-b), $\dot{\tilde{a}}_i = -\dot{\hat{a}}_i$, $\dot{\tilde{b}}_i = -\dot{\hat{b}}_i$

From (4-10) and (4-11), $\hat{a}_i$ and $\hat{b}_i$ are obtained after some calculation.

$$\dot{\hat{a}}_i^{\,T} = \alpha_1 (y - \hat{y}) h_i(\hat{x})\,x^T$$

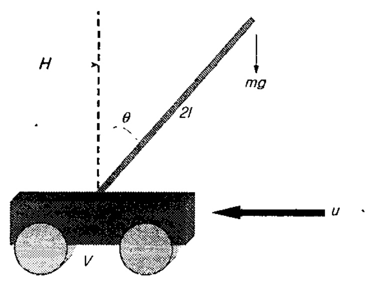
$$\dot{\hat{b}}_i = \alpha_2 (y - \hat{y}) h_i(\hat{x}) u^T$$

V. SIMULATION

Let's consider the problem of balancing and swing-up of an inverted pendulum on a cart. The state equation of motion for the pendulum is as follows.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = f(x) + g(x)u + d(t) = \frac{g\sin(x_1) - aml\,x_2^2\sin(2x)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \tag{5-1}$$



<Fig. 2> Inverted Pendulum System

where

x_1 angle θ (in radians) of the pendulum from the vertical;

x_2 the angular velocity;

g the gravity constant, $9.8\ m/s^2$;

m the mass of the pendulum;

M the mass of the cart;

$2l$ the length of the pendulum;

u the control force applied to the cart (in Newtons);

$a = \frac{1}{m + M}$.

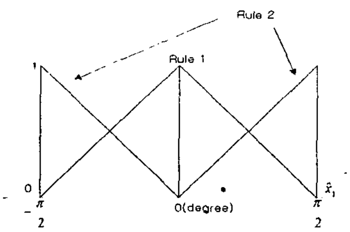
We choose $m = 2.0kg$, $M = 8.0kg$, $2l = 1.0m$ in the simulation.

As a model for the pendulum, we use the following Takagi-Sugeno fuzzy model with two rules. Membership functions are shown in Fig. 2.

Rule 1 : *IF* $\hat{x}_1$ *is about* 0 *THEN* $\hat{x}_2 = a_1{}^T\hat{x} + b_1u$

Rule 2 : *IF* $\hat{x}_1$ *is about* $\pm\frac{\pi}{2}$ $(|\hat{x}_1|<\frac{\pi}{2})$

THEN $\hat{x}_2 = a_2{}^T\hat{x} + b_2u$

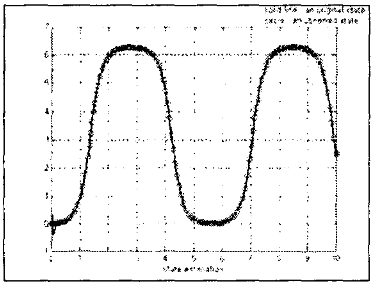


<Fig. 3> Membership Function

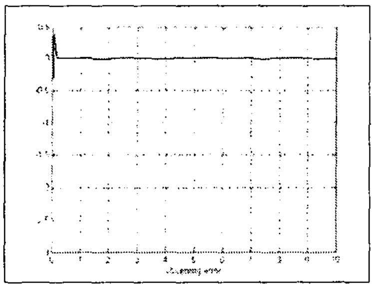
For computer simulation, the following observer gain and adaptive gain are used.

$$L=[\,70 \quad 1250\,],\ \alpha_1=\alpha_2=1$$

Note that the observer gain can be every value, which satisfies $L=\left[\,L_1 \quad \frac{L_1{}^2}{4}\,\right]$. The only difference between a high gain and a low gain is how fast observer states catch up with original states.



<Fig. 4> Estimation of state x_1



<Fig. 5> Observation error

Figs.4 and 5 show the simulation results of the proposed observer. From Fig. 4, it can be seen that the observer state, $\hat{x}_1$ observes the original state x_1 very well as expected. There shows that the observation error disappears within very short time in Fig. 5.

VI. CONCLUSION

This paper proposed an indirect adaptive fuzzy observer design method based on T-S fuzzy model and applied it to an inverted pendulum on a cart in order to show the performance of the proposed observer. T-S fuzzy model was adapted to construct the structure of the proposed observer system. Using indirect adaptive law, system parameters were estimated. In the process of deriving adaptive law, the Lyapunov theory and Lipchitz condition are used. Therefore, the proposed observer system were able to deal with not only unknown parameters but also unknown states. At last, simulation results confirmed that the proposed algorithm could achieve the observation problem of unknown states together with the estimation problem of unknown parameters.

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APPENDIX

Remark :

$$k_f\|B\| < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \quad B^T P = C$$

$$\eta = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$$

$$\Pi = \begin{bmatrix} A - LC & 0_{n \times 1} \\ 0_{1 \times n} & -\sigma \end{bmatrix}$$

where, η denotes the maximum convergence rate of Π and cannot be increased arbitrarily for a fixed Π in general. In this case, however, it can be shown that the maximum convergence rate η can be increased arbitrarily by the appropriate choice of elements of Π. Consider the characteristic equation of Π.

$$\begin{aligned} det(sI - \Pi) &= (s+\sigma)det(sI - A + LC) \\ &= (s+\sigma)(s^n + l_1 s^{n-1} + \cdots + l_n) = 0 \end{aligned}$$

In addition, since the maximum convergence rate η can be interpreted as the negative value of the maximum real part of the eigenvalues of Π as follows.

$$\eta = -\max(-\sigma, Re(\lambda_i)), \quad i = 1, \ldots, n$$

$$\lambda_i^n + l_1 \lambda_i^{n-1} + \cdots + l_n = 0$$

It can be shown that, by increasing σ and choosing $L = [l_n \cdots l_1]^T$ such that all roots of the polynomial $d(s) = s^n + l_1 s^{n-1} + \cdots + l_n$ move leftwards, η can be increased arbitrarily and, consequently.