A Study on Constructing Digital Logic Systems based on Edge-Valued Decision Diagram

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Abstract:

This paper presents a method of constructing the digital logic systems(DLS) using edge-valued decision diagrams(EVDD). The proposed method is as following. The EVDD is a new data structure type of decision diagram(DD) that is recently used in constructing the digital logic systems based on the graph theory. Next, we apply EVDD to function minimization of digital logic systems. The proposed method has the visible, schematical and regular properties.

Key Words : Digital Logic Systems, Decision Diagram, Edge-Valued Decision Diagram, Function Minimization

1. INTRODUCTION

In recently, the method coming up important method that constructing the digital logic systems[1,2] based on graph theory[3-5].

S.B.Aker[6] defined concept firstly of decision diagram(DD) that is directed acyclic graph(DAG) type, also derived binary decision diagram(BDD) from above DD.

R.E.Bryant[7] research firstly the Boolean function construction and function minimization using BDD that was proposed by S.B.Akers.

Also, Yung-Te Lai etal[8] and S.B.K. Vrudhula etal[9] propose binary edge-valued decision diagram(EVBDD) that improved BDD's drawback.

That has advantage higher state instead of Boolean function representation expression, also hierarchical verification. Therefore EVBDD has canonical representation characteristics.

Also, D. M. Miller[10] propose firstly multiple-valued[11,12] decision diagram which extended the binary decision diagram.

This paper propose algorithm of constructing the multiple-valued edge-valued decision diagram(MVEVDD) that extended EVBDD, also propose a method of constructing the digital logic systems after minimizing the p-valued n variable function.

This paper's organization is as following.

In section 2, we discuss properties of graph in order to analysis and synthesis decision diagram, also discuss the concept function minimization using decision diagram.

In section 3, we discuss the properties and variable valued extension of binary decision diagram.

In section 4, we discuss mathematical background for edge-valued decision diagram.

In section 5, we discuss the algorithm of multiple-valued edge-valued decision diagram from binary edge-valued decision diagram.

Also, in section 6, we discuss multiple variable extension based on above section 4 and section 5.

In section 7, we apply our proposed algorithm to some example, then we compare and investigate result with reference.

In section 8, we summary characteristics of our edge-valued decision diagram, also we prospect future research fields.

2. PROPERTIES OF DECISION DIAGRAM

In this section, we discuss properties of graph in order to analysis and synthesis decision diagram, also discuss the concept function minimization using decision diagram.

2.1. Properties of Graph

The graph represented as following expression (2-1) in generally.

$$G(V,E) \qquad (2\text{-}1)$$

Where, V is finite set of nonempty node, and E is set of two subset branch in node set.

Also, $|V|$ is order of graph and means number of node, $|E|$ is size of graph and means number of branch.

In case of set for all node that placed adjacently node we call neighbored N(V), also we call degree of node deg(V) in case of number of node that happened in node V.

Specially, we call $dsg_G(V)$ that happened in graph G.

If node V placed in level k, we call child that placed in adjacently in level k+1, also we call parent that placed in adjacently in level k-1.

You can see references[3-5] in case of any other properties besides above properties.

2.2. Decision Diagram

Decision diagram is directed acyclic graph, also use parent node and children node, and node means logic variable and branch means input variable. The node of end was called terminal node, and means logic constant. Also, the node of starting was called root node and the other nodes are called non-terminal node and mean variables.

Each branch from each node have logic variable in relation to its node and branch. The node in relation to any logic variable be able to multiple presence, and branch from each node is equal to number of input variable.

Decision diagram has tree property, node of 1st level must one, also sub-tree occurred as much as input variable from root node. Decision diagram was changed according to sequence of variable that mean node, also multiple decision diagram exit for one logic variable. The terminal node's value equal to function's value. We refer to references besides above properties of decision diagram.

2.3. Function Minimization using by Decision Diagram

The decision diagram is visible for representation of logic function, and function minimization or simplification is easy using decision diagram.

The number of nodes over BDD is equal to gate when we design logic circuit, therefore reduction of node over decision diagram is function minimization.

In generally, BDD that was constructed with n variables have 2^{n+1}-1 node and 2^n terminal nodes.

Therefore, minimization processing and ordering variable are important factors.

3. BINARY EDGE-VALUED DECISION DIAGRAM

In this section, we discuss properties of edge-valued binary decision diagram(EVBDD) and variable extension of EVBDD.

3.1. Edge-Valued Binary Decision Diagram

EVBDD have advantage for handling integer operation and Boolean expression but BDD have handling only Boolean expression.

3.2. Variable value Extension of EVBDD

EVBDD' node have each variable allocation, then edge was defined according to variable value.
Variable have all $\{0,1\}^n$'s value, if variable have constant input instead of binary value, variable must represent binary vector.

4. MATHEMATICAL BACKGROUND

In this section, we discuss important mathematical properties that used in opened this paper.

4.1. Literal

We define following definitions in order that we extend from binary over BDD to multiple-valued.

[Definition 4-1]

Let X_i is multiple-valued variable which have any value among set Pi={0, 1, 2, ... , (P-1)}.

For any subset $S_i \subseteq P_i$, X_i^{Si} is literal which represent the following expression (4-1).

$$X_i^{Si} = 1 \ \text{if} \ S_i \in X_i$$
$$0 \ \text{if} \ S_i \notin X_i \qquad (4\text{-}1)$$

4.2. Reed-Muller Expansion

In case of general P-valued n variable extension, we use Reed-Muller expansion(RME) which extended Boolean arithmetic operation using by mod arithmetic operation,. Also we derived general expression of multiple-valued edge-valued decision diagfam from that.

The Boolean arithmetic operation is represented by following expression (4-2), because that logic sum and logic product in Boolean arithmetic operation be able to replace with mod2 sum and mod2 product,

$$F(X_1, X_2, \ldots\ldots, X_{n-1}, X_n)$$
$$= \sum_{i=0}^{K-1} C_i X_1^{ei,1} X_2^{ei,2}, \ldots, X_{n-1}^{ei,n-1} X_n^{ei,n} \qquad (4\text{-}2)$$

where, $K=2^n$ and $\sum$ is mod2, $e_{i,j}$(i=0,1,......,2n-2 ; j=1,2,......,n) is 0 or 1, also C_i is constant.

On the other hand, C_i obtained using by following expression (4-3)

$$T\begin{bmatrix} Tn\text{-}1 & 0 \\ Tn\text{-}1 & Tn\text{-}1 \end{bmatrix} \qquad (4\text{-}3)$$

Where, $T_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

5. MULTIPLE-VALUED EDGE-VALUED DECSION DIAGRAM

5.1. Definition of MVEVDD

[Definition 5-1]

MVEVDD was constructed with pair <C, F>, where C is constant and F has a node following two case.

(1)one terminal node which was represented by **0**

(2)Non terminal node V was constructed P+2 tuple <variable(V), $child_1$, $child_{m,1}(V)$, ... , $child_{m,P-2}(V)$, $child_r(V)$, value>, where variable(V) is multiple-valued variable.

[Definition 5-2]

P-valued edge-valued decision diagram <C, F> represent arithmetic operation C+ Ψ, where Ψ is function which represent by F.

Also, **0** means constant function), <X, l, (m,1), (m,2), ... , (m,P-2), r, v> is arithmetic operation function $X(l+v)+(X-1)_{m,1}+ \ldots\ldots +\{X-(P-2)\}_{m,P-2}+\{X-(P-1)\}$

5.2. Conversion Expression

For example, the following expression is P-valued 3 variable conversion expression.

$$F(X_1, X_2, X_3)=\beta+X_1[\delta_1+X_2\{\zeta_1+(\zeta_2+\zeta_4)X_3\}]+\sum_{j=0}^{P-1}(X_2-j)$$

$$\delta_3 X_3+\sum_{i=0}^{P-1}\sum_{j=0}^{P-1}(X_1-j)[X_2\{\delta_2+\zeta_2\}+(X_2-j)\,\delta_3 X_3]$$

5.3. Algorithm of generation MVEVDD from EVBDD

In this section, we discuss the algorithm of generation multiple-valued edge-valued decision diagram from edge-valued binary decision diagram.

[STEP 1] Insert middle edge for the purpose to make node which P tuple <X, l, m1, ... ,mp-2, r,v> which represent arithmetic operation of MVEVDD.

[STEP 2] After insert middle edge, convert arithmetic operation function at each node to $X(l+v)+(X-1)_{m,1}+ \ldots\ldots +[X-(P-2)]_{m,p-2}+[X-(P-1)]_r$.

[STEP3] Apply threshold detector to each node, the result was represented in following expression (5-1).

$$X = \begin{cases} 1 & \text{if } X=m \\ 0 & \text{Otherwise} \end{cases} \qquad (5\text{-}1)$$

The expression (5-1) has logic 1 in case of input variable's value is P-valued, and has 0 in any other case.

6. MVEVDD EXTENSION IN MULTIPLE VARIABLE

The number of coefficient for each degree in n-variable function expansion is as following expression (6-1).

$$_nC_0+{}_nC_1+ \ldots + {}_nC_{n-1}+{}_nC_n = \sum_{i=0}^{n} {}_nC_i = 2^n-1 \qquad (6\text{-}1)$$

therefore, expression (4-2) can expressed in following expression (6-2).

$$F(X_1, X_2, \ldots\ldots , X_{n-1}, X_n) =$$

$$C_0 + \sum_{i=0}^{K-1} C_i X^{ei,1} X^{ei,2}, \ldots , X^{ei,n-1} X^{ei,n} \qquad (6\text{-}2)$$

The processing of edge-valued according variable X(i=1, 2, ... ,n) is as following.

$$F|_{Xi}(i=1, 2, \ldots ,n) \qquad (6\text{-}3)$$

First, we extended P-valued case.

$$F|_{Xi}= C_0+X_1(C_1+\sum_{i=2}^{K-1} C_i\ X_2^{ei,1}\ X_3^{ei,2},\ \ldots\ldots ,\ X_{n-1}^{ei,n-1}$$

$$X_n^{ei,n}$$

$$+\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n})$$

$$+(1-X_1)(\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots, X_{n-1}^{ej,n-1} X_n^{ej,n}) \qquad (6\text{-}4)$$

$$F|_{Xi}= C_0+X_1(C_1+\sum_{i=2}^{K-1} C_i\ X_2^{ei,1}\ X_3^{ei,2},\ \ldots\ldots ,\ X_{n-1}^{ei,n-1}$$

$$X_n^{ei,n}$$

$$+\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n}$$

$$+(X_1-1)(\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n})$$

$$+(X_1-2)(\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n})$$

$$+(X_1-(P-1))(\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n})$$

$$= C_0+X_1(C_1+\sum_{i=2}^{K-1} C_i X_2^{ei,1} X_3^{ei,2}, \ldots\ldots , X_{n-1}^{ei,n-1} X_n^{ei,n}$$

$$+\sum_{j=2}^{K-1} C_j X_2^{ej,1} X_3^{ej,2}, \ldots\ldots , X_{n-1}^{ej,n-1} X_n^{ej,n})$$

⋮

$$+\sum_{g=1}^{m-1}(X_1-g)\left(\sum_{j=2}^{K-1}C_iX_2^{ej,1}X_3^{ej,2},\ldots,X_{n-1}^{ej,n-1}X_n^{ej,n}\right) \quad (6\text{-}5)$$

where, $K=2^n$

Here, we put $F|_{X0}=F|_0=C_0$ for constant term, generalized n-variable expression is as following.

$$F(X_1, X_2, .., X_{n-1}, X_n)=\sum_{i=0}^{m-1}F|_{X1} \quad (6\text{-}6)$$

Here,

$F|_{X0}=F|_0=C_0$

$$F|_{X1}=\sum_{i=1}^{K-1}C_iX_1^{ei,1}X_2^{ei,2},\ldots\ldots,X_{n-1}^{ei,n-1}X_n^{ei,n}$$

$$F|_{X2}=\sum_{i=2}^{K-1}C_iX_2^{ei,1}X_3^{ei,2},\ldots\ldots,X_{n-1}^{ei,n-1}X_n^{ei,n}$$

:

:

$$F|_{Xn-1}=\sum_{i=n-1}^{K-1}C_iX_{n-1}^{ei,n-1}X_n^{ei,n}$$

$$F|_{Xn}=\sum_{i=n}^{K-1}C_iX_n^{ei,n}$$

Where, $K=2^n$

$$F|_{X1}=X_1(C_1+F|_{X2}+F^*|_{X2})+\sum_{g=1}^{P-1}(X_1-g)F^*|_{X2}$$

$$F|_{X2}=X_2(C_2+F|_{X3}+F^*|_{X3})+\sum_{g=1}^{P-1}(X_2-g)F^*|_{X3}$$

:

:

$$F|_{Xn-1}=X_{n-1}(C_{n-1}+F|_{Xn}+F^*|_{Xn})+\sum_{g=1}^{P-1}(X_{n-1}-g)F^*|_{Xn} \quad (6\text{-}7)$$

Here, $F|_{X0}=F|_0=C_0$, and $F^*|_{Xi}$ is function which have non-edged branch.

Therefore, the generalized P-valued n-variable edged function can be represented as following.

$F(X_1, X_2, \ldots\ldots, X_{n-1}, X_n)=$

$$F|_{Xi}=X_i(C_i+F|_{Xi+1}+F^*|_{Xi+1})+\sum_{g=1}^{P-1}(X_{n-1}-g)F^*|_{Xi+1}$$

$(i=1, 2, \ldots, n)$ (6-8)

The following figure 6-1 depicted above MVEVDD.

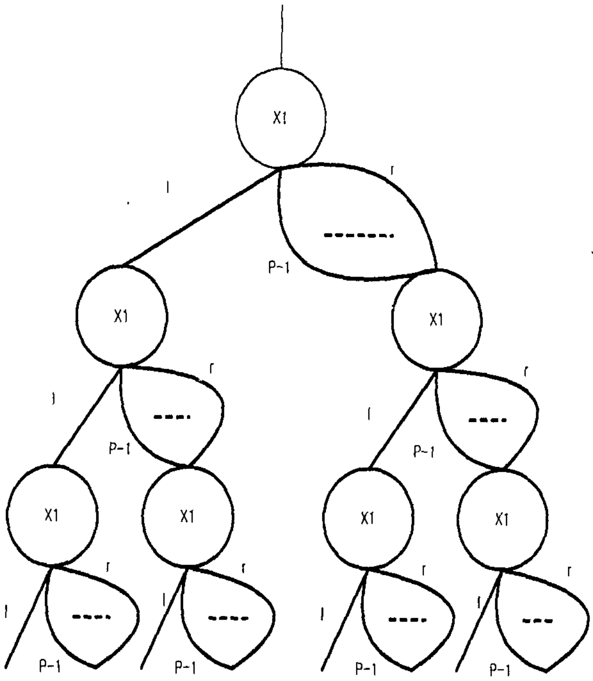

Fig. 6-1. General structure of P-valued n-variable MVEVDD.

7. COMPARISON AND DISCUSSION

In this section, we apply our proposed method to some example, we compare with result and investigate its result.

For example, we apply our algorithm for obtaining edge-valued decision diagram to Function $F(X_1, X_2, X3)=-2+5X_2+X_2X_3+3X_1X_2+4X_1X_2X_3-2X_1X_3+X_3$.
Next, we obtain the transformation expression for EVBDD and EVTDD.

First, we transfer function to EVBDD, it is as following.
$F(X_1,X_2,X3)=-2+5X_2+X_2X_3+3X_1X_2+4X_1X_2X_3-2X_1X_3+X_3$
$=-2+X_2[(5+X_1(3+X_3(4))+(1-X_1)(X_3(2)))+(1-X_2)[X_1(X_3(-1))+(1-X_1)(X_3(1))]$ (7-1)

Next, we transfer function to EVTDD, it is as following.
$F(X_1, X_2, X3)=-2+X_2[5+X_1(3+X_3(4))+(X_1-1)(X_3(2))+(X_1-2)(X_3(2))+(X_2-1)[X_1(X_3(-1))+(X_1-1)(X_3(1))+(X_1-1)(X_3(1))]+(X_2-2)[X_1(X_3(-1))+(X_1-1)(X_3(1))+(X_1-2)(X_3(1))]$ (7-2)

Table 7-1. The comparison table for each DDs

	BDD	EVBDD	Constant input EVBDD	Constant input EVTDD
Number of Node	$2^{n+1}-1$	2^n	$N\bullet(2^n-1)$	$(n-1)\bullet(2^n-1)$

Table 7-2. The comparison table between this paper method and previous methods

	Edge-valued DD	Applied Function	Regularity & Schematics
Y.T.Lai etal	EVBDD	Boolean Function	▲
S.B.K. Vrudhula etal	EVBDD	Boolean Function	▲
This paper	MVEVDD	P-Valued n-variable Function	●

Remarks 1 :
EVBDD : Edge-Valued Binary Decision Diagram
EVTDD : Edge-Valued Ternary Decision Diagram
MVEVDD : Multiple-Valued Edge-Valued Decision Diagram

Remarks 2 :
▲ : Available part
⦿ : Some good
● : Good

8. CONCLUSION

This paper present a method of constructing P-valued n-variable digital logic systems using multiple-valued edge-valued decision diagram which extended the concept of binary decision diagram and its new data structure edge-valued decision diagram.
The proposed method is more regularity
For the future, it require the more general type of MVEVDD, also we apply proposed method to any other digital logic design techniques.

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