Generalization of ALOHA with capture effect in case of two power levels

DINH CHI HIEU
Radio Frequency Department, Ministry of Posts & Telematics, S.R. of Vietnam
Tel: +84-4-8582662 E-mail: hieusdc@yahoo.com

Abstract: This paper proposed a systematic analysis for slotted ALOHA with capture effect. This is a generalization for slotted ALOHA system. Based on this model, we can increase the maximum throughput of slotted ALOHA system with two power levels. Lee's algorithm is consider to be an extension of ALOHA system with capture effect. In this paper, we showed that, the choice of Lee's algorithm is not an optimum one. Based on the previous experimental results, we proposed here a more practical analysis for slotted ALOHA system. The result is very accurate and can be applied to other wireless systems which also employed capture effect.

Keywords: Slotted ALOHA, capture effect.

1. INTRODUCTION

1.1. ALOHA system

The ALOHA system devised by Abramson [6] and others at the University of Hawaii employs a satellite repeater that broadcasts the packets received from the various users who access the satellite. This method allows many user access the channel without coordination [8]. The idea here was that whenever a packet arrived at a transmitter, it would simply be transmitted, ignoring all other transmitters in the network. If another transmitter was transmitting in an overlapping interval then we have collision and those packets will be retransmitted later. This is call pure/unslotted ALOHA. The performance of pure ALOHA is quite low:

$$S = Ge^{-2G} \quad (1)$$

Where S is throughput of the system and G is the offered load. We observer that $S_{max} = 1/2e \approx 0.18$ [packets/slot] when $G = 1/2$.

Slotted version of ALOHA was constructed by only allowing packets to transmit in time slot. By this modification, we can have a better performance:

$$S = Ge^{-G} \quad (2)$$

In this case the improvement is $S_{max} = 1/2e \approx 0.37$ [packets/slot] when $G = 1$.

Over the years, this basic strategy has been improved, generalized, and analyzed in many ways [7]. In this paper, we only refer to slotted version and variation of ALOHA system.

1.2. Capture effect

It is generally accepted that capture effect can increase performance of the system. Here we only consider captured effect in case of two power levels. The extension for many power levels is possible but of little real application to wireless communication, although this problem is still interest in the theoretical point of view. In this paper, we accept those rules:

- If two or more packets at the same power lever are transmitted at the same time then collision occurred.
- If there are multiple packets are transmitted at the same time but there is only one packet that has high power level and less than C packets with low power level then the high power level packet will be received successfully. This effect is called "*capture effect*". This model is base on the real measurement results in [5] and was employed in [4].

1.3. Slotted ALOHA with capture effect

By employing capture effect, one can greatly improve performance of the system. The question here is how to find the best strategy? It seems natural to randomly choose the power level for each transmission equally and independently. Most researchers followed this trend [1], [3],... Recently, Sarker, ect. [2] proposed to choose high power level with different probability. Unfortunately, [2] have two week points:

1. Supposed perfect capture effect, that means high power packets can always get through the channel successfully.
2. The choice of p, probability to transmit a packet in low power level, is arbitrary.

In Lee's algorithm [1], the author choose $p = 0.5$. In this paper, we propose an algorithm to systematically choose p in order to maximize the throughput of the system. By these results, we show that the Lee's choice is not an optimum one.

Figure 1 is an example of this channel model. Here

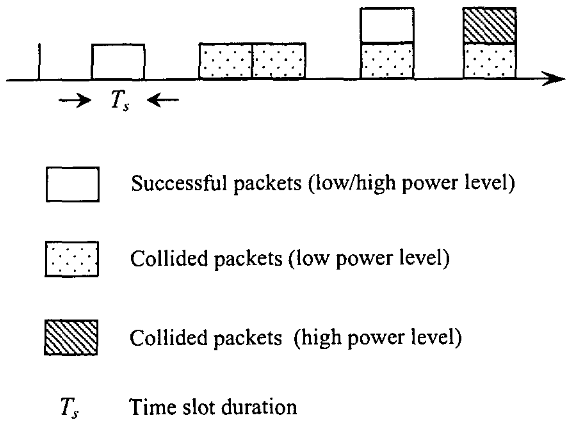

Fig. 1 An example of the channel model

we also consider that low power packet(s) that is/are captured by high power packet is/are collided packet(s).

2. SYSTEM MODEL

Let us consider in case of two power levels. In order to obtain generic throughput expression, we make use the following notations:

- p probability that a mobile terminal selects low power level ($0 \leq p \leq 1$), then 1-p is the probability of that terminal to select high power level.
- λ mean packet arrival at low power level [packets/s].
- T_s Time slot duration [s].
- $G = \lambda T_s$ Offeres load .

Arrival rate follows Poison model:

$$P(X = k) = (\lambda^k/k!) \exp(-\lambda) \quad (3)$$

$P(X = k)$ is the probability that there are k packets transmitted at low power level.

C number of low level packets that capture effect can occur.

The transmission follows renewal process. Under these conditions, throughput of system can be expressed as in (4).

3. THROUGHPUT ANALYSIS

Equation (4) can be written as in (5). Equation (5) means that throughput of two power levels ALOHA system always greater than throughput of original ALOHA system no matter what the choice of p is. In the limited case, when high power packet can always get through the channel, we have:

$$\begin{aligned} S_{pc} &= \lim_{C \to \infty} S_c \\ &= Ge^{-G}\left((1-p)e^{Gp} + p\right) \end{aligned} \quad (6)$$

S_{pc} in (6) is called throughput of perfect power capture effect. We can see clearly that there are big different in the form of S_c and S_{pc}.

3.1. Maximum throughput

First, let we consider the case of perfect capture, $C = \infty$, and then we will move to the more practical case of $C \geq 2$.

a. *Perfect power capture effect case:*

To have maximum throughput, we have to solve equation :

$$\begin{aligned} S_c &= P_{high}(X=1) \times P_{low}(X \leq C) + P_{high}(X=0) \times P_{low}(X=1) \\ &= G(1-p)e^{-G(1-p)}\left(\sum_{n=0}^{C}\frac{(Gp)^n}{n!}\right)e^{-Gp} + e^{-G(1-p)}\left(Gpe^{-Gp}\right) \\ &= Ge^{-G}\left((1-p)\sum_{n=0}^{C}\frac{(Gp)^n}{n!} + p\right) \end{aligned} \quad (4)$$

$$\begin{aligned} S_c &= Ge^{-G}\left((1-p)\sum_{n=0}^{C}\frac{(Gp)^n}{n!} + p\right) \\ &= Ge^{-G}\left((1-p)\sum_{n=1}^{C}\frac{(Gp)^n}{n!} + 1\right) > Ge^{-G} \end{aligned} \quad (5)$$

$$\frac{dS_{pc}}{dG} = 0 \qquad (7)$$

Taking derivation of S_{pc} we will have:

$$\frac{dS_{pc}}{dG} = e^{-G}\left((1-p)e^{Gp}+p\right) - Ge^{-G}\left((1-p)e^{Gp}+p\right) + Ge^{-G}\left(p(1-p)e^{Gp}\right) \qquad (8)$$

Simplify (8) to suite our need we can get a simpler equation:

$$(1-p)e^{Gp}\left(1-(1-p)G\right) + p(1-G) = 0 \qquad (9)$$

(9) does not have explicit solution. Here we can find numerical solution of G_{max}. Substitute G_{max} to (6) we can find S_{cp}*. Because $p \in [0, 1]$, we can search in this range to find the $max\{S^*_{pc}\}$. This result is shown graphically in figure 2. We can see from that figure that our method can yield much better result than Lee's choice.

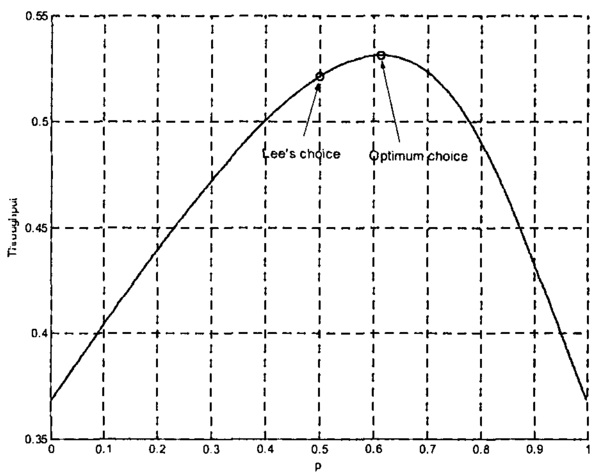

Fig. 2 Maximum comparison in case of perfect power capture effect (C = ∞)

b. Power capture effect with C = 2 case

Although the results in case of perfect captured effect case as above is interesting, it is very hard to get these results in real condition because: we need *very high* transmission power for high power packets transmission, that is impractical for most equipments (even for the case of $C = 2$, we will need at least 9 dB difference [4]). In the case of $C = 2$, equation (5) becomes: (10). Derivation of S_c is given in (11). Repeat the same procedure for finding $max\{S_c^*\}$, we have to solve equation (12):

$$\frac{dS_c}{dG} = 0 \qquad (12)$$

Rewrite (12) to get a polynomial form, we got (13). Equation (13) is a third degree equation. For convenience, we will present (13) in the appendix part.

Solving this equation, we get three solutions, two of them are complex solutions because of the constraint

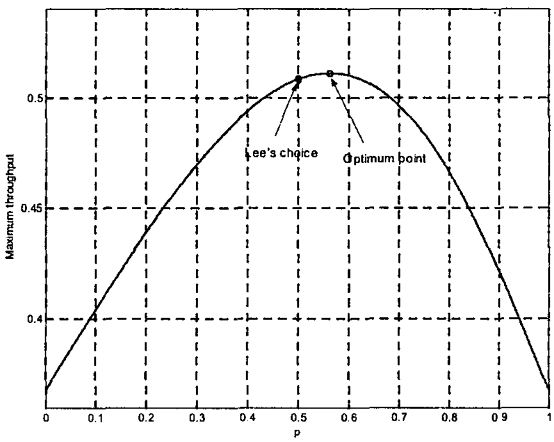

Fig. 3 Maximum comparison in case of power capture effect $C = 2$

of p. The real solution is very complicated and is show in the appendix of this paper. The result is show in figure 3, we also see that, we get improvement comparing to Lee's algorithm.

4. SIMULATION RESULT

These results are verified by stochastic computer simulation program. Simulation program was run for the case of $p = 7/8$ and show as an example, it shows excellence agreements with the theoretical analysis. For other value of p we have the same results, these results are not shown here because of the limitation of the paper.

At $p = 7/8$ then we have:

$$S_c = Ge^{-G}\left\{(1-p)\left[\frac{(Gp)^2}{2!} + Gp + 1\right] + p\right\} \qquad (10)$$

$$\frac{dS_c}{dG} = e^{-G}\left\{(1-p)\left[\frac{(Gp)^2}{2} + Gp + 1\right] + p\right\} - Ge^{-G}\left\{(1-p)\left[\frac{(Gp)^2}{2} + Gp + 1\right] + p\right\} + Ge^{-G}(1-p)\left(Gp^2 + p\right) \qquad (11)$$

$$G_{max} \approx 1.2319 \quad \text{and} \quad S_{c\max} \approx 0.4339$$

Results is shown in figure 4 in case of slotted ALOHA without capture effect; $p = 7/8$ with power capture effect in both $C = 2$ and $C = \infty$ for comparison purpose.

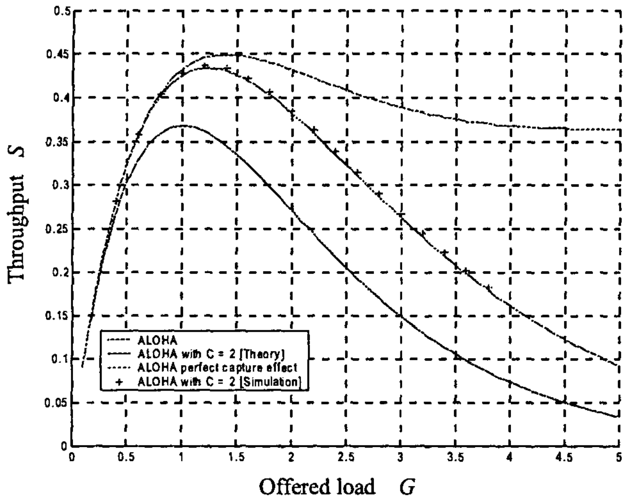

Fig. 4 System performance comparison at $p = 7/8$

From figure 4, we can see that our proposed algorithm is very accurate. It can predict the performance of the system with high accuracy. From here, we can also get the big difference of the throughput of the system in case of perfect, but unpractical, capture effect and in more real case.

Comparison of Lee's algorithm and proposed algorithm:

Table 1 Comparison of maximum throughput

Maximum Throughput	C = 2	C = ∞
Lee's algorithm	0.5100	0.5200
Proposed algorithm	0.5111	0.5315
$p_{optimum}$	0.563	0.6125

Figure 5 is a comparison of Lee's algorithm and our proposed algorithm. In Lee's algorithm p is *always* equal to 0.5, but in this case, by our calculation, the optimum performance should be with $p \approx 0.6125$. We can see that, S_{max} shifts to the right and the system get better throughput when G increase. .

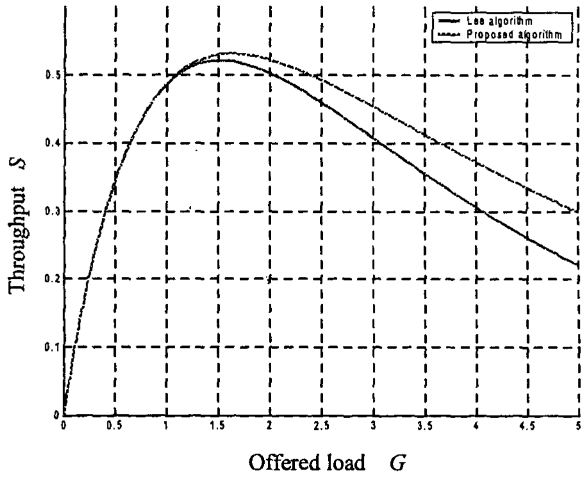

Fig 5. Throughput comparison in case of perfect power captured effect

5. CONCLUSION

A general frame work for analyzing slotted ALOHA with capture effect is proposed. The results show that this approach is more practical and very accurate.

The gain of using capture effect is significant, even in the case of two power level. The results of this proposed algorithm is more practical model for wireless system using capture effect than previous algorithms. This result also correct some misunderstanding in the previous papers.

Base on the algorithm here, one can apply it to systematically analyze performance of wireless communication system that employ capture effect.

The case of more than two power levels are now under investigation.

6. APPENDIX

Taking derivation of S_c and make some simple manipulation we can get the expression of equation (13) is as below.

This is a solvable equation. For practical purpose, we only pay attention to the real solution of (13). Here is the real solution of (13), other two complex solutions are not shown here:

$$\left\{\frac{1}{2}p^2(1-p)\right\}G^3 + \left\{p(1-p) + \frac{3}{2}p^2(1-p)\right\}G^2 + \{2p(1-p) - 1\}G + 1 = 0 \quad (13)$$

$$A = \sqrt{\frac{12 + 12p - 12p^2 + 24p^3 - 18p^4 + 12p^5 - 114p^6 + 108p^7}{1 - p}} \qquad (14)$$

$$B = \sqrt[3]{\left(-10 - 8p - 27p^3 + 27p^4 + 3A\right)\left(1 - p\right)^2} \qquad (15)$$

$$C = \left(\frac{2 + 4p - 9p^2 + 9p^3}{pB}\right) \qquad (16)$$

$$G_{max} = \frac{B}{p(p-1)} + C + \frac{3p-2}{3p} \qquad (17)$$

References

[1] C. C. Lee, *Random signal levels for channel access in packet broadcast networks*, IEEE Journal on Selected Areas in Communications, Vol. SAC-5, No 6, Jul. 1987

[2] J. H. Sarker, M. Hassan, S. J. Halme, *Power level selection schemes to improve throughput and stability of slotted ALOHA under heavy load*, Computer Communications, No 25, 2002

[3] S. Shimamoto, Y. Onozato, Y. Teshigawara, *Performance evaluation of power level division multiple access (PDMA) scheme*, ICC '92

[4] S. Shimamoto, H. Kudo, Y. Onozato, *Fixed Assignment based window access with capture (FAWAC) for personal satellite communications*, GLOBECOM '94

[5] S. Shimamoto, H. Kubota, H. Kuwabara, Y. Onozato, *A study on satellite network configuration control employing transmission power control*, Electronics and Communications in Japan, Vol. 83, No. 7, 2000

[6] N. Abramson, *The throughput of packet broadcasting channels*, IEEE Trans. Commun., Vol. COM-25, No. 1, 1977

[7] Gallager R. G., *A perspective on multiple access channels*, IEEE Trans. On Information Theory, IT-31, 1985

[8] John G. Proakis, *Digital communications*, 4th edition, McGraw-Hill, 2001