정보 및 제어 심포지엄(ICS'04) 2004. 5. 22

적응 확장 칼만 필터를 이용한 3차원 자세 추정

Attitude Estimation using Adaptive Extended Kalman Filter

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Abstract – This paper is concerned with attitude estimation using low cost, small-sized accelerometers and gyroscopes. A two step extended Kalman filter is proposed, which adaptively compensates external acceleration. External acceleration is the main source of estimation error. In the proposed filter, direction of external acceleration is estimated. According to the estimated direction, the accelerometer measurement covariance matrix of the two step extended Kalman filter is adjusted. The proposed algorithm is verified through experiments.

Key Words : Attitude estimation, inertial sensors, Kalman filter, adaptive filter

I. INTRODUCTION

Attitude estimation is necessary in many different applications. Probably the most extensively studied area is attitude estimation in inertial navigation systems (INS) [1]. In INS, attitude is very accurately estimated using expensive accelerometers and gyroscopes. Accelerometers and gyroscopes are too expensive and too large for most applications. However, due to recent electro-mechanical technical advance, in particular due to micro electro mechanical systems, low cost, small-sized accelerometers and gyroscopes have been developed [2]. Basically attitude can be estimated using accometers only by measuring the gravitational field. However, due to disturbances (most notably external acceleration), gyroscopes are also used to reduce effects of disturbances. Thus the key issue is how to combine accelerometers and gyroscopes to obtain good attitude estimation. Almost all papers use the Kalman filter to do this : attitude estimation for mobile robots [3], for a walking robot [4], and for a head-tracker [5].

In our paper, we propose the two step extended Kalman filter, which adaptively compensates external acceleration. External acceleration, which affects attitude estimation based on accelerometers, is the major source of attitude estimation error. Similar approaches, which also adaptively compensate external acceleration, are used in [5] and [6].

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II. INERTIAL SENSORS FOR ATTITUDE ESTIMATION

Attitude in the paper means pitch angle (θ) and roll angle (ϕ) of the Euler angles. The heading (yaw angle) is not considered in the paper. The Euler angles are the angular rotation between the body axis (x_b, y_b, z_b) and the inertial axis (x_f, y_f, z_f): we follow the standard aeronautics convention in [7].

To estimate attitude, we use 6 measurement variables:

- (a_x, a_y, a_z) : accelerometer outputs in the body axis.
- (g_x, g_y, g_z) : gyroscope outputs, which measure angular rates around the body axis.

Attitude can be estimated using accelerometers only by measuring the gravitational acceleration. From simple geometry, we have

$$\theta = \sin^{-1}(a_x) \quad \text{and} \quad \phi = \sin^{-1}(a_y / \cos\theta) \tag{1}$$

where all accelerometer outputs are normalized with the gravitational acceleration constant g. Note that only a_x and a_y are needed for attitude estimation. Although a_z is not directly used for attitude estimation, a_z plays an important role in the proposed algorithm. Attitude estimation error in (1) could be large when there are external acceleration: accelerometers cannot tell difference between the gravitational acceleration and external acceleration. Attitude can be estimated by integrating gyroscope outputs. However, the integration error inevitably accumulates as time goes by; thus gyroscope-based attitude estimation is reliable only for the short time.

III. STANDARD EXTENDED KALMAN FILTER

In this section, we introduce the standard Kalman filter for attitude estimation.

The state $x(t)$ and the measurement $z(t)$ are defined as follows:

$$\begin{aligned} x(t) &= [\theta\ \phi\ w_x\ w_y\ w_z]' \\ z(t) &= [a_x\ a_y\ g_x\ g_y\ g_z]' \end{aligned} \tag{2}$$

The system equation is given by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + w(t) \\ z(t) &= f(x(t)) + v(t) \end{aligned} \tag{3}$$

$$A(t) = \begin{bmatrix} 0\,0\,0 & \cos\phi(t) & -\sin\phi(t) \\ 0\,0\,1 & \sin\phi(t)\tan\theta(t) & \cos\phi(t)\tan\theta(t) \\ 0\,0\,0 & 0 & 0 \\ 0\,0\,0 & 0 & 0 \\ 0\,0\,0 & 0 & 0 \end{bmatrix}, \quad f(x(t)) = \begin{bmatrix} \sin\theta \\ \sin\phi\cos\theta \\ w_x \\ w_y \\ w_z \end{bmatrix}$$

Process and measurement noise $w(t)$ and $v(t)$ are assumed to be uncorrelated zero-mean white Gaussian processes satisfying

$$Q = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&q_1&0&0 \\ 0&0&0&q_1&0 \\ 0&0&0&0&q_1 \end{bmatrix} = E\{w(t)w(t)'\}, \quad R = \begin{bmatrix} r_1&0&0&0&0 \\ 0&r_2&0&0&0 \\ 0&0&r_3&0&0 \\ 0&0&0&r_3&0 \\ 0&0&0&0&r_3 \end{bmatrix} = E\{v(t)v(t)'\}.$$

Note that there is a singularity in A at $\theta = \pm\pi/2$: $\tan(\pm\pi/2)$ is not defined. In most attitude estimation problems, this poses little problem since θ and ϕ are in the range of $|\theta| \ll \pi/2$ and $|\phi| \ll \pi/2$. Once the system model (3) is chosen, the remaining thing is to select the covariance matrix Q and R. Since the estimation error depends on how Q and R are selected, it is important to choose correct Q and R values. In (3), the first two rows represent the standard relationship between $(\dot{\theta}, \dot{\phi})$ and (w_x, w_y, w_z). Since this relationship is exact, the first 2×2 block of Q is a zero matrix. The last three rows imply that we assume the derivatives of (w_x, w_y, w_z) are uncorrelated white noises and its covariances are all q_1. This assumption is made because of simplicity although it is possible that the assumption is not true for many real situations. The covariance value q_1 reflects our knowledge about $(\dot{w}_x, \dot{w}_y, \dot{w}_z)$. For example, small q_1 value means that we assume that (w_x, w_y, w_z) of the object is slowly changing. The measurement noise covariance R is a diagonal matrix, which means that all sensors are assumed to be uncorrelated. Usually, R is chosen from sensor characteristics. In this case, r_i indicates how good or bad the given sensor is: for example, large r_3 value means that the gyroscope output noise is large. In addition to indicators of each sensor's accuracy, R plays another important role when two sensors are fused to estimate the same quantity: note that attitude can be estimated from either accelerometers or gyroscopes. The role is to decide amount of each sensor's contribution to the estimation. Thus the ratio between (r_1, r_2) and r_3 is a weighting of two sensor's contribution. This weighting function role is automatically achieved if R is chosen according to each sensor's accuracy. Since the measurement output is sampled, a discretized system of (3) is used. Suppose the sampling period is T. An exact discretized system is a highly nonlinear system; thus to obtain a simplified discretized system, we assume that $A(t)$ is constant during the sampling period. Then the discretized system is given by [8]

$$\begin{aligned} x_{k+1} &= \Phi_k x_k + w_k \\ x_k &= f(x_k) + v_k \end{aligned} \tag{4}$$

where $x_k = x(kT)$ and $z_k = z(kT)$,

$$\Phi_k = \exp(A(kT)T) = \begin{bmatrix} I & W(kT)T \\ 0 & I \end{bmatrix}$$

$$W(kT) = \begin{bmatrix} 0 & \cos\phi(kT) & -\sin\phi(kT) \\ 1 & \sin\phi(kT)\tan\theta(kT) & \cos\phi(kT)\tan\theta(kT) \end{bmatrix}.$$

Process noise covariance matrix Q_k of the discretized system is given by

$$\begin{aligned} Q_k &= E\{w_k w_k'\} \\ &\approx \int_0^T \exp(A(kT)s)\,Q\exp(A(kT)'s)\,ds \\ &= \begin{bmatrix} \frac{1}{3}q_1 T^3 W(kT)W(kT)' & \frac{1}{2}q_1 T^2 W(kT) \\ \frac{1}{2}q_1 T^2 W(kT)' & q_1 TI \end{bmatrix}. \end{aligned}$$

The measurement noise covariance matrix of the discretized system is the same as that of (3). The standard extended Kalman filter for (4) is given as follows [8]:

- Initialization
  - $\hat{x}_0$: Initial attitude
  - P_0 : set 0
- Time Update
  - $\hat{x}_{k+1}^- = \Phi_k \hat{x}_k$
  - $P_{k+1}^- = \Phi_k P_k \Phi_k' + Q_k$
- Measurement update(Joseph form)

$$\begin{aligned} K_k &= P_k^- C_k'(C_k P_k^- C_k' + R)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k(z_k - f(\hat{x}_k^-)) \\ P_k &= (I - K_k C_k)P_k^-(I - K_k C_k)' + K_k R K_k'. \end{aligned}$$

$$C_k = \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}_k^-} = \begin{bmatrix} \cos\hat{\theta}(kT) & 0 & 0\,0\,0 \\ 0 & \cos\hat{\phi}(kT)\cos\hat{\theta}(kT) & 0\,0\,0 \\ 0 & 0 & 1\,0\,0 \\ 0 & 0 & 0\,1\,0 \\ 0 & 0 & 0\,0\,1 \end{bmatrix} \tag{5}$$

IV. TWO STEP EXTENDED KALMAN FILTER

The main drawback of the standard extended Kalman filter is that the estimation error becomes large if the object is experiencing external acceleration. The Kalman filter contains the model (1), which is not valid when there is external acceleration. This problem cannot be avoided even if very accurate accelerometers are used. Thus when there is external acceleration, gyroscope outputs should be trusted more and this can be done by making r_1 and r_2 large. Similar ideas are employed in [5] [6], though different system models are used. In [5], if gyroscope outputs are zero and inclinometer outputs are not changing, then it is assumed that external acceleration does not exist and accelerometer measurement noise covariance (corresponding to r_1 and r_2) is adjusted to small value. In [6], the following observation is used to detect existence of external acceleration.

Observation: A necessary condition for acceleration free movements is

$$a_x^2 + a_y^2 + a_z^2 = 1 \tag{6}$$

The above observation states that if external acceleration does not exists, then acceleration sensed by 3 axis accelerometer should be the gravitational acceleration only. Recall that the accelerometer outputs are normalized with the gravitational acceleration so that (6) is satisfied. In [6], if (6) is not satisfied, only gyroscope outputs are used to estimated attitude. In our framework, this can be interpreted as selecting very large r_1 and r_2 when (6) is not satisfied. In this paper, we use the same method to check existence of external acceleration: it is assumed that there is external acceleration if

$$f(a_x, a_y, a_z) = |a_x^2 + a_y^2 + a_z^2 - 1| > \delta \qquad (7)$$

where δ is a scalar parameter depending on accelerometer measurement noise characteristics. When existence of external acceleration is detected by (7), we use more sophisticated method to adjust r_1 and r_2. The direction of external acceleration is estimated and according to the direction, r_1 and r_2 are adjusted. To do this, we propose the two step extended Kalman filter:

- Initialization
  - $\hat{x}_0$: Initial attitude
  - P_0 : set 0
- Time Update

$$\hat{x}_{k+1}^- = \Phi_k \hat{x}_k$$
$$P_{k+1}^- = \Phi_k P_k \Phi_k' + Q_k$$

- Measurement Update Step 1 : Gyroscope only

$$\begin{aligned} K_{k,g} &= P_k^- C_2' (C_2 P_k^- C_2' + R_2)^{-1} \\ \hat{x}_{k,g} &= \hat{x}_k^- + K_{k,g}(z_{k,2} - C_2 \hat{x}_k^-) \\ P_{k,g} &= (I - K_{k,g} C_2) P_k^- (I - K_{k,g} C_2)' + K_{k,g} R_2 K_{k,g}' \end{aligned} \qquad (8)$$

where $C_2 = [0\; I_3]$, $R_2 = \begin{bmatrix} r_3 & 0 & 0 \\ 0 & r_3 & 0 \\ 0 & 0 & r_3 \end{bmatrix}$, $z_k = \begin{bmatrix} z_{k,1} \\ z_{k,2} \end{bmatrix} \in \begin{bmatrix} R^2 \\ R^3 \end{bmatrix}$

- Accelerometer noise covariance adjustment
  - if (7) is satisfied, then

$$\begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} = \max\left(\alpha_1 \begin{bmatrix} r_{1,k-1} \\ r_{2,k-1} \end{bmatrix} + \alpha_2 (z_{k,1} - C_{1,k} \hat{x}_{k,g}), \begin{bmatrix} r_{1,nom} \\ r_{2,nom} \end{bmatrix} \right) \qquad (9)$$

$\alpha_1 (|\alpha_1| < 1)$ and α_2 are scalar parameters and $C_{1,k}$ can be obtained from the following partition of C_k:

$$C_k = [C_{1,k} \;\; C_2]'.$$

  - if (7) is not satisfied, then

$$\begin{bmatrix} r_{1,k} \\ r_{2,k} \end{bmatrix} = \alpha_1 \begin{bmatrix} r_{1,k-1} \\ r_{2,k-1} \end{bmatrix} + \begin{bmatrix} r_{1,nom} \\ r_{2,nom} \end{bmatrix} \qquad (10)$$

- Measurement Update Step 2 : Acceleration now

$$\begin{aligned} K_{k,a} &= P_{k,g}^- C_{1,k}' (C_{1,k} P_{k,g}^- C_{1,k}' + R_{1,k}^{-1}) \\ \hat{x}_k &= \hat{x}_{k,g}^- + K_{k,a}(z_{k,1} - C_{1,k} \hat{x}_{k,g}^-) \\ P_k &= (I - K_{k,a} C_{1,k}) P_{k,g}^- (I - K_{k,a} C_{1,k})' + K_{k,a} R_{1,k} K_{k,a}' \end{aligned} \qquad (11)$$

where $R_{1,k} = \begin{bmatrix} r_{1,k} & 0 \\ 0 & r_{2,k} \end{bmatrix}$.

In (8), only gyroscope outputs are used to estimate the state. Note that (8) is nothing but the standard Kalman filter equation when only gyroscope outputs ($z_{k,2}$) are available. In (9), $r_{1,k}$ and $r_{2,k}$ are adjusted if there is external acceleration. Note that $z_{k,1}$ is the accelerometer output and $C_{1,k}\hat{x}_{k,g}$ is the estimated accelerometer output using only gyroscope outputs. The difference between these values should be small when there is no external acceleration. When there is external acceleration, $z_{k,1} - C_{1,k}\hat{x}_{k,g}$ is proportional to external acceleration. Thus the role of (9) is that accelerometer output in the direction of external acceleration is not trusted. The level of trust is reflected in $r_{1,k}$ and $r_{2,k}$. Note that in (9) and (10), a low pass filter is used so that $r_{1,k}$ and $r_{2,k}$ are not changed abruptly. Also note that $r_{1,k}$ and $r_{2,k}$ are always greater than $r_{1,nom}$ and $r_{2,nom}$ since too small r_1 and r_2 values may cause the Kalman filter divergence problem. In the measurement update 2, acceleration measurements only are used to estimate state.

We note that if the same C_k is used and R_k is constant (i.e., the adaptive algorithm is not used), the standard Kalman filter (5) and the two step Kalman filter (8) and (11) are identical. In the two step extended Kalman filter, the standard extended Kalman filter is divided into two step to estimate and compensate external acceleration.

V. CONCLUSION

In this paper, we have proposed the two step extended Kalman filter for general purpose attitude estimation. The main contribution is external acceleration, which is the main source of estimation error, is estimated and compensated. To verify the proposed algorithm, the sensor system consisting of 3 axis accelerometers and 3 axis gyroscopes is constructed and tested while intentional external acceleration is generated.

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