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Networked servo motor control systems

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Abstract -An H_2 servo controller is proposed for networked control systems. The network-induced delay is assumed to be time-varying and vary in the known range. The proposed controller guarantees stability and H_2 performance for all time-varying delay in the known range. The proposed controller is verified using a simple networked motor control system.

Key Words :networked control system, servo control, LMI

1. INTRODUCTION

Control systems in which control loops are closed through a serial network is called networked control systems (NCSs). Recently, NCSs have received a lot of attention due to their flexibility and easy maintenance [1], [2]. The main disadvantage of NCSs is network-induced time delay in the control loop. Since the time delay problem is unavoidable in NCSs, the problem has been studied extensively. Depending on the network type and scheduling methods, the time delay characteristics in NCSs can be modelled as constant, time-varying, and stochastic. In the case of constant time delay [3], it is relatively easy to design controllers. In [4], dynamic scheduling methods are proposed and network-induced delay is assumed to be time-varying. And maximum allowable delay bound (MADB) for a given controller is derived: if the network-induced time delay is smaller than MADB, the closed-loop system is stable. The derived bound is rather conservative and less conservative bound is derived in [5]. In both cases, controller synthesis problems are not considered. In [6], an LQG controller is proposed for a NCS where time delay is a stochastic process. It is assumed that the network-induced time delay is measurable, for example, by using time-stamped packet. In this paper, we propose a controller for a NCS with time-varying delay, where the delay is known to vary in the known range. The time-varying delay is treated as parameter variation in the system and robust control technique is used to design a controller. An H_2 servo control problem is formulated in the framework of NCSs.

2. PROBLEM FORMULATION

Consider a networked control system in Fig. 1, where sampled outputs and actuator commands are sent through a single serial communication channel. The configuration in Fig. 1 should be interpreted as generic and other configurations are also possible. For example, sensor 1 and actuator 3 can be in the same hardware board and in that case they are connected to the network through a single network interface.

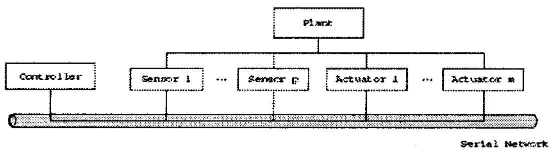

Fig. 1. Networked Control Systems

A timing diagram of the networked control system is shown in Fig. 2, where sensor outputs are periodically transmitted to the controller (with the period T). Then the controller computes actuator commands and transmits them to actuators.

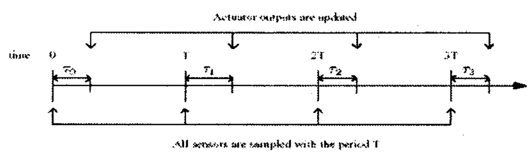

Fig. 2. Timing diagram of the networked control system

Time-varying delay τ_k includes communication delay (sensors-to-controller plus controller to actuators) and controller computation time . Controller computation time can be considered constant; however, communication delay is time-varying depending on the network traffic. In this paper, the following network assumptions are made.

(N1) Plant outputs are sampled with the fixed period T and the sampling is synchronized.

(N2) Delay τ_k is time -varying and its bounds are known:

$$\tau_{\min} \le \tau_k \le \tau_{\max} < T \tag{1}$$

(N3) Actuator updates are synchronized.

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The networked servo control problem can be formulated in a non-standard sampled-data control framework. We assume that the continuous time plant $G(s)$ is a linear, time-invariant system given by

$$G(s): \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \qquad (2)$$

where $x \in R^n$ is the state, $y \in R^p$ is the output and $u \in R^m$ is the control. As the controller $C(z)$, we use a linear time-invariant discrete controller:

$$C(z): \begin{aligned} \zeta_{k+1} &= A_c \zeta_k + B_c e_k \\ u_k &= C_c \zeta_k + D_c e_k \end{aligned} \qquad (3)$$

where $\zeta \in R^{n_c}$. The ideal sampler is assumed with the period T. If the delay τ and the hold time is constant, the problem is just a standard sampled-data control problem. However, τ_k is time-varying and the hold-time is also time-varying depending on τ_k. We formulate the control problem in the discrete-time framework: i.e., the closed-loop system of $C(z)$ and $\tilde{G}$ The discrete-time system . $\tilde{G}$ includes $G(s)$, the sampler, hold and network delay τ_k . Defining

$$y_k \cong y(kT),\ u_k \cong u(kT),\ \tilde{x}_k \cong \begin{bmatrix} x(kT) \\ u_{k-1} \end{bmatrix},$$

we have a state-space representation of $\tilde{G}$:

$$\tilde{G}: \begin{aligned} \tilde{x}_{k+1} &= \tilde{A}_k \tilde{x}_k + \tilde{B}_k u_k \\ y_{k+1} &= \tilde{C}_k \tilde{x}_k \end{aligned}$$

The servo control objective in this paper is to find a stabilizing controller $C(z)$, which minimizes

$$\sum_{k=0}^{\infty} (\|e_k\|_2^2 + \beta^2 \|u_{k-1}\|_2^2)$$

when a step input is applied as a reference command. This problem can be formulated into a discrete time-varying H_2 control problem.

main problem : find a comprehensively stabilizing controller $C(z)$, which minimizes $\|T_{zw}\|_2$, where T_{zw} is a system from w to z and $\alpha \in R^p$ is a constant vector.

Definition 1 [9]: If the feedback system $(C(z), G)$ is internally stable and T_{zw} is stable, the overall system is said to be comprehensively stable. Such $C(z)$ is called a comprehensively stabilizing controller. H_2 norm in this paper is defined by

$$\|T_{zw}\|_2^2 \equiv \sum_{k=0}^{\infty} \|z_k\|_2^2$$

we will find a controller minimizing an upper bound of H_2 norm. The generalized plant for the H_2 problem is given by

$$G_g: \begin{aligned} \overline{x}_{k+1} &= \overline{A}_k \overline{x}_k + \overline{B}_1 w_k + \overline{B}_{2,k} u_k \\ z_k &= \overline{C}_1 \overline{x}_k \\ y_k &= \overline{C}_2 x_k \end{aligned} \qquad (7)$$

Note that $(\overline{A}_k, \overline{B}_{2,k})$ is not stabilizable: pole 1 of G_g is not controllable. Thus if T_{zw} is to be stable, the pole 1 should not be observable: that is, pole 1 should be cancelled out by a zero 1. To achieve this, we will use techniques in [11], where a controller $C(z)$ is designed to have a discrete integrator so that the pole 1 of G_g is a cancelled out by the controller pole 1. Note that the closed- loop system T_{zw} is given by

$$T_{zw}: \begin{bmatrix} \overline{x}_{k+1} \\ \zeta_{k+1} \end{bmatrix} = A_{cl,k} \begin{bmatrix} \overline{x}_k \\ \zeta_k \end{bmatrix} + B_{cl} w_k,\ z_k = C_{cl} \begin{bmatrix} \overline{x}_k \\ \zeta_k \end{bmatrix} \qquad (8)$$

where

$$A_{cl,k} \equiv \begin{bmatrix} \overline{A_k} + \overline{B_{2,k}} \underline{D_c} \overline{C_2} & \overline{B_{2,k}} C_c \\ B_c \overline{C_2} & A_c \end{bmatrix}, B_{cl} \equiv \begin{bmatrix} \overline{B_1} \\ 0 \end{bmatrix}, C_{cl} \equiv \begin{bmatrix} \overline{C_1} & 0 \end{bmatrix}.$$

Lemma 1 and 2 are technical results to derive the constraints on the controller so that T_{zw} is stable.

Lemma 1: There exists a constant $[u_0' \ u_1']' \neq 0$ such that

$$\begin{bmatrix} \overline{A_k} & \overline{B}_{2,k} \\ \overline{C}_{11} & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} u_0 \\ 0 \end{bmatrix} \quad \text{for all } k. \qquad (9)$$

and furthermore, if we partition u_0 as follows:

$$u_0 = \begin{bmatrix} u_{0.1} \\ u_{0,2} \\ u_{0,3} \end{bmatrix} \in \begin{bmatrix} R^{1 \times (m+1-p)} \\ R^{n \times (m+1-p)} \\ R^{m \times (m+1-p)} \end{bmatrix}, \qquad (10)$$

then u_0 and u_1 can be obtained from the following : $u_1 = u_{0,3}$.

$$\begin{bmatrix} 0 & \exp(AT) - I & \int_0^T \exp(Ar) B dr \\ \alpha & -C & 0 \end{bmatrix} u_1 = 0 \qquad (11)$$

Lemma 2: Let T be defined by

$$T \equiv \begin{bmatrix} u_0 & u^{\perp} & 0 \\ -u_0 & -u_0^{\perp} & I \end{bmatrix} \qquad (12)$$

where u_0 and $u_0^{\perp}$ are from (9) and satisfy

$$\begin{bmatrix} u_0' \\ u_0^{\perp'} \end{bmatrix} \begin{bmatrix} u_0 & u_0^{\perp} \end{bmatrix} = \begin{bmatrix} I_{m+1-p} & 0 \\ 0 & I_{n-m+p} \end{bmatrix}.$$

If the controller (3) satisfies the following constrains:

$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \begin{bmatrix} -u_0 \\ 0 \end{bmatrix} = \begin{bmatrix} -u_0 \\ u_1 \end{bmatrix}, \qquad (13)$$

then $\qquad T_{zw} = (S_2' A_{cl,k} T_2, S_2' B_{cl}, C_{cl} T_2), \qquad (14)$

Theorem 1: Let F_1, F_2, $F_{1,cl}$ and $F_{2,cl}$ be defined by

$$F_1 \equiv \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, F_2 \equiv \begin{bmatrix} 0 & 0 & I \end{bmatrix}, F_{1,cl} \equiv \begin{bmatrix} F_1 \\ 0 \end{bmatrix}, F_{2,cl} \equiv \begin{bmatrix} F_2 - D_c \overline{C}_2 & -C_c \end{bmatrix}$$

If (A_c, B_c, C_c, D_c) satisfies (13) and there exist $P = P'$ and $W = W'$ satisfying

$$\begin{bmatrix} -P & PS_2' A_{cl,nom} T_2 & PS_2' B_{cl} & PS_2' & 0 \\ T_2' A_{cl,nom}' S_2 P & -P & 0 & 0 & T_2^1 F_2' \\ \overline{B}_{cl}' S_2 P & 0 & -I & 0 & 0 \\ F_1' S_2 P & 0 & 0 & -B_{\Delta}^{-1} & 0 \\ 0 & F_2 T_2 & 0 & 0 & -I \end{bmatrix} < 0 \qquad (15)$$

$$\begin{bmatrix} P & T_2' C_{cl}' \\ C_{cl} T_2 & W \end{bmatrix} > 0, \qquad (16)$$

then the system is comprehensively stable and

$$\|T_{zw}\|_2^2 \leq T_r(W) \quad \text{for all } \tau_{\min} \leq \tau_k \leq \tau_{\max}. \qquad (17)$$

Now we are ready to derive an H_2 controller in the next theorem.

Theorem 2: If there exist $X = X'$, $Z = Z'$, $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$ satisfying

$$\begin{bmatrix} -X & -u_0' & (1,3) & (1,4) & u_0' B_1 & u_0' F_1 & 0 \\ \star & -Z & A & (2,4) & ZB_1 & ZF_1 & 0 \\ \star & \star & -X & -u_0' & 0 & 0 & X u_0' F_2 + \hat{C}' \\ \star & \star & \star & -Z & 0 & 0 & u_0 u_0' F_2 + u_0 u_1' - u_0 u_0' C_2 \hat{D}' \\ \star & \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \star & -B_{\Delta}^{-1} & 0 \\ \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \qquad (18)$$

$$\begin{bmatrix} X & u_0^{\perp\prime} & Xu_0^{\perp\prime}\overline{C}_1{}' \\ \star & Z & u_0^{\perp}u_0^{\perp\prime}\overline{C}_1{}' \\ \star & \star & W \end{bmatrix} > 0 \tag{19}$$

where
$$(1,3) \equiv u_0^{\perp\prime}\overline{A}_{nom}u_0^{\perp\prime}\overline{B}_{2,nom}\hat{C}$$
$$(1,4) \equiv u_0^{\perp\prime}\overline{A}_{nom}u_0^{\perp\prime}\overline{B}_{2,nom}u_1u_0{}' + u_0^{\perp\prime}\overline{B}_{2,nom}\hat{D}C_2$$
$$(2,4) \equiv Z\overline{A}_{nom}u_0^{\perp}u_0^{\perp\prime} - Z\overline{B}_{2,nom}u_1u_0{}' + Zu_0u_0{}' + \hat{B}C_2$$

then there exists a controller(A_c, B_c, C_c, D_c) satisfying(13),(15)and (16). we eliminate the equality constraint by (13) parametrizing all controllers satisfying (13):

$$\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \begin{bmatrix} u_0u_0{}' & 0 \\ -u_0u_0 & 0 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} u_0^{\perp\prime} & 0 \\ C_2 & I \end{bmatrix} \tag{20}$$

where $K_{11} \sim K_{22}$ are free parameters. To perform linearizing change of variables, partition P and P^{-1}as:

$$P \equiv \begin{bmatrix} Y & N \\ N' & U \end{bmatrix},\ P^{-1} \equiv \begin{bmatrix} X & M \\ M' & V \end{bmatrix}.$$

Let Z be defined by $Z \equiv \begin{bmatrix} u_0^{\perp} & I \end{bmatrix} P \begin{bmatrix} u_0^{\perp\prime} \\ I \end{bmatrix}$. (21)

Let the change of controller variables be defined as:

$$\begin{aligned} \hat{D} &\equiv K_{22} \\ \hat{C} &\equiv u_1u_0{}'M' - K_{21}(-X + u_0^{\perp\prime}M') - K_{22}C_2M' \\ \hat{B} &\equiv (u_0^{\perp}N + U)K_{21} + Z\overline{B}_{2,nom}K_{22} \\ \hat{A} &\equiv Z\overline{A}_{nom}u_0^{\perp}X - Z\overline{B}_{2,nom}u_1u_0{}'M' + Zu_0u_0{}'M' + (u_0^{\perp}N + U)K_{11}(-X + u_0^{\perp\prime}M') \\ &\quad + (u_0^{\perp}N + U)K_{12}C_2M' + Z\overline{B}_{2,nom}K_{21}(-X + u_0^{\perp\prime}M') + Z\overline{B}_{2,nom}K_{22}C_2M'. \end{aligned} \tag{22}$$

$P = P' > 0$ satisfying (21) can be parametrized as follows:

$$P = F^{\dagger\prime}ZF^{\dagger} + F^{\perp\prime}RF^{\perp},\ R = R' > 0 \tag{23}$$

where $F = \begin{bmatrix} u_0^{\perp\prime} \\ 0 \end{bmatrix}$. By choosing any $R = R' > 0$, we have P; thus Y, N and U are computed. From the fact $PP^{-1} = I$, we have $YX + NM' = I$ and we have $M' = N^{\dagger}(I - YX)$. Since we have X, N, U and M, we can compute $K_{11} \sim K_{22}$ from (22) as follows:

$$\begin{aligned} K_{22} &\equiv \hat{D} \\ K_{21} &\equiv (u_1u_0{}'M' - K_{22}C_2M' - \hat{C})(-X + u_0^{\perp\prime}M')^{-1} \\ K_{12} &\equiv (u_0^{\perp}N + U)^{-1}(\hat{B} - Z\overline{B}_{2,nom}K_{22}) \\ K_{11} &\equiv (u_0^{\perp}N + U)^{-1}(\hat{A} - Z\overline{A}_{nom}u_0^{\perp}X + Z\overline{B}_{2,nom}u_1u_0{}'M' \\ &\quad - Z\overline{B}_{2,nom}K_{22}C_2M')^{-1} - K_{12}C_2M'(-X + u_0^{\perp\prime}M')^{-1} \\ &\quad - (u_0^{\perp}N + U)^{-1}Z\overline{B}_2,nom\,K_{21}. \end{aligned}$$

Finally, a controller(A_c, B_c, C_c, D_c)can be computed from (20).

3. CONCLUSION

In this paper, we proposed a servo controller for networked control systems with time-varying delays. In networked control systems, there is inevitable time delay in data transmission and the delay in many cases is time-varying depending on the network delay. The proposed servo controller guarantees the closed-loop stability for all time- varying delays belonging to a certain interval. As the performance index, H_2 norm is used. The controller can be computed easily by solving linear matrix inequalities.

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