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매개화된 민감도 해석에 의한 PM MRI의 Pole Piece 형상 최적화

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Shape Optimization for Magnetic Pole Piece of PM MRI using Nonlinear Parameterized Sensitivity Analysis

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Abstract - The ferromagnetic pole piece of permanent magnet assembly for magnetic resonance imaging(MRI) is optimally designed to get high homogenious magnetic field, taking into account the non-linearity of the magnetic materials. In the design, the pole face is kept smooth and axis-symmetric by using B-spline parameterization, and nonlinear design sensitivity analysis is used for search direction.

1. Introduction

The permanent MRI magnet is a viable alternative to the resistive and superconducting MRI magnet because of the low operating cost [1]-[4]. One of the most important aspects determining the quality of the MRI device is the homogeneity of the magnetic flux density in the diameter of spherical volume(DSV).

Since the permanent MRI magnet assembly employs very strong Nd-Fe-B permanent magnet, the device is magneticallysaturated locally even though the magnetic circuit has large air gap. Therefore, the magnetic non-linear property of the material have to be considered to get an accurate solution of field computation. However, most of the previous research on the design of permanent MRI magnet has been confined to linear magnetostatic model. D.Kim *et al.* independently derived the linear design sensitivity formulae based on boundary element method [3]. Y.Yao et al. derived the design sensitivity formula for 3D linear problem taking account into eddy current based on finite element method [4].

In this paper, the shape of the magnetic pole piece of the permanent MRI magnet is optimized taking into account the non-linearity of the materials to get a required homogeneity in the DSV by using 3D nonlinear finite element method and sensitivity analysis combined with steepest descent method. For the manufactur- ability, the shape is kept axis-symmetric and smooth by using B-spline parameterization. During the optimization process finite element meshes are deformed by using mesh relocation method without regeneration. This method guarantees the constant mesh topology during the optimization [4].

2. Non-linear and parameterized Design Sensitivity Analysis

The shape optimization problem for nonlinear magneto-static system can be generally expressed as follows[3],[4]:

$$\text{Minimize} \quad F(p) = f([p],[A(\nu,p)]) \tag{1-a}$$

$$\text{Subject to} \quad [p]_L \le [p] \le [p]_U \tag{1-b}$$

where F is the objective function, $[p]$ is the movable points on the design surface, and $[A]$ is the magnetic vector potential in 3D non-linear problems, $[p]_L$ and $[p]_U$ are the lower and upper limitations of the movable points, respectively.

The governing equation for the 3D non-linear magneto-static problems is, with the magnetic vector potential as the state variable, given as follow:

$$\nabla\times\nu(\nabla\times\vec{A}-\vec{B_r})=\vec{J} \tag{2}$$

where ν is the magnetic resistivity, B_r is the residual magnetic flux density of permanent magnet, and J is the exiting current density. Applying Galerkin's approximation to (2), the residual vector is defined as follow:

$$[R]=[K(\nu)][A]-[Q] \tag{3}$$

where the magnetic reluctivity is non-linear function of magnetic flux density.

The design sensitivity is defined as the total derivative of the objective function with respect to the design variable as follow [5]:

$$\frac{dF}{d[p]^T}=\frac{\partial F}{\partial[p]^T}+\frac{\partial F}{\partial[A]^T}\cdot\frac{d[A]}{d[p]^T} \tag{4}$$

where superscript T denotes the transpose. After differentiating both sides of (3), considering the non-linearity of the material, with respect to [p] and multiplying an adjoint variable $[\lambda]^T$, we obtain

$$[\lambda]^T([K]+[\overline{K}])\frac{d[A]}{d[p]^T} = -[\lambda]^T\left(\frac{\partial[R]}{\partial[p]^T}\bigg|_{\nu=C}-\frac{\partial[R]}{\partial\nu}\frac{\partial\nu}{\partial B^2}\frac{\partial B^2}{\partial[p]^T}\right) \tag{5}$$

where the matrices are defined as follows:

$$[\overline{K}]=\frac{\partial[K(\nu)]}{\partial\nu}\frac{\partial\nu}{\partial B^2}\frac{\partial B^2}{\partial[A]^T}\cdot[\overline{A}] \tag{6}$$

$$\frac{\partial[R]}{\partial[p]^T}\bigg|_{\nu=C}=\frac{\partial[K(\nu)]}{\partial[p]^T}[\overline{A}]-\frac{\partial[Q]}{\partial[p]^T} \tag{7}$$

$$\frac{\partial[R]}{\partial\nu} = \frac{\partial[K]}{\partial\nu}\cdot[\overline{A}] \quad (8)$$

where $[\overline{A}]$ is the converged solution of (3) and in computing (7) ν is the converged magnetic reluctivity of the material. If $[\lambda]$ is chosen so that the coefficients of the terms involving $d[A]/d[p]^T$ in (4) and (5) are equal, the adjoint variable can be computed as follows:

$$[K+\overline{K}]^T[\lambda] = \frac{\partial F}{\partial[A]} \quad (9)$$

The design sensitivity, finally, is derived using (4) and (9) as:

$$\frac{dF}{d[p]^T} = \frac{\partial F}{\partial[p]^T} - [\lambda]^T\left(\left.\frac{\partial[R]}{\partial[p]^T}\right|_{\nu=c} - \frac{\partial[R]}{\partial\nu}\frac{\partial\nu}{\partial B^2}\frac{\partial B^2}{\partial[p]^T}\right) \quad (10)$$

The design surface composed of the movable nodal points should be smooth in order to be manufactured using NC machine like lathe. Moreover, the design surface has often unsmoothed jagged shape in the optimization process because the nodal points on the design surface are usually taken as the design variables and allowed to move independently. For this reason, the design surface is parameterized using spline technique, and the optimal design surface is found by optimizing the control points of the spline.

With the $(n\times m)$ control points in (u, v) parametric plane are given, the non-rational B-spline surface is defined as follows [6]:

$$S(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,k}(u)N_{j,l}(v)C_{ij} \quad (11)$$

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } x_i \le u < x_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$N_{i,k}(u) = \frac{(u-x_i)N_{i,k-1}(u)}{x_{i+k-1}-x_i} + \frac{(x_{i+k}-u)N_{i+1,k-1}(u)}{x_{i+k}-x_{i+1}} \quad (13)$$

where k and l are the orders of B-spline basis function, and x_i is an element of knot vectors. The control points of B-spline have their influence only over the limited region of the design surface.

When the design surface is parameterized using spline technique, the relationship between the nodal points on the design surface and the control points can be expressed as [6]:

$$[p] = [J][C] \quad (14)$$

where $[J]$ is Jacobian matrix determined by the basis functions of the spline and $[C]$ is the control point vector. The design sensitivity for the control points can be computed using (14) as follows:

$$\frac{dF}{d[C]^T} = \frac{dF}{d[p]^T}\frac{d[p]}{d[C]^T} = \frac{dF}{d[p]^T}[J] \quad (15)$$

After the control points are updatedby steepest descent algorithm., the coordinates of the nodal points on the design surface are computed using (14).

3. Shape Optimization Results

The shape of the magnetic pole piece of an open permanent MRI magnet with two columns, shown in Fig.1, is optimized to get the uniform magnetic field in the 30cm DSV. The diameter of the magnetic pole and the permanent magnet are 50cm and 60cm, respectively. The dimensions of the analysis model are shown in Fig. 2. The residual magnetic flux density of Nd-Fe-B magnet is 1.21 T. The B-H characteristicof pole piece is shown in Fig. 3. A half of the model is discretized into 34,750 nodes and 198,000 tetrahedral elements.

Fig. 4 shows the variation of the objective function values, where the linear design sensitivity analysis gives a little bigger objective function value than non-linear sensitivity analysis. After 15 iterations of optimization, the objective function values are almost converged.

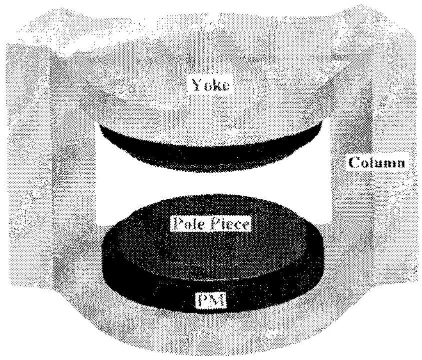


Fig. 1 Shape of the MRI

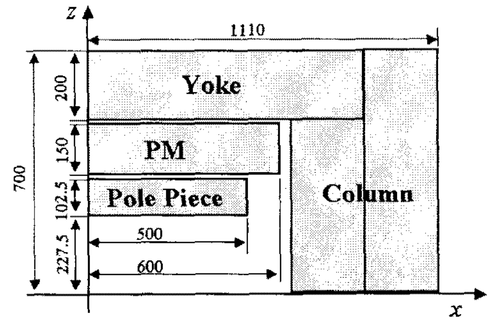


Fig. 2 Dimensions of PM MRI model (unit: mm)

Through optimization the objective functions for linear case and non-linear case are reduced to 3.21% and 1.95% of the each initial value.

The optimized pole shapes are compared in Fig. 5. The optimized pole shape by linear sensitivity analysis is deeper and the center hill

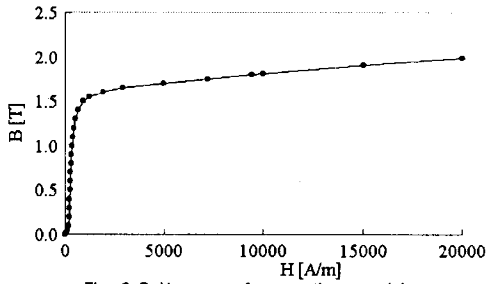


Fig. 3 B-H curve of magnetic material

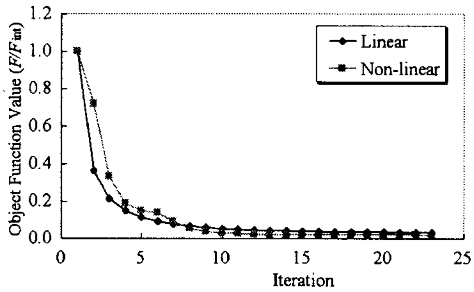
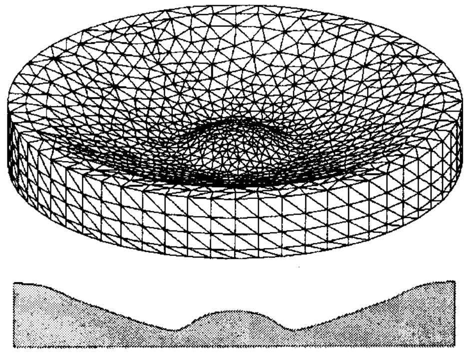


Fig. 4 B-H curve of magnetic material

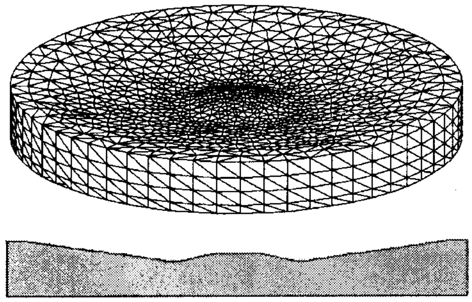
is higher than that of non-linear sensitivity optimization result. Because the magnetic flux density can be saturated locally in linear computation, the sharp shape of the pole piece is obtained. The field homogeneity for the initial flat pole piece shape in the DSV is 4761 ppm and for the optimal shape it is reduced to 550 ppm.

4. Conclusion

The magnetic pole piece of permanent magnet assembly for MRI is shape optimized considering the magnetic non-linearity of the material. In the optimization, the surface of pole piece is parameterized using B-spline to be manufactured using NC machine. Through the numerical example, it is shown the magnetic non-linearity of the material has to be considered to get more accurate solution, and the optimal shape of the pole, which gives the uniform field distribution with less then 2% of the initial object function, is designed.



(a) linear optimization



(b) non-linear optimization

Fig. 5 Comparison of the optimized pole pieces

[참 고 문 헌]

[1] T. Miyamoto, H. Sakurai, H. Takabayashi, M. Aoki, *IEEE Trans. Magn.*, Vol.25, No.5, pp.3907-3909, 1989

[2] A. Podol'skii, *IEEE Trans. Magn.*, Vol.36, No.2, pp.484-490, 2000

[3] D.H. Kim, B.S. Kim, J.H. Lee, W.S. Nah, I.H. Park, *IEEE Transaction on Applied Superconductivity*, Vol.12, No.1, pp.1467-2002, 2002

[4] Y. Yao, C.S. Koh, and X. Dexin, *IEEE Trans. Magn.*, Vol.40, No.2, pp.1164-1167, 2004

[5] H. Park, B. T. Lee, S. Y. Hahn, *IEEE Trans. on Magn.*, Vol.28, No.2, pp.1533-1536, 1992

[6] D. F. Rogers and J. A. Adams, *Mathematical Elements for Computer Graphics*, McGraw-Hill, 1990