R상의 Intuitionistic 퍼지수의 성질

Some Properties of Intuitionistic Fuzzy Numbers on R

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Abstract

In this paper, we find some properties of intuitionistic fuzzy numbers and scribe the properties of intuitionistic fuzzy sets in R.

1. Introduction

In 1979, Mizumoto and Tanaka discussed the four arithmetic operation of fuzzy numbers in R which are defined by the extension principle and investigated the algebraic structures and the ordering of fuzzy numbers. In 1986, Atanassov [2] defined intuitio -nistic fuzzy set with the membership and the non-membership functions. In 2000, Liu and Shi [1] defined the cut-sets of intuitionistic fuzzy sets, and then, the decompositions of intuitionistic fuzzy set are established.

In this paper, we find some properties of intuitionistic fuzzy numbers and describe the properties of intuitionistic fuzzy sets in R.

2. The definition and properties of intuitionistic fuzzy sets

We denote intuitionistic fuzzy sets in X by IFS(X).

In the class of intuitionistic fuzzy sets, the following relations for the membership and the non-membership functions [1],[2],[3],[4],[5].

(1) $A \subseteq B \leftrightarrow$

$$\mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x),\ x \in X.$$

(2) $A = B \leftrightarrow A \subseteq B$ and $B \subseteq A$.

(3) $A' = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$.

(4) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >: x \in X\}$.

(5) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) >: x \in X\}$.

are defined, where $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$.

And for an intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ if $\mu_A(x) = 0$,

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then $\nu_A(x)=1$ and if $\mu_A(x)=1$, then $\nu_A(x)=0$.

For $A_t \in IFS(X), t \in T$, we define the operations

(1) $\cap_{t\in T}A_t = <x, \wedge_{t\in T}\mu_{A_t}(x), \vee_{t\in T}\nu_{A_t}(x)>: x \in X$

(2) $\cup_{t\in T}A_t = <x, \vee_{t\in T}\mu_{A_t}(x), \wedge_{t\in T}\nu_{A_t}(x)>: x \in X$

Throughout this paper, let $I=[0,1]$, R be the set of all real numbers.

$<I> = \{<a,b>: a+b\le 1, a,b\in I\}$

$<I>_0 = <I> - \{<0,1>\}$

Definition 2.1 ([1]). For any $<a_t,b_t> \in <I>$, $t\in T$, we define operations on $<I>$ as follows.

$\vee_{t\in T}<a_t,b_t> = <\vee_{t\in T}a_t, \wedge_{t\in T}b_t>$,

$\wedge_{t\in T}<a_t,b_t> = <\wedge_{t\in T}a_t, \vee_{t\in T}b_t>$,

$<a_t,b_t>' = <b_t,a_t>$

Definition 2.2 ([1]). For any $<a_i,b_i> \in <I>, i=1,2$, we define relations as follows.

$<a_1,b_1> = <a_2,b_2> \leftrightarrow a_1=a_2,\ b_1=b_2$,

$<a_1,b_1> \le <a_2,b_2> \leftrightarrow a_1\le a_2,\ b_1\ge b_2$,

$<a_1,b_1> < <a_2,b_2> \leftrightarrow <a_1,b_1> \le <a_2,b_2>, <a_1,b_1> \ne <a_2,b_2>$.

$<a_1,b_1> \ne <a_2,b_2>$.

Theorem 2.1([1]). Let $\alpha, \alpha_t \in <I>$, $t\in T$. Then $\alpha\wedge(\vee_{t\in T}\alpha_t) = \vee_{t\in T}(\alpha\wedge\alpha_t)$,

$\alpha\vee(\wedge_{t\in T}\alpha_t) = \wedge_{t\in T}(\alpha\vee\alpha_t)$.

Definition 2.3([1]). For $A\in IFS(X), <\lambda_1,\lambda_2> \in <I>$, we call

$A_{<\lambda_1,\lambda_2>} = \{x\in X: \mu_{A(x)} \ge \lambda_1, \nu_{A(x)} \le \lambda_2\}$.

$A_{<\lambda_1,\lambda_2>} = \{x\in X: \mu_{A(x)} > \lambda_1, \nu_{A(x)} < \lambda_2\}$

cut-set and strong cut-set of A, respectively.

Definition 2.4([1]). Let $A\in IFS(X)$, $<\lambda_1,\lambda_2> \in <I>$. We define the operation

$<\lambda_1,\lambda_2> \cdot A = \{<x, \lambda_1\wedge\mu_{A(x)}, \lambda_2\vee\nu_{A(x)}>: x\in X\}$

Theorem 2.2 ([1]). If $A,B\in IFS(X)$, $<\lambda_1,\lambda_2>$, $<\lambda_3,\lambda_4> \in <I>$, then

(1) $<\lambda_1,\lambda_2> \le <\lambda_3,\lambda_4> \rightarrow <\lambda_1,\lambda_2>\cdot A \subseteq <\lambda_3,\lambda_4>\cdot A$,

(2) $A\subseteq B \rightarrow <\lambda_1,\lambda_2>\cdot A \subseteq <\lambda_1,\lambda_2>\cdot B$.

Theorem 2.3 ([1]). (Decomposition Theorem) Let $A\in IFS(X)$, then

(1) $A = \cup_{<\lambda_1,\lambda_2>\in<I>} <\lambda_1,\lambda_2>\cdot A_{<\lambda_1,\lambda_2>}$,

(2) $A = \cup_{<\lambda_1,\lambda_2>\in<I>} <\lambda_1,\lambda_2>\cdot A_{<\lambda_1,\lambda_2>}$.

3. Intuitionistic fuzzy numbers and their extended operations

Definition 3.1 ([1]). Let $A\in IFS(X)$, we call A a intuitionistic normal fuzzy set, if there exists $x_0\in X$, such that $\mu_A(x_0)=1$.

Definition 3.2 ([1]). Let $A\in IFS(X)$, A is called an intuitionistic convex fuzzy set on S, if $A_{<\lambda_1,\lambda_2>}$ is an ordinary convex set for any $<\lambda_1,\lambda_2> \in <I>$.

Theorem 3.1 ([1]). Let $A\in IFS(X)$, then A is an intuitionistic convex fuzzy set if and only if the following conditions hold:

$\mu_A(kx_2+(1-k)x_1) \ge \min\{\mu_A(x_1), \mu_A(x_2)\}$,

$\nu_A(kx_2+(1-k)x_1) \le \max\{\nu_A(x_1), \nu_A(x_2)\}$

where $x_1,x_2\in X$ 이고 $k\in I$.

Definition 3.3 ([1]). Let $A\in IFS(R)$. A is called an intuitionistic fuzzy number (IFN for short) on R, if A is normal and $A_{<\lambda_1,\lambda_2>}$ is a

losed bounded interval for arbitary
$< \lambda_1, \lambda_2 > \in < I >_0$.

Let $IFN(R)$ denote the set of all intuitionistic fuzzy numbers on R.

efinition 3.4 ([1]). Let $A \in IFN(R)$.
1) For any $x \in A_{<0,1>}$, A is called positive FN if $x > 0$.
2) For any $x \in A_{<0,1>}$, A is called negative FN if $x < 0$.
3) For any $x \in A_{<0,1>}$, A is called zero IFN f $x = 0$.

efinition 3.5 ([1]). Let $*$ be a binary peration on S, $A, B \in IFN(R)$, we define the xtended operations as follows

$$A * B = \vee_{z=x*y}\{< z, \mu_A(x) \wedge \mu_B(y), \nu_A(x) \vee \nu_B(y) >: z \in R\}$$
$$= \{< z, \vee_{z=x*y}(\mu_A(x) \wedge \mu_B(y)), \wedge_{z=x*y}(\nu_A(x) \vee \nu_B(y)) >: z \in R\}$$

specially, we call

$$A \oplus B = \vee_{z=x+y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$
$$A \ominus B = \vee_{z=x-y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$
$$A \otimes B = \vee_{z=x\cdot y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$
$$A \oslash B = \vee_{z=x/y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$
$$A \vee B = \vee_{z=x\vee y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$
$$A \wedge B = \vee_{z=x\wedge y}\{z, (\mu_A(x) \wedge \mu_B(y)), (\nu_A(x) \vee \nu_B(y)) >: z \in R\},$$

xtended addition, extended subtraction, xtended multiplication, extended division, xtended maximum and extended minimum, espectively.

4. The algebraic properties of intuitionistic fuzzy numbers

Theorem 4.1 ([1]). Let $A, B \in IFN(R)$, then $A \oplus B$, $A \ominus B$, $A \otimes B$, $A \vee B$, $A \wedge B$ are all normal. Especially, $A \oslash B$ is normal whenever B is a positive IFN or a negative IFN.

Theorem 4.2 ([1]). If $A, B \in IFN(R)$, then $A_\alpha \pm B_\alpha, A_\alpha \times B_\alpha$, $A_\alpha \vee B_\alpha$, $A_\alpha \wedge B_\alpha$ are closed bounded interval for any $\alpha \in < I >_0$.

Theorem 4.3 ([1]). If $A, B \in IFN(R)$, then $A \oplus B, A \ominus B, A \otimes B, A \vee B, A \wedge B \in IFN(R)$ and $A \oslash B \in IFN(R)$ whenever B is a positive IFN or a negative IFN.

Theorem 4.4 ([1]). If $A, B, C \in IFN(R)$, then $A \oplus B = B \oplus A, A \otimes B = B \otimes A$.

Theorem 4.5 ([1]). If $A, B, C \in IFN(R)$, then

$$(A \oplus B) \oplus C = A \oplus (B \oplus C),$$
$$(A \otimes B) \otimes C = A \otimes (B \otimes C).$$

Theorem 4.6. Any $A, B, C \in IFN(R)$ are shown not to satisfy the distributive law.

Let $IFN_p(R)$ denote the set of all $IFN(R)$ with positive $A_{<0,1>}$.

Theorem 4.7. The following distributive law is satisfied for $A, B, C \in IFN_p(R)$.

$$A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C).$$

Theorem 4.8. For any $A \in IFN(R)$, there exists the following identity.
$A \oplus 0 = A$, $A \otimes 1 = A$
where 0 and 1 are ordinary number.

Theorem 4.9. For $A \in IFN(R)$, there exist no inverse under $\oplus, \otimes$.

Definition 4.2. The algebraic system $R = < R; +, \cdot >$ is called a commutative semiring with unity if it satisfies (1)~(5).

(1) $a, b \in R \rightarrow a+b,\ a \cdot b \in R$

(2) $(a+b)+c = a+(b+c)$,

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

(3) $a+b = b+a,\ a \cdot b = b \cdot a$

(4) $a \cdot (b+c) = (a \cdot b)+(a \cdot c)$

(5) There exists a zero 0 and a unity 1 such that $a+0 = a,\ a \cdot 1 = a$.

Theorem 4.10. $< IFN_p(R), \oplus, \otimes >$ have the structure of commutative semiring.

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