비선형 시스템의 TSK 퍼지모델 기반 하이브리드 적응제어

TSK Fuzzy Model Based Hybrid Adaptive Control of Nonlinear Systems

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Abstract

In this thesis, we present the Takagi-Sugeno-Kang (TSK) fuzzy model based adaptive controller and adaptive identification for a general class of uncertain nonlinear dynamic systems. We use an estimated model for the unknown plant model and use this model for designing the controller. The hybrid adaptive control combined direct and indirect adaptive control based on TSK fuzzy model is constructed. The direct adaptive law can be showed by ignoring the identification errors and fails to achieve parameter convergence. Thus, we propose an TSK fuzzy model based hybrid adaptive (HA) law combined of the tracking error and the modeling error to adjust the parameters. Using a Lyapunov synthesis approach, the proposed hybrid adaptive control is proved. The hybrid adaptive law (HA) is better than the direct adaptive (DA) method without identifying the modeling error in terms of faster and improved tracking and parameter convergence. In order to show the applicability of the proposed method, it is applied to the inverted pendulum system and the performance is verified by some simulation results.

Key words: adaptive control, TSK fuzzy model, adaptive identification, Lyapunov synthesis

I. Introduction

Many dynamic systems to be controlled have constant or slowly-varying uncertain parameters. Adaptive control [1],[2] is an approach to the control of such systems. The basic idea in adaptive control is to estimate the uncertain plant parameters (or equivalently, the corresponding controller parameters) on-line based on the measured system signals and use the estimated parameters in the control input computation. Generally, the basic objective of adaptive control is to maintain consistent performance of a system in the presence of uncertainty or unknown variation in plant parameters.

Fuzzy control [3],[4] is another approach that suits for controlling imprecisely defined systems and the controller is composed of a collection of fuzzy IF-THEN rules. Fuzzy logic controllers provide a systematic and efficient framework for incorporating linguistic fuzzy information from human experts. In this paper, the fuzzy controller considered is constructed only from fuzzy modeling rules. We use a hybrid adaptive fuzzy control, which combines adaptive fuzzy identification and adaptive fuzzy control for adjusting the parameters. We propose the hybrid adaptive law combined of modeling error and tracking error for adjusting the parameters.

This paper is organized as follows. In Section Ⅱ, we introduce Takagi-Sugeno-Kang fuzzy systems. In Section Ⅲ, we formulate a certain problem and design a self-tunning controller using adaptive fuzzy systems. We propose a hybrid adaptive law using TSK fuzzy model for adjusting its parameters. In Section Ⅳ, in order to show the applicability of the proposed method, it is applied to the inverted pendulum system and the performance is verified by some simulation results. Finally, the conclusion of the paper is presented in Section Ⅴ.

Ⅱ. Takagi-Sugeno-Kang Fuzzy Model

TSK fuzzy model can be represented as IF-THEN form or Input-Output form [5].

★ **IF-THEN form**

Plant rule i:

IF x *is* M_{i1} and $\dot{x}$ *is* M_{i2} and $\cdots$ and $x^{(n-1)}$ *is* M_{in}

THEN $x^{(n)} = a_i^T \underline{x} + b_i u, \quad i = 1, 2, \cdots, r$

where $\underline{x} = [x \ \dot{x} \cdots x^{(n-1)}]^T \in R^n, a_i^T \in R^n, b_i \in R$, M_{ij} is the fuzzy set and r is the number of rules.

★ **Input-Output form**

$$x^{(n)} = \frac{\sum_{i=1}^{r} w_i(x)\{a_i^T \underline{x} + b_i u\}}{\sum_{i=1}^{r} w_i(x)} = \sum_{i=1}^{r} h_i(x)\{a_i^T x + b_i u\}$$

where

$$w_i(x) = \prod_{j=1}^{n} M_{ij}(x^{(j-1)}), \quad h_i(x) = \frac{w_i(x)}{\sum_{i=1}^{r} w_i(x)}, M_{ij}(x^{(j-1)})$$

is the grade of membership of $x^{(j-1)}$ in M_{ij}. Assum that $\sum_{i=1}^{r} w_i(x) > 0, \ w_i(x) \geq 0, \ (i = 1, 2, \cdots, r)$.

Hence, $h_i(x) \geq 0, \quad \sum_{i=1}^{r} h_i(x) = 1$.

Ⅲ. Hybrid Adaptive Controller Using TSK Fuzzy Model

In direct adaptive control, the controller parameters are directly adjusted without identifying the plant parameters. Control of the unknown plant without identification may result in poor response. Therefore, we use the hybrid adaptive control combined of the direct and the indirect adaptive control.

The direct adaptive law can be showed by ignoring the identification errors and fails to achieve parameter convergence. Thus, we propose an TSK fuzzy model based hybrid adaptive (HA) law combined of the tracking error and the modeling error to adjust the parameters. The proposed method showed not only enhanced parameter convergence but also fast tracking error convergence.

A. Problem Specification

Consider the nth-order nonlinear system of the controllability canonical form [2],[6]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x_1, x_2, \cdots, x_n) + g(x_1, x_2, \cdots, x_n)u \\ y = x_1 \end{cases} \quad (3\text{-}1)$$

or, equivalently

$$\begin{cases} x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}) + g(x, \dot{x}, \cdots, x^{(n-1)})u \\ y = x \end{cases} \quad (3\text{-}2)$$

where f and g are *unknown* real continuous functions, $u \in \mathbb{R}$ are the input and output of system, and $x = (x_1, x_2, \cdots, x_n)^T = (x, \dot{x}, \cdots, x^{(n-1)})^T \in \mathbb{R}^n$ is the state vector of the system which is assumed to be available for measurement. The controllability of (3-2) requires that $g(x) \neq 0$ for all x in a certain controllability region $U_c \subset \mathbb{R}^n$. Since $g(x)$ is continous, without loss of generality, we assume that $g(\underline{x}) > 0$ for all $x \in U_c$. In addition, we assume that the function f and g are bounded. Then, we design TSK fuzzy model of the given n-th nonlinear system.

In this paper, TSK fuzzy model was used to identify the unknown nonlinear system (3-2) and estimates $\hat{f}$ and $\hat{g}$ represented as follows

$$\hat{f}(\underline{x} \mid \theta_a) = \sum_{i=1}^{r} h_i(\hat{\underline{x}}) \hat{a}_i^T \hat{\underline{x}} = \theta_a^T \xi_a \quad (3\text{-}3)$$

$$\hat{g}(\underline{x} \mid \theta_b) = \sum_{i=1}^{r} h_i(\hat{\underline{x}}) \hat{b}_i = \theta_b^T \xi_b \quad (3\text{-}4)$$

B. Structure of Controller

The controller is designed based on the plant model in a nonadaptive control design. If the plant model is not known, it is reasonable to replace it by an estimated model and this model for designing the controller. If the plant dynamics is known, we can solve the control problem by the so-called feedback linearization method [2]. In this method, the functions f and g are used to construct the following feedback control law:

$$u = u(x, t) = \frac{1}{g(\underline{x})}[-f(x) + y_m^{(n)}(t) + k^T e] \quad (3\text{-}5)$$

where $e(t) = y_m(t) - y(t)$ is the tracking error, $e(t) = (e, \dot{e}, \cdots, e^{(n-1)})^T$, and $k = (k_n, \cdots, k_2, k_1)^T \in \mathbb{R}^n$ is chosen such that all roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \cdots + k_n$ are in the open left-half of the complex plane. Applying the control law (3-5) to the system (3-2) results in the following error dynamics:

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0 \quad (3\text{-}6)$$

This implies that starting from any initial conditions, we have $\lim_{t\to\infty} e(t) \mid = 0$. However, since f and g are unknown, we cannot use them for constructing the control law (3-5). Therefore, we replace them by their estimates $\hat{f}$ and $\hat{g}$ to construct controller

$$u_c = u_c(x, t \mid \theta_a, \theta_b)$$

$$= \frac{-\sum_{i=1}^{r} h_i(\hat{x}) \hat{a}_i^T \hat{x} + y_m^{(n)} + k^T e}{\sum_{i=1}^{r} h_i(\hat{x}) \hat{b}_i} \quad (3\text{-}7)$$

where θ_a and θ_b are parameters of the approximating systems $\hat{f}$ and $\hat{g}$, respectively.

By defining the minimum approximation error

$$w = w(\underline{x}, \theta_a, \theta_b, t)$$
$$= \sum_{i=1}^{r} h_i(\hat{\underline{x}}) a_i^{*T} \underline{x} - f(\underline{x}) + \sum_{i=1}^{r} h_i(\hat{\underline{x}}) b_i^* - g(\underline{x})\, u_c \quad (3\text{-}8)$$

Applying the control law (3-7) to the system (3-2), after some manipulation using (3-8), results in error dynamic equation

$$\dot{\underline{e}} = \Lambda_c \underline{e} + \underline{b}_c w + \underline{b}_c \left\{ \sum_{i=1}^{r} h_i(\hat{\underline{x}})(\hat{a}_i^T - a_i^{*T}) \hat{\underline{x}} \right\} + \left\{ \sum_{i=1}^{r} h_i(\hat{\underline{x}})(\hat{b}_i - b_i^*) \right\} u_c \quad (3\text{-}9)$$

where

$$\Lambda_c = \begin{bmatrix} 0 & 1 & 0 & \cdots 0 \\ \vdots & \ddots & \ddots & \ddots 0 \\ 0 & \cdots & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & -k_1 \end{bmatrix} \qquad \underline{b}_c = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\alpha = \left\{ \sum_{i=1}^{r} h_i(\hat{\underline{x}})(\hat{a}_i^T - a_i^{*T}) \hat{\underline{x}} \right\} + \left\{ \sum_{i=1}^{r} h_i(\hat{\underline{x}})(\hat{b}_i - b_i^*) \right\} u_c \quad (3\text{-}10)$$

C. Adaptive Law And Stability Analysis

We consider the following Lyapunov function candidate:

$$V(\underline{e}, \tilde{a}_i, \tilde{b}_i) = \frac{1}{2} \underline{e}^T p \underline{e} + \frac{1}{2} \sum_{i=1}^{r} \frac{\tilde{a}_i^T \tilde{a}_i}{\gamma_{1i}} + \frac{1}{2} \sum_{i=1}^{r} \frac{\tilde{b}_i \tilde{b}_i}{\gamma_{2i}} \quad (3\text{-}11)$$

where, V is positive definite function and radially unbounded.

$$\tilde{a}_i = \hat{a}_i - a_i^*, \quad \tilde{b}_i = \hat{b}_i - b_i^* \quad (3\text{-}12)$$

Λ_c : $n \times n$ symmetric positive definite matrix

γ_1, γ_2 : positive adaptation constant gains

After differentiating V, an adaptation law to make $\dot{V} \leqq 0$ is constructed. By replacing $\dot{e}$ and $\dot{e}^T$ with (3-9), $\dot{V}$ is expressed as follows.

$$\dot{V} = -\frac{1}{2} e^T Q e + e^T p b_c w + e^T p b_c \alpha + \sum_{i=1}^{r} \frac{\tilde{a}_i^T \dot{\tilde{a}}_i}{\gamma_{1i}} + \sum_{i=1}^{r} \frac{\dot{\tilde{b}}_i \tilde{b}_i}{\gamma_{2i}}$$

$$= -\frac{1}{2} e^T Q e + e^T p b_c w + e^T p b_c \left[\sum_{i=1}^{r} h_i(\hat{x}) \tilde{a}_i^T x + \sum_{i=1}^{r} h_i(\hat{x}) \tilde{b}_i u_c \right] + \sum_{i=1}^{r} \frac{\tilde{a}_i^T \dot{\tilde{a}}_i}{\gamma_{1i}} + \sum_{i=1}^{r} \frac{\dot{\tilde{b}}_i \tilde{b}_i}{\gamma_{2i}}$$

$$\underline{e}^T p\underline{b}_c \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{a}_i^T\underline{x} + \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{b}_i u_c + \sum_{i=1}^{r}\frac{\tilde{a}_i^T\dot{\tilde{a}}_i}{\gamma_{1i}} + \sum_{i=1}^{r}\frac{\dot{\tilde{b}}_i\tilde{b}_i}{\gamma_{2i}} = 0 \quad (3\text{-}13)$$

Divide (3-13) by the part with respect to a and b.

with respect to a,

$$\sum_{i=1}^{r}\frac{\tilde{a}_i^T\dot{\tilde{a}}_i}{\gamma_{1i}} = -\underline{e}^T p\underline{b}_c \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{a}_i^T\underline{x}$$

$$\dot{\tilde{a}}_i = -\gamma_{1i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})\hat{\underline{x}}$$

$$\dot{\hat{a}}_i = -\gamma_{1i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})\hat{\underline{x}} \quad (3\text{-}14)$$

with respect to b,

$$\sum_{i=1}^{r}\frac{\dot{\tilde{b}}_i\tilde{b}_i}{\gamma_{2i}} = -\underline{e}^T p\underline{b}_c \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{b}_i u_c$$

$$\dot{\tilde{b}}_i = -\gamma_{2i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})u_c$$

$$\dot{\hat{b}}_i = -\gamma_{2i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})u_c \quad (3\text{-}15)$$

In conclusion, we can arrange the adaptive law as follows.

$$\begin{cases} \dot{\hat{a}}_i = -\gamma_{1i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})\hat{\underline{x}} \\ \dot{\hat{b}}_i = -\gamma_{2i}\underline{e}^T p\underline{b}_c h_i(\hat{\underline{x}})u_c \end{cases} \quad (3\text{-}16)$$

where $\underline{e}$ is the tracking error vector. Since the adaptive law (3-16) directly adjusts the controller parameters using the output tracking error, we call it the *direct adaptive law* (DA law). Note that the DA law adjusts the parameters as far as $\underline{e} \neq 0$. Using the above DA law, we have

$$\begin{aligned} \dot{V}_D &= -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T p\underline{b}_c w + \underline{e}^T p\underline{b}_c \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{a}_i^T\underline{x} + \sum_{i=1}^{r} h_i(\hat{\underline{x}})\tilde{b}_i u_c \\ &\quad + \sum_{i=1}^{r}\frac{\tilde{a}_i^T\dot{\tilde{a}}_i}{\gamma_{1i}} + \sum_{i=1}^{r}\frac{\dot{\tilde{b}}_i\tilde{b}_i}{\gamma_{2i}} \\ &= -\frac{1}{2}\underline{e}^T Q\underline{e} + \underline{e}^T p\underline{b}_c w \\ &\approx -\frac{1}{2}\underline{e}^T Q\underline{e} \le 0 \end{aligned} \quad (3\text{-}17)$$

where the index "D" in $\dot{V}_D(t)$ stands for the DA law, and the approximation is made assuming that w is small. It can be seen from (3-17) that the tracking error e causes the Lyapunov function (3-11) to decrease.

We define the following series-parallel identification model [6] for identifying the nonlinear plant (3-1):

$$\begin{cases} \dot{\hat{x}}_1 = x_2 \\ \dot{\hat{x}}_2 = x_3 \\ \quad\vdots \\ \dot{\hat{x}}_n = \hat{f}(\underline{x} \mid \underline{\theta}_a) + \hat{g}(\underline{x} \mid \underline{\theta}_b)u \end{cases} \quad (3\text{-}18)$$

where $u \in \mathbb{R}$ is the input of the identification model, $\hat{\underline{x}} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n)^T \in \mathbb{R}^n$ and $\underline{x} = (x_1, x_2, \cdots, x_n)^T = (x, \dot{x}, \cdots, x^{(n-1)})^T \in \mathbb{R}^n$ are the state vectors of the identification model and the plant. We set $u = u_c$ in (3-18) and define the modeling error ϵ as follows:

$$\begin{aligned} \epsilon &= \dot{\hat{x}}_n - x^{(n)} \\ &= \sum_{i=1}^{r} h_i(\hat{\underline{x}})\hat{a}_i^T\underline{x} - f(\underline{x}) + \sum_{i=1}^{r} h_i(\hat{\underline{x}})\hat{b}_i - g(\underline{x})\, u_c \end{aligned} \quad (3\text{-}19)$$

Using (3-3), (3-4) and (3-8), the modeling error rewrite as

$$\epsilon = [(\hat{a}_i^T - a_i^{*T})\underline{x} + (\hat{b}_i - b_i^*)u_c]\sum_{i=1}^{r} h_i(\hat{\underline{x}}) + w \quad (3\text{-}20)$$

The modeling error can be used for adjusting the model parameters, thus adjusting the controller parameters. Since we have two sources of information for adjusting one set of adjustable parameters, we expect combination of the tracking error and the modeling error.

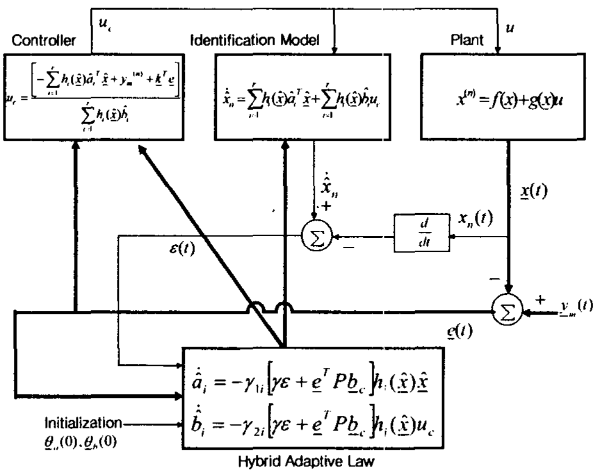

Fig. 3-1 The overall scheme of the TSK fuzzy model based hybrid adaptive control system

Based on the above discussions, we

propose the following *hybrid adaptive law* (HA law), which uses both the tracking error and the modeling error to adjust the parameters:

$$\begin{cases} \dot{\hat{a}}_i = -\gamma_{1i}[\gamma\epsilon + e^T p b_c] h_i(\hat{x})\hat{x} \\ \dot{\hat{b}}_i = -\gamma_{2i}[\gamma\epsilon + e^T p b_c] h_i(\hat{x}) u_c \end{cases} \tag{3-21}$$

where γ is a positive constant. The overall control scheme of this algorithm is shown in Fig. 3-1. Computing the time derivative of the Lyapunov (3-11) along the trajectories of (3-9) and the proposed HA law (3-21), we have

$$\dot{V}_H = -\frac{1}{2}e^T Q e + e^T p b_c w - \gamma\left[(\hat{a}_i^T - a_i^{*T})\hat{x} + (\hat{b}_i - b_i^*)u_c\right]\sum_{i=1}^{r} h_i(\hat{x}) w - \gamma\left[(\hat{a}_i^T - a_i^{*T})\hat{x} + (\hat{b}_i - b_i^*)u_c\right]\sum_{i=1}^{r} h_i(\hat{x})^2 \tag{3-22}$$

where the index "H" in $\dot{V}_H(t)$ stands for the HA law. By defining the parameter deviation index

$$m = m(x, \theta_a, \theta_b, t) = \left[(\hat{a}_i^T - a_i^{*T})\hat{x} + (\hat{b}_i - b_i^*)u_c\right]\sum_{i=1}^{r} h_i(\hat{x}) \tag{3-23}$$

since $\epsilon = m + w$, we can rewrite (3-22) as

$$\dot{V}_H = -\frac{1}{2}e^T Q e + (e^T p b_c - \gamma m)w - \gamma m^2 \approx -\frac{1}{2}e^T Q e - \gamma m^2 \tag{3-24}$$

or

$$\dot{V}_H = -\frac{1}{2}e^T Q e + (e^T p b_c + \gamma\epsilon)w - \gamma\epsilon^2 \approx -\frac{1}{2}e^T Q e - \gamma\epsilon^2 \tag{3-25}$$

where the approximation is made assuming that w is small.

Therefore, from (3-24) and (3-25), either $e \neq 0$ or $\epsilon \neq 0 \, (m \neq 0)$ can cause the Lyapunov function V to decrease.

Actually, we use the following adaptive law to adjust the parameter vector θ_a, θ_b in on-line adaptation.

$$\dot{\hat{a}}_i = -\gamma_{1i}[\gamma\epsilon + e^T p b_c] h_i(\hat{x})\hat{x} \tag{3-26}$$

$$\dot{\hat{b}}_i = \begin{cases} max\{0, -\gamma_{2i}[\gamma\epsilon + e^T p b_c] h_i(\hat{x}) u_c\} & \text{if } \hat{b}_i \leq \delta \\ -\gamma_{2i}[\gamma\epsilon + e^T p b_c] h_i(\hat{x}) u_c & otherwise \end{cases} \tag{3-27}$$

where δ is a positive constant specified by the designer and determines $\Omega_b \triangleq \{\theta_b = (\hat{b}_1, \hat{b}_2, \cdots, \hat{b}_r)^T \in R^r \mid \hat{b}_i \geq \delta > 0; i = 1, 2, \cdots, r\}$. The role of δ is to not only keep $\hat{g}(x \mid \theta_b) > 0$ for all $x \in U_c$ to prevent the denominator of (3-7) from crossing zero but also provide good performance. The δ should be chosen a small positive constant to assure a nonrestrictive Ω_b.

Ⅳ. Simulation Example

Let's consider the application of the direct adaptive fuzzy controller and the hybrid adaptive fuzzy controller to regulate of the inverted pendulum system on a cart from a certain initial condition. The state equation of the inverted pendulum system is as follows.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g\sin(x_1) - amlx_2^2\cos(x_1)\sin(x_1) + a\cos(x_1)u}{l(4/3 - am\cos^2(x_1))} \end{cases}$$

where

- x angle θ(radian) of the pendulum from the vertical
- $\dot{x}$ the angular velocity
- g the gravity constant, $9.8\, m/s^2$
- m the mass of the pendulum
- M the mass of the cart
- $2l$ the length of the pendulum
- u the control force applied to the cart
- $a = \dfrac{1}{m+M}$.

We choose $m = 2.0kg$, $M = 8.0kg$, and $2l = 1.0m$ in the following simulations.

As a model for the pendulum, we use the following TSK fuzzy model with two rules.

Rule 1 : *IF* x *is about* 0 *THEN* $\ddot{x} = a_1^T x + b_1 u$

Rule 2 : *IF* x *is about* $\pm\frac{\pi}{2}$ $(x < \frac{\pi}{2})$ *THEN* $\ddot{x} = a_2^T x + b_2 u$

We choose $k_1 = 1000$, $k_2 = 100$, $P = [15 \;\; 5 ; 5 \;\; 5]$, $Q = diag(10, 10)$, $\delta = 0.5$, $\gamma_{11} = 12$, $\gamma_{12} = 10$, $\gamma_{21} = 1$, $\gamma_{22} = 0.5$ and $\gamma = 5$ for the hybrid adaptive fuzzy controller.

The following simulation results is obtained for the reference signal.

$$\underline{y}_m(t) = (y_m(t), \dot{y}_m(t))^T = ((\pi/20)sin(t), (\pi/20)cos(t))^T$$

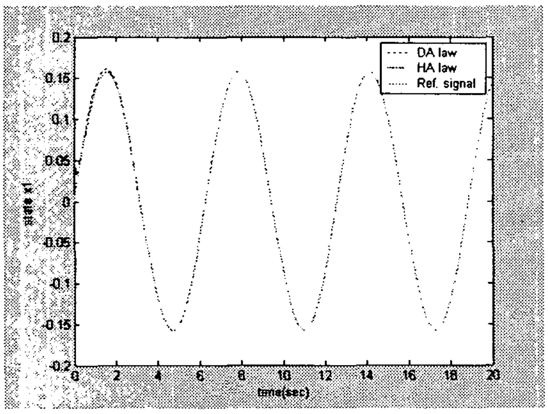

Fig.5-1 State $x_1(t) = \theta(t)$

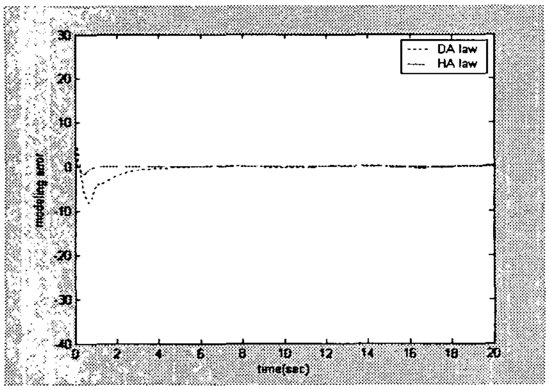

Fig. 5-2 Modeling error

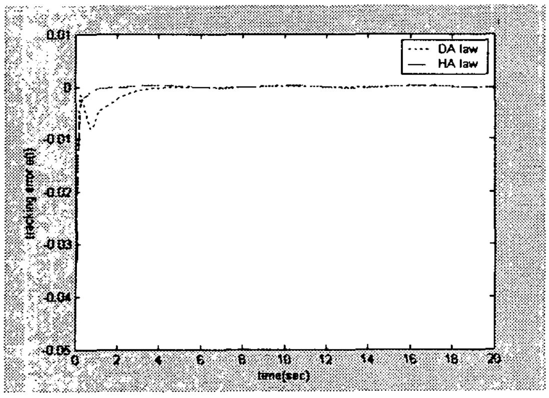

Fig. 5-3 Tracking error

We showed the regulation results of state $x_1(t)$ to verify the performance of the proposed hybrid adaptive fuzzy controller in Fig. 5-1. The modeling error and tracking error of hybrid adaptive fuzzy controller converge to zero faster than the direct adaptive fuzzy controller in Fig. 5-2 and Fig. 5-3. Thus, we can conclude that the proposed method showed better performance than the direct adaptive controller.

V. Conclusions

In this paper, we proposed a hybrid adaptive scheme which combines adaptive fuzzy identification and adaptive fuzzy control for a class of unknown nonlinear dynamic systems. In direct adaptive control, the controller parameters are directly adjusted without identifying the plant parameters. In indirect adaptive control, the controller parameters are adjusted by the estimated plant parameters. The hybrid adaptive control is a combination of the direct and the indirect adaptive control. The hybrid adaptive fuzzy control system combines adaptive fuzzy control and adaptive fuzzy identification. The direct adaptive law can be showed by ignoring the identification errors and fails to achieve parameter convergence. The proposed hybrid adaptive law showed not only enhanced parameter convergence but also fast tracking error convergence. Performance analysis using a Lyapunov function proved to decrease faster along the adaptive system trajectories.

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