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유한요소모델 개선을 위한 자동화된 매개변수 선정법 : 이론

An Automated Parameter Selection Procedure for Updating Finite Element Model : Theory

***(This paper was also presented in the 22nd IMAC held in Dearbon MI in Feb. 2004.)**

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Key Words : Finite Element Model(유한 요소 모델), Model Updating(모델 개선), Parameter Selection(매개 변수 선정)

ABSTRACT

Finite element model updating is an inverse problem to identify and correct uncertain modeling parameters that leads to better predictions of the dynamic behavior of a target structure. Unlike other inverse problems, the restrictions on selecting parameters are very high since the updated model should maintains its physical meaning. That is, only the regions with modeling errors should be parameterized. And the variations of the parameters should be kept small while the updated results give acceptable correlations with experimental data. To avoid an ill-conditioned numerical problem, the number of parameters should be kept as small as possible. Thus it is very difficult to select an adequate set of updating parameters which meet all these requirements. In this paper, the importance of updating parameter selection is illustrated through a case study, and an automated procedure to guide the parameter selection is suggested based on simple observations. The effectiveness of the suggested procedure is tested with two example problems, ones is a simulated case study and the other is a real engineering structure.

1. Introduction

The predicted dynamic behavior of a finite element (FE) model often differs from experimental results of a target structure. Thus, an FE model needs to be verified and, if necessary, updated for further applications. FE model updating is an inverse process to identify and correct uncertain modeling parameters that leads to better predictions of the dynamic behavior of the structure. Although all real structures have infinite numbers of degrees of freedom (DOFs) and modes, the data that can be obtained from modal tests are quite limited for practical reasons. Experimental modal analysis rarely uses more than a couple of hundred sensors. Thus, the number of measured DOFs is very small, and also the available transducers and hardware limit the frequency range of measurements. On the other hand, FE models consist of many finite elements, easily extending in many cases to several thousands. Thus, due to the inherent limitations of experimental data, the number of parameters which can be used to modify an FE model far exceeds that of the measured data of a target structure. There can be numerous modified or updated FE models that agree well with the incomplete test data[1]. But, if the aim of model updating is not simply to mimic the incomplete test results, there must be some restrictions on the selection of updating parameters and their allowable changes so that the updated model retains its physical foundation.

Updating parameters should be selected with the aim of correcting modeling errors. So, only the regions containing modeling errors should be parameterized and allowed to change in correction process[2]. And the criteria to be minimized for model improvement should be sensitive to chosen parameters. Otherwise, the updating parameters easily deviate far from their initial values and lose their physical meaning[2]. If only the sensitivity is concerned, the best way of parameter selection is to assign an updating parameter to each of the finite elements having modeling errors. But, usually an FE model for a real

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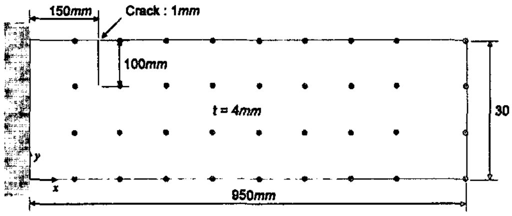
structure has modeling errors in so many finite elements, it is impractical to allocate an updating parameter for each of the finite elements. This is because the updated parameter values of neighboring elements can be oscillatory, which are physically meaningless[3]. And, in numerical point of view, too many updating parameters cause ill-conditioned problems or trapping in many local minima[2]. This paper suggests an idea to select suitable FE model updating parameters among the many candidates set.

(a)



(b)

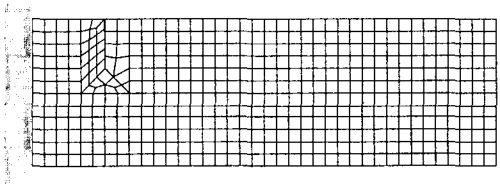


Figure 1: Cracked clamped plate: (a)simulated vibration measurement points: (b)fine FE model (3126 DOFs).

TABLE 2: Comparison of modal properties of cracked plate and updated FE model

Natural frequency (Hz)				
Mode	Simulated experiment	Updated model[a]	Error (%)	MAC
1	3.6011	3.4526	−4.1231	0.9999
2	22.7184	22.0003	−3.1606	0.9694
3	23.7103	23.7655	0.2327	0.9694
4	65.0973	62.4133	−4.1231	0.9912

2.2 Case study

A clamped plate having a crack is provided to demonstrate the effects of updating parameter selections on updated results (Figure 1(a)).

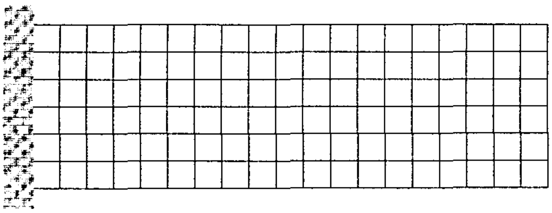


Figure 2: Initial FE model (840 DOFs).

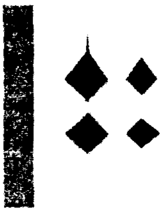


Figure 3: Error location of the initial FE model utilizing force balance method

To simulate experimental data, a fine FE model with 3126 DOFs, is constructed (Figure 1(b)). It is assumed that out-of-plane (z-direction) vibrations are measured at 36 points as marked in Figure 1(a). Figure 2 shows an initial FE model with 840 DOFs. Due to the crack of the test plate, the modal properties from the initial FE model show deviations from those of the test model as summarized in Table 1. Here, the experimental and analytical modes are paired using the MAC. The 2nd and 3rd mode pairs are poorly correlated and the initial FE model needs to be updated for a better correlation. Using an error location technique[5], the region with dominant modeling errors are checked as shown in Figure 3. The plot shows dominant errors in the initial model around the cracked area. In this case study, the finite elements around the dominant error region are grouped into two as in Figure 4. And it is assumed that the mass matrix of the initial FE model is correct and only the stiffness matrix needs to be updated. Thus, the stiffness correction matrix is expressed when we setting two updating parameters p_{k_1} and p_{k_2}.

$$\Delta K = \sum_{i=1}^{2} p_{k_i} K_i \qquad (1)$$

where k_i is the stiffness matrix of the i^{th} region, and the coefficient p_{k_i} is the updating parameter. Among the correlations shown in Table 1, the natural frequency error of the 2nd mode pair and the MAC values of 2nd and 3rd mode pairs, which show the most undesirable correlations, are set as the multiobjective function to be minimized:

$$\{F_1, F_2, F_3\} = \{((f_{a_2} - f_{x_2})\ f_{x_2})^2, 1-MAC_{22}, 1-MAC_{33}\} \qquad (2)$$

where MAC_{ii} is the MAC value of i^{th} mode pair, and f_{x_i} and f_{a_i} are the i^{th} experimental and analytical natural frequencies respectively.

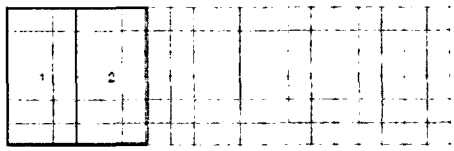


Figure 4: Case study – setting two updating parameters.

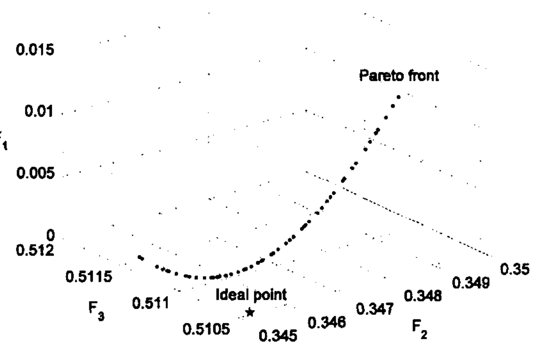


Figure 5: Case study – Pareto front and ideal point.

(a) (b)

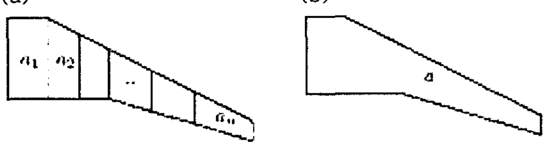


Figure 6: Substructure with modeling errors : (a) n updating parameter; (b) one updating

parameters.
To prevent the other values moving to poor optimization results, they are bounded with constraints:

$$\left(\frac{f_{a_i} - f_{x_i}}{f_{x_i}}\right)^2 \le 0.0025,\ i = 1,3,4$$
$$MAC_{ii} \ge 0.9,\ i = 1,4.$$

The maximum allowable change of the updating parameters, p_{k_1} and p_{k_2}, are set as 0:7. To evaluate the effectiveness of the selected parameters, the multiobjective optimization problem defined by Eqs. (2) and (3) is solved using a multiobjective evolutionary algorithm[6]. The resulting Pareto front is plotted as in Figure 5. It should be noted that the objective functions, F2 and F3, seldom vary compared to their initial values, although F1 changes drastically. The ideal point of Eq. (2) is calculated as

$$\{F_1, F_2, F_3\} = \{0.0000,\ 0.3453,\ 0.5105\} \quad (4)$$

The ideal point[7] is obtained by minimizing each of the objective functions in Eq. (2) individually subject to the constraints (3). Note that the ideal point corresponds to the lower bound of the Pareto front, which is not realizable.
Although the parameters are selected in the regions of large modeling errors, even the lower bound of the Pareto front is not satisfactory. Thus it can be concluded that the parameter selection is not appropriate. Then, how can we obtain an appropriate parameter set? This usually requires a considerable physical insight into the target structure, and trial-and-error approaches are commonly used. But in this work, an idea to get appropriate parameters will be suggested in the following section.

3. Updating Parameter Selection Procedure

3.1 Basic observations

Consider a substructure with n updating parameters (, , ...,) as in Figure 6(a). Assume that modeling errors are correctly located and the updating parameters are associated with them. An objective function or criterion F which defines a difference between analytical and experimental results is modified utilizing the updating parameters. In a linear approximation, the maximum possible variation of F is given by

(5)

where is the maximum allowable change of the updating parameters. Thus, the absolute sum of the sensitivities, , represents the effectiveness of the selected updating parameters in modifying the objective function and is defined as total sensitivity.

Figure 7: Schematic of the 1

The equality in Eq. (7) holds only when the signs of $\frac{\partial F}{\partial a_1}, \frac{\partial F}{\partial a_2}, \ldots, \frac{\partial F}{\partial a_n}$ are the same. Thus, it can be said that the two requirements of updating parameters in section 2.1, number of parameters and their sensitivities, are competitive. That is, by grouping updating parameters into larger parameters, the number of updating parameters can be reduced, but the total sensitivity decreases in general.

From this basic observations, we can construct a set of updating parameters such that the objective functions of primary concern are most sensitive to the selected updating parameters. The parameter selection procedure suggested in this study is accomplished by a sequence of two different selection phases. After the 1st phase of parameter selection procedure, the analyst can stop the parameter selection procedure if the resulting number of parameters are acceptable. Otherwise, he or she can proceeds to the 2nd phase so that the number of parameters can be further reduced.

3.2 Updating parameter selection

3.2.1 1ST PHASE OF PARAMETER SELECTION

Among the two requirements of updating parameters which are dealt with in this study, if only the sensitivities of updating parameters are concerned, the best way of selecting updating parameters is to assign an updating parameter to each of the finite element with modeling errors. By grouping the individual elements into several substructures and assigning an updating parameter to each substructure, the number of parameters can be reduced at the cost of total sensitivity decrease. But, examining Eq. (7), the number of parameters can be lowered without sacrificing the total sensitivity by merging the neighboring elements as long as the sensitivities of the merging elements are the same. Based on this fact, the 1st phase of updating parameter selection for multiple objective functions $F = \{F_p, F_q, \ldots, F_r\}$ is stated as :

STEP 1 Assign an updating parameter a to each finite element with modeling errors and calculate for each parameter,

STEP 2 Merge two neighboring parameters ai and aj into one parameter if . where Note the sensitivity of the objective functions with respect to the merged parameter is simply equal to . Repeat this until no neighboring parameters have the same sensitivity sign.

3.2.2 2nd PHASE OF PARAMETER SELECTION

As a result of the 1st phase of the parameter selection procedure, a list of updating parameters are obtained. Obviously, none of the neighboring parameters have the same sign of the sensitivities. When the number of the resulting parameters are still large and unacceptable, the 2nd phase can be processed. In this case, sacrifice of total sensitivity should be accepted to some extend.

Consider two neighboring parameters a_i and a_j. By merging the two parameters, the total sensitivity is changed from

(8)

to

(9)

where and n is the total

number of the updating parameters. Thus, the decrement of the total sensitivity by this grouping is expressed as

$$\left|\frac{\partial F}{\partial a_i}\right| + \left|\frac{\partial F}{\partial a_j}\right| - \left|\frac{\partial F}{\partial a_i} + \frac{\partial F}{\partial a_j}\right|. \qquad (10)$$

In other words, the required sacrifice for reducing one parameter is equal to Eq. (10). Thus, it is quite reasonable to search two neighboring parameters that minimize Eq. (10) and merge them as one parameter. This results in one parameter reduction at the minimal cost. Note that $F = \{F_p, F_q, \ldots, F_r\}$ is a vector quantity. Thus, there can be various methods to evaluate the vector sacrifice (Eq. (10)). In this study, a scalar index, assuming that every objective function is equally important, is presented as

$$\sum_{k=p,q,\ldots,r} \Delta F_k^{'}, \qquad (11)$$

where the normalized sacrifice $\Delta F_k^{'}$ is defined as the sacrifice of the objective function F_k divided by its total sensitivity at the beginning of the 2nd phase (or before any decrement). From these observations, the 2nd phase of the updating parameter selection procedure is suggested:

STEP Find two neighboring substructures which minimize Eq. (11) and merge them as one parameter. Repeat this procedure until some ending criteria, such as the finial number of parameters, the maximum allowed sacrifice or both, are met.

3.2.3 PROGRAM IMPLEMENTATION

For simple FE models, the parameter selection procedure can be performed manually as illustrated in Figure 7. But for complex structures, this can be a tedious or difficult work. For a program implementation, the computer needs to know whether two finite elements or substructures are neighboring or not. As a finite element consists of a group of nodes, two neighboring elements must share some nodes. For example, an 1D element must share one node with the other element if they are neighboring. For each combination of three different kinds of elements, the number of sharing nodes of two neighboring elements is summarized in Figure 8. Pseudo codes implementing the parameter selection procedure as well as the neighborhood test are provided in [8]

	1D element	2D element	3D element
1D element	1	1	1
2D element	1	2	2
3D element	1	2	3 or 4

Figure 8: Number of sharing nodes of two neighboring elements.

(a)

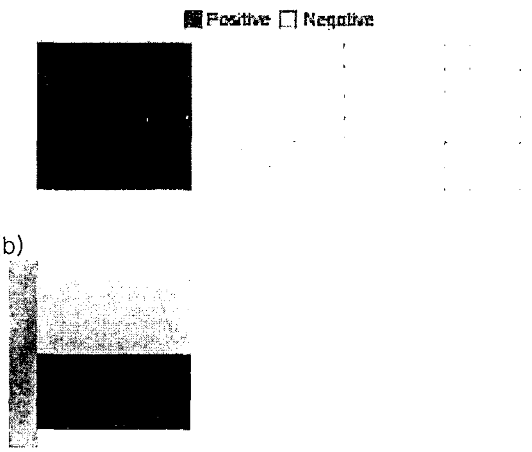


Figure 9: Sign maps of the sensitivities of $\{F_1, F_2, F_3\}: (a) \frac{\partial F_1}{\partial p_{k_i}}; (b) \frac{\partial F_2}{\partial p_{k_i}}$ *and* $\frac{\partial F_3}{\partial p_{k_i}}$.