Obstacle Avoidance in the Chaos Mobile Robot

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***Abstract*— In this paper, we propose a method to avoid obstacles that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When a chaos robot meets an obstacle in a Lorenz equation or Hamilton equation trajectory, the obstacle reflects the robot.**

We also show computer simulation results for avoidance obstacle which fixed obstacles and hidden obstacles of Lorenz equation and Hamilton equation chaos trajectories with one or more Van der Pol obstacles

I. INTRODUCTION

CHAOS theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Lorenz equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a method to avoid obstacles using unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol(VDP) equation with an unstable limit cycle. When chaos robots meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Lorenz equation or Hamilton equation, the obstacles reflect the chaos robots.

Computer simulations also show multiple obstacles can be avoided with an Lorenz equation or Hamilton equation.

II. CHAOTIC MOBILE ROBOT EQUATION

A. Mobile Robot

As the mathematical model of mobile robots, we assume a two- wheeled mobile robot as shown in Fig. 1.

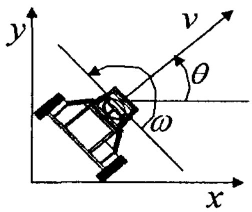


Fig. 1 two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity ω[rad/s] be the inputs in the system. The state equation of the two-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot.

B. Chaos Equations and Embedding of Chaos Circuit in the Robot

In order to generate chaotic motions for the mobile robot, we apply chaos equations such as an Lorenz equation or Hamilton circuit equation.

1) Lorenz equation

We define the Lorenz equation as follows:

$$\begin{aligned} \dot{x} &= \sigma(y-x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= xy - \beta z \end{aligned} \quad (2)$$

Using equation (2), we can get the state equation of the mobile robot as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \sigma(y-x) \\ \gamma x - y - xz \\ xy - \beta z \\ v\cos x_3 \\ v\sin x_3 \end{bmatrix} \quad (3)$$

2) Hamilton Equation

Hamilton equation is one of the simplest physical models that has been widely investigated by mathematical, numerical methods. One of the main attractions of Hamilton circuit is that it can be easily built with periodic motion and subperiodic motion by computer simulation. We can derive the state equation of Hamilton circuit following as from equation (4)

$$\begin{aligned} \dot{x}_1 &= x_1(13 - x_1^2 - y_1^2) \\ \dot{x}_2 &= 12 - x_1(13 - x_1^2 - y_1^2) \end{aligned} \quad (4)$$

Using equation (4), we can get the state equation of the mobile robot as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1(13-x_1^2-y_1^2) \\ 12-x_1(13-x_1^2-y_1^2) \\ v\cos x_3 \\ v\sin x_3 \end{bmatrix} \quad (5)$$

C. Mirror Mapping.

Equation (3) and (5) assume that the mobile robot moves in a smooth state space without boundaries. However, real robots move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the robots approach walls or obstacles using Eq. (6) and (7). Whenever the robots approach a wall or obstacle, we calculate the robots' new position by using Eq. (6) or (7).

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (6)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (7)$$

We can use equation (6) when the slope is infinity, such as $\theta = 90$, and use equation (7) when the slope is not infinity.

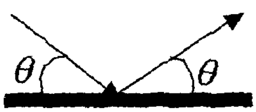


Fig. 2 Mirror mapping

III. NUMERICAL ANALYSIS OF THE BEHAVIOR OF THE CHAOS ROBOT

We investigated by numerical analysis whether the mobile robot with the proposed controller actually behaves in a chaotic manner. In order to computer simulation, we applied mirror mapping and have shown it in Fig. 2. The parameters and initial conditions are used as follows:

A. Lorenz equation case

Coefficients:

v= 1[m/s], σ= 10, γ=28, β=8/3

Initial conditions:

$x_1 = 0.10$, $x_2 = 0.265$, $x_3 = 0.27$,

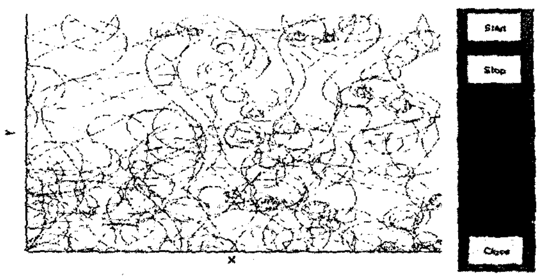
$x = 0.1$, $y = 0.5$

B. Hamilton equation case

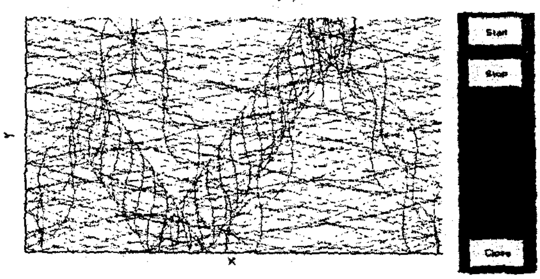
Initial conditions:

$x_1 = 4$, $x_2 = 3.5$, $x = 0$, $y = 0$

Fig. 3 shows the trajectories in which mirror mapping was applied only on the outer wall. In this case, the chaos robot has no obstacles, and we can confirm that the robot is adequately meandering along the trajectories of Lorenz and Hamilton equation, and are covering the whole space in their chaotic manner.



(a)



(b)

Fig. 3 Trajectory of the mobile robot. (a)Lorenz equation, (b) Hamilton equation

IV.THE CHAOTIC BEHAVIOR OF CHAOS ROBOT WITH MIRROR MAPPING AND OBSTACLE

In this section, we will study avoidance behavior of a chaos trajectory with obstacle mapping, relying on the Lorenz equation and Hamilton equation respectively.

Fig. 4 and 5 shows that a chaos robot trajectories to which mirror mapping was applied in the outer wall and in the inner obstacles as well using Eq. (6) and (7), relying on Lorenz equation (3) and Hamilton equation (5). The chaos robot has two fixed obstacles, and we can confirm that the robot adequately avoided the fixed obstacles in the Lorenz and Hamilton chaos robot trajectories.

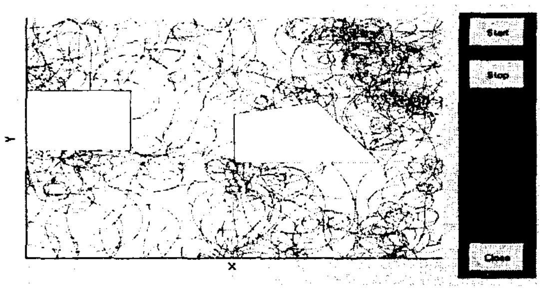


Fig. 4 Lorenz equation trajectories of chaos robot with fixed obstacle

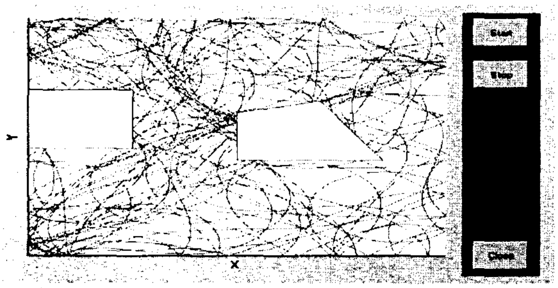


Fig.5 Hamilton equation trajectories of chaos robot with fixed obstacles

V.THE MOBILE ROBOT WITH VAN DER POL EQUATION OBSTACLE.

In this section, we will discuss the mobile robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot can not move close to the obstacle and the obstacle is avoided.

A. VDP equation as an hidden obstacle

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2)y - x \end{aligned} \tag{8}$$

From equation (8), we can get the following limit cycle as shown in Fig. 11.

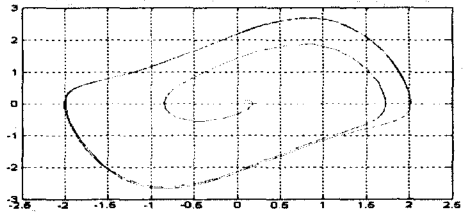


Fig. 6 Limit cycle of VDP

B. Magnitude of Distracting force from the obstacle

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \tag{9}$$

where D_k is the distance between each effective obstacle and the mobile robot.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \dot{x}_k \\ \dot{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1 - (y_o - y)^2)(y_o - y) - (x_o - x) \end{bmatrix} \tag{10}$$

where (x_o, y_o) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual robot (I) and the enlarged coordinates (I/2L) of the magnitude of the virtual robot in VDP(x_k', y_k') as follows:

$$\begin{aligned} L &= \sqrt[2]{(\dot{x}_{vdp}^2 + \dot{y}_{vdp}^2)} \\ I &= \sqrt{(x_r^2 + y_r^2)} \\ x_k' &= \frac{\dot{x}_k}{L}\frac{I}{2}, \quad y_k' = \frac{\dot{y}_k}{L}\frac{I}{2} \end{aligned} \tag{11}$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{bmatrix} \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\dot{x} + \frac{D_k}{D_0}\dot{x}_k)}{n} \\ \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\dot{y} + \frac{D_k}{D_0}\dot{y}_k)}{n} \end{bmatrix} \tag{12}$$

Using equations (9)-(12), we can calculate the avoidance method of the obstacle in the Lorenz equation and Hamilton equation trajectories with one or more VDP obstacles.

In Fig. 7, the computer simulation result shows that the chaos robot has two robots and a total of 5 VDP obstacles with 2 fixed obstacles, including VDP obstacles at the origin in the Lorenz equation trajectories. We can see that the robot sufficiently avoided the obstacles in the Lorenz equation trajectories.

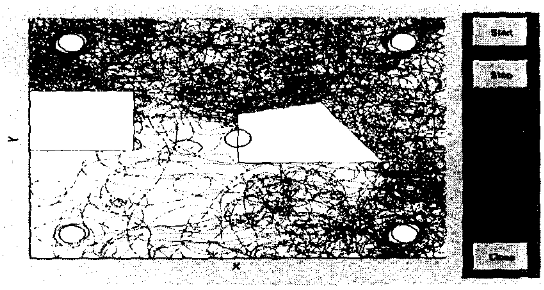


Fig. 7 Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles with 2 fixed obstacles in Lorenz equation trajectories.

In Fig. 8, the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles with 2 fixed obstacle, including VDP obstacles at the origin in the Hamilton equation trajectory. We can see that the robot sufficiently avoided the obstacles in the Hamilton equation trajectory.

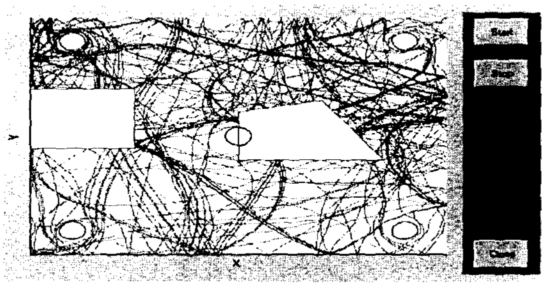


Fig. 8 Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles with 2 fixed obstacles in Hamilton equation trajectory.

D. The Cooperation between the 2 Lorenz mobile robots and 2 Hamilton mobile robots

In this section, we will study avoidance behavior of a chaos trajectory with obstacle mapping, relying on the cooperation of between 2 Lorenz chaos robots and 2 Hamilton chaos robots.

Fig. 9 and 10 shows that a chaos robot trajectories to which mirror mapping was applied in the 2 fixed obstacles and in the hidden obstacles as well using Eq. (6) and (7) that, relying on together with Lorenz equation (3) and Hamilton equation (5). The cooperative chaos robot between Lorenz and Hamilton has two fixed obstacles and 5 hidden obstacles, and we can confirm that the robot adequately avoided the fixed obstacles and hidden obstacles as well as together with fixed obstacles and hidden obstacles in the Lorenz and Hamilton chaos robot trajectories.

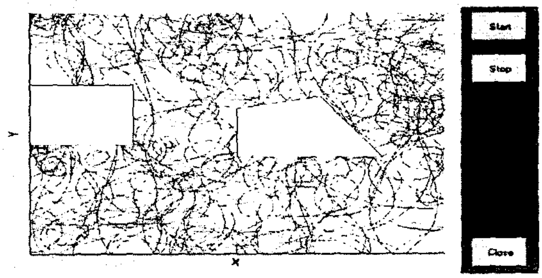


Fig. 9 Computer simulation result of obstacle avoidance with 2 robots and 2 fixed obstacles in Lorenz equation trajectory and Hamilton equation trajectory.

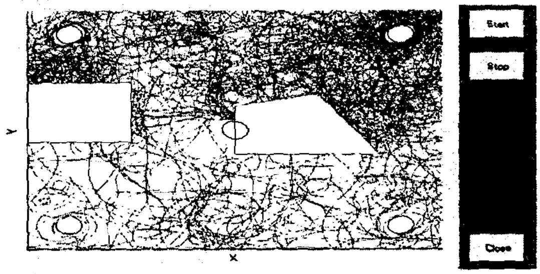


Fig. 10 Computer simulation result of obstacle avoidance with 2 robots and 5 hidden obstacles with 2 fixed obstacles in Lorenz equation trajectory and Hamilton equation trajectory.

C. The relationship between two mobile robots

At this point, we consider two mobile robots that have VDP trajectories. If we do not consider the distance of the two mobile robots, they may happen to collide. So we embedded the VDP equation in the movements of the mobile robot.

If the two robots approach each other, because they have a VDP equation with an unstable limit cycle, the two robots repel each other. As a result, the two robots never happen to collide.

We assume that if the distance between the two robots is less than 0.5 m, the possibility of collision is higher than over 0.5 m. Thus, we can say that two robots with less than 0.5m between them have collided.

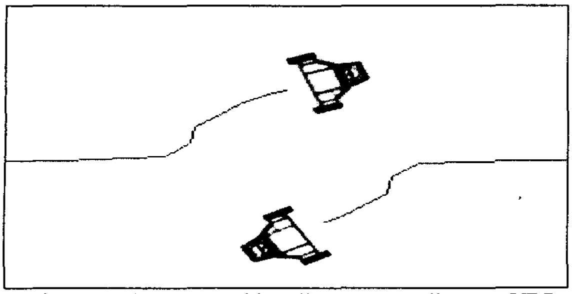
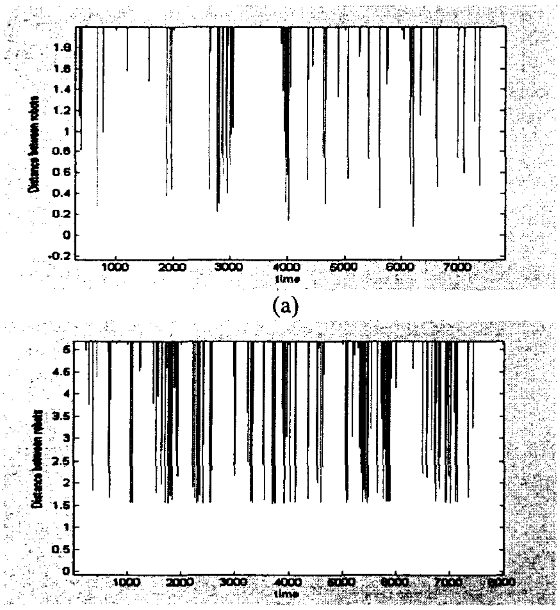


Fig. 9 Robot to avoid collision according to VDP equation

1) Lorenz equation

In Fig 11(a), we can see that when no other action was taken, to the Lorenz equation trajectory of each robot, the robots collide at several instances (700S, 1800S, 2800S, 4000S, 6000S, 6300S etc.).

In order to avoid collision, we applied a VDP equation to the Lorenz equation trajectory of each robot. In Fig .11(b), we can see that there is no point when the robots collided. So, in order to avoid collision between the two robots, we need to apply a VDP equation in the Lorenz equation trajectories.



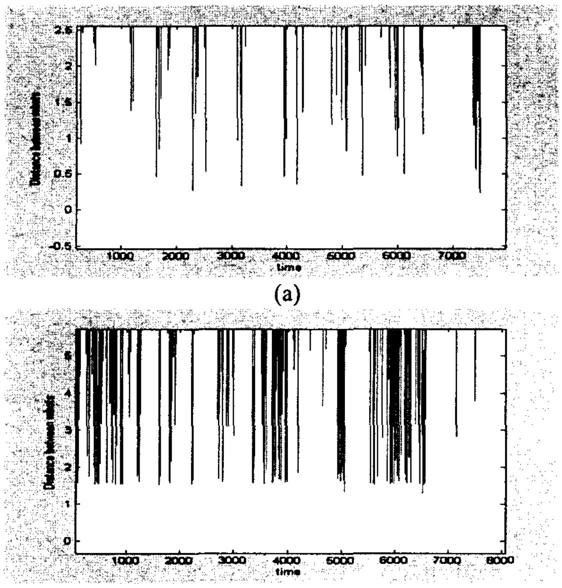
(a)

(b)

Fig. 11 Inter-robot distance (a) when no action taken, (b) when VDP equation applied to the Lorenz equation trajectories of each robot

2) Hamilton equation

In Fig. 12(a), we can see that when VDP trajectories are not applied to the Hamilton equation trajectory of each robot, the robots collided at several instances (1600S, 2300S, 3200S, 4500S etc.). In order to avoid collision between the two robots, we applied VDP trajectories in each robot with a Hamilton equation. In Fig. 12(b), we can see that there were no collisions. So, in order to avoid collision between the two robots, we can also apply a VDP equation in the Hamilton equation chaos robot system.



(a)

(b)

Fig. 12 Inter-robot distance in which were not applied (a) and were applied (b) VDP trajectories with Hamilton equation

VI. CONCLUSION

In this paper, we proposed a chaotic mobile robot, which employs a mobile robot with Lorenz equation and Hamilton equation trajectories, and also proposed an obstacle avoidance method in which we assume that the obstacle has a Van der Pol equation with an unstable limit cycle.

We designed robot trajectories such that the total dynamics of the mobile robots was characterized by an Lorenz equation or Hamilton equation, and we also designed the robot trajectories to include an obstacle avoidance method. By the numerical analysis, it was illustrated that obstacle avoidance methods with a Van der Pol equation that has an unstable limit cycle gave the best performance.

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