

## DIANA를 이용한 콘크리트 균열 진전 시뮬레이션

### Simulation of crack propagation of concrete with the DIANA

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#### ABSTRACT

This paper discusses 2D lattice models of beams for simulating the fracture of brittle materials. A simulation of an experiment on a concrete beam subjected to bending, in which two overlapping cracks occur, is used to study the effect of individual beam characteristics and different arrangements of the beams in the overall lattice.

It was found that any regular orientation of the beams influences the resulting crack patterns. Methods to implement a wide range of poisson's ratios are also developed, the use of the lattice to study arbitrary micro-structures is outlined. The crack pattern that are obtained with lattice are in good agreement with the experimental results. Also, numerical simulations of the tests were performed by means of a lattice model, and non-integer dimensions were measured on the predicted lattice damage patterns.

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#### 1. Introduction

Since concrete is a highly heterogeneous material and concrete cracking is a localized phenomenon (resulting in stress redistributions during cracking), the implementation of concrete cracking into finite element codes is not straightforward. Iterative calculation techniques have to be adopted for correct predictions of the highly non-linear material behavior. In this respect two main approaches to modeling concrete can be distinguished, namely by continuum or discrete methods. When a continuum model is adopted for "predicting" fracture processes in concrete, the macro-level is usually chosen as the level of modelling. Consequence of this choice is that the constitutive relation in the model is mostly non-linear. When, on the other hand, the material's micro-level is addressed, and the material is treated as three phase material (aggregate, matrix and interface between matrix and aggregates), a brittle constitutive relation seems satisfactory. In spite of the individual elastic-brittle behavior of the material constituents, non-linearity will be observed at the macro-level. In the model presented in this paper, the second (discrete or lattice) approach is used for modelling concrete cracking at the meso-level. In such a model

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the material is discretized by a network of beams or trusses, and cracking is modelled by removing a beam from the mesh. Through the years, considerable attention has been given to lattice models. Lattice type models were first developed in theoretical physics. Then, Fracture in disordered materials, the model has been applied for simulating percolation problem and electrical conductivity in a lattice. The original model proposed by Herrmann incorporated a regular square lattice illustrated in Fig. 1a. For simulating concrete fracture the regular triangular lattice (Fig. 1b) was proposed by Schlangen & Van Mier as original. A random lattice, proposed by Moukarzel & Herrmann, has been used for similar purposes. In this random lattice, the connectivities of the beams are determined by the Voronoi construction of a set of nodes. The random lattice is illustrated in Fig.1c. Most of the simulations of concrete fracture presented in this paper were carried out with the regular triangular lattice that was mentioned before.

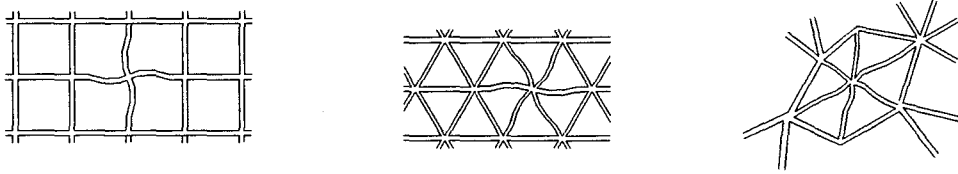


Fig.1 Lattice types: regular square lattice (a), regular triangular lattice (b) and random lattice(c)

## 2. Principle of the Lattice Model

In the adopted lattice model integrated beam elements with three degrees of freedom are directly used. The constitutive relation of an element is linear-elastic, and the stress in an element is calculated as a combination of the normal force and the bending moments acting on the element. This "effective" stress causes failure as soon as the strength of the beam element is exceeded. Because of the linear-elastic behavior of the lattice, the failure load of a beam is calculated within a single step of a finite element analysis. This procedure of loading the mesh and consequently removing an element is repeated until complete failure of the lattice has been obtained. In order to reduce the computational effort, generally only the area of the specimen where cracks are expected to grow, is modelled with a lattice. The remainder of the specimen is modelled with continuum elements. The simulations presented in this paper are carried out with the finite element package DIANA, mainly because of the availability of several types of continuum elements besides the beam elements required for the lattice. At the boundary between lattice continuum elements, the beam nodes are tied to the nodes of the continuum elements. In order to reduce the computer time, the lattice model is currently being implemented as a "special" module in the DIANA finite element code. Although each individual element in a lattice fails brittle, the(global) softening behavior of heterogenous materials like concrete can be simulated with the model. When a regular lattice(Fig. 1b) is used, heterogeneity has to be implemented in the model in order to obtain realistic crack patterns observed in concrete experiments. Heterogeneity is introduced by varying the Young's modulus and the strength of the beam elements. Considering the material structure of concrete, a realistic strength and stiffness distribution strength and stiffness of the distinctive phase are assigned to the beams falling inside the aggregates (A), in the matrix (M) or at the interfacial zone between the aggregate and the matrix (bond, B)

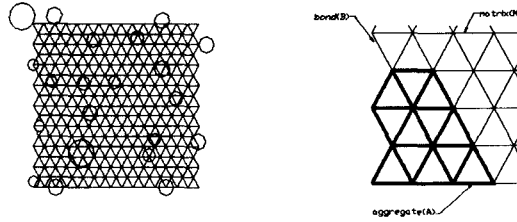


Fig. 2 Aggregate structure projected on top of a lattice (a) and assigning properties to the beam in the three phases of the material (b)

### 3. Parameter determination

To describe the global elastic behavior of the lattice, the Young's modulus ( $E$ ) and Poisson's ratio ( $\nu$ ) of the material which is to be modelled are available as input. They have to correspond to the global behavior of the lattice, which can be adjusted by changing the geometrical properties (height  $h$  and thickness  $t$ ) and the global Young's modulus of the beams ( $E_{\text{beam}}$ ). For two dimensional simulations, it seems obvious to choose the beam thickness equal to the thickness of the simulated specimen. When a regular lattice is adopted, the remaining beam properties ( $h$  and  $E_{\text{beam}}$ ) can be determined in a very straight forward manner, since the Poisson's ratio of the lattice is directly related to the height over length ratio of the beams. For a regular triangular lattice without particle overlay and consisting of prismatic beams it was found that:

$$\nu = \frac{1 - \frac{12l}{A^2}}{3 + \frac{12l}{A^2}} = \frac{1 - (\frac{h}{l})^2}{3 + (\frac{h}{l})^2}$$

This function is shown graphically in Fig. 3, with the results for the lattice with varying  $P_k$  (the ratio of the aggregate volume to the total volume of the concrete). The Poisson's ratios for the lattices given in Fig. 3a are the average values resulting from calculations on 175 meshes measuring  $50 \times 50$  nodes. When the height of the beams is fixed, the local Young's modulus of the beams ( $E_{\text{beam}}$ ) can be determined from the global stiffness of the lattice, which has to be coincided with the stiffness of the material that is modelled. In Fig. 3b the relation between the ratios  $E/E_{\text{beam}}$  and  $h/s$  is shown by the five different value of the  $A$ . The stiffness of the lattice can be varied by changing the Young's modulus of the beams. The values of the ratios, however, must be kept constant in order to maintain a linear relation between  $E$  and  $E_{\text{beam}}$ .

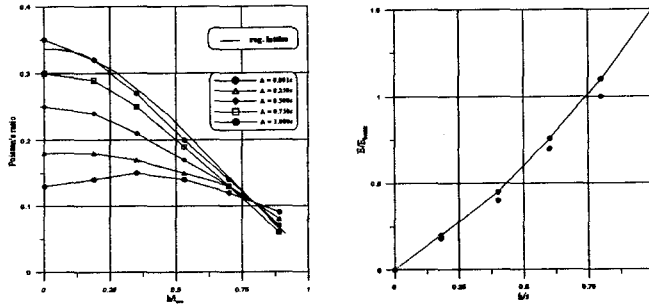


Fig.3 Relation between  $\nu$  and  $h/l_{avg}$  for a regular triangular lattice with varying  $P_k$

Then the parameter related to the global elastic behavior of the mesh, parameters related to the fracture law is required. Cracking is obtained by removing one beam from the mesh in each step of a lattice analysis. The choice of the beam is based on a very simple fracture law and the "effective" stress  $f$  is calculated following:

$$f_{eff} = \beta \cdot \left( \frac{F}{A} + \alpha \cdot \frac{|M_i, M_j|_{max}}{W} \right) < f_t$$

where  $F$  is the normal force in the beam,  $A$  is the cross sectional area,  $M_i$  and  $M_j$  are the moments in the two respective nodes of the beam and  $W=1/6 \cdot b \cdot h^2$  is the sectional moment of the beam. Fig.4a shows that the bond fraction approaches an asymptotic value with increasing beam length. At a certain moment only the largest particles are present in the mesh. The number of such large-sized particles is limited in the fuller distribution that was used. Fig.4b shows that the force that can be carried by the global lattice hardly varies with beam length. This indicates that, for the present example with  $P_k = 0.75$ , failure is governed by the weakest elements, i.e. the bond beams, will become clear further on. In the Fig.4c the phase fractions are shown for aggregate sizes 25mm for varying  $P_k$  values.

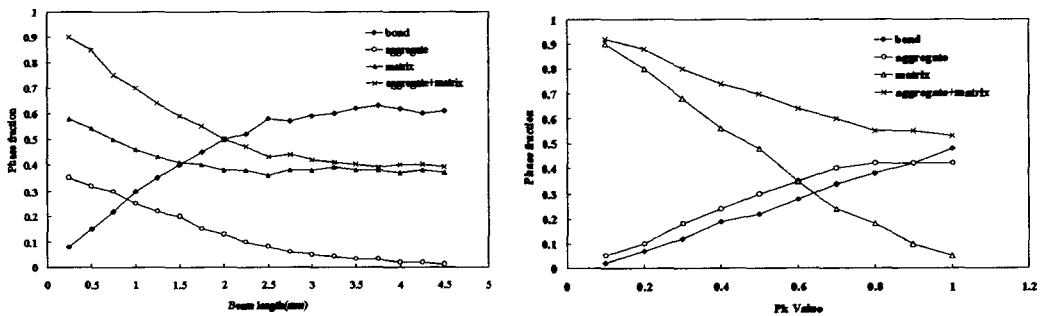


Fig.4 Phase fractions of bond, matrix and aggregate beams as a function of beam length (a), and fraction of bond, matrix and aggregate phases with varying  $P_k$  (b)

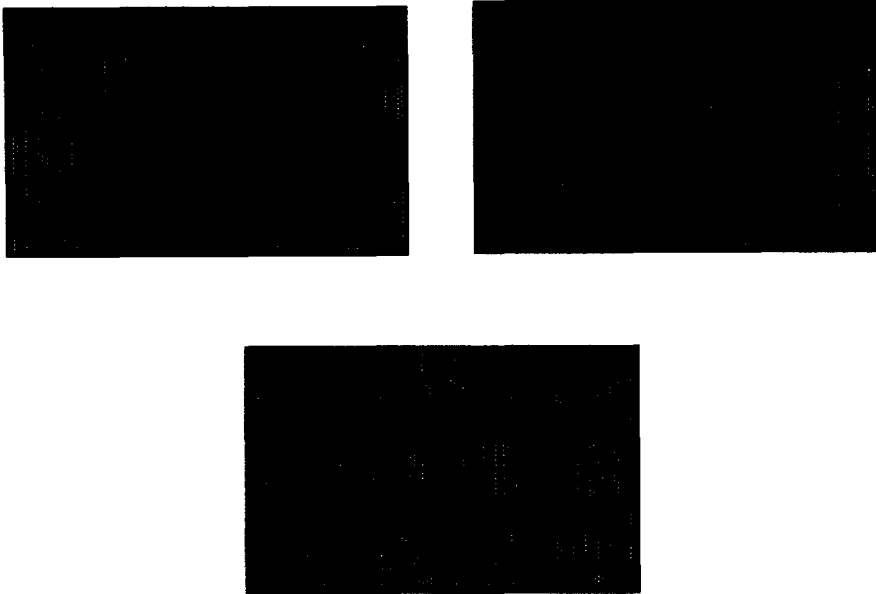


Fig. 5 Modelling of cement based concrete with varying  $P_k$  values

#### 4. Application of the Lattice Model

In order to apply to the lattice model, the model has been used for simulating test. Main purpose of these calculations was to validate the model by means of comparing the numerical results to those of experiments. A lattice simulations of a bending test show another influence on the tail of the softening curve, now depending on the maximum aggregate size however(Fig. 6).

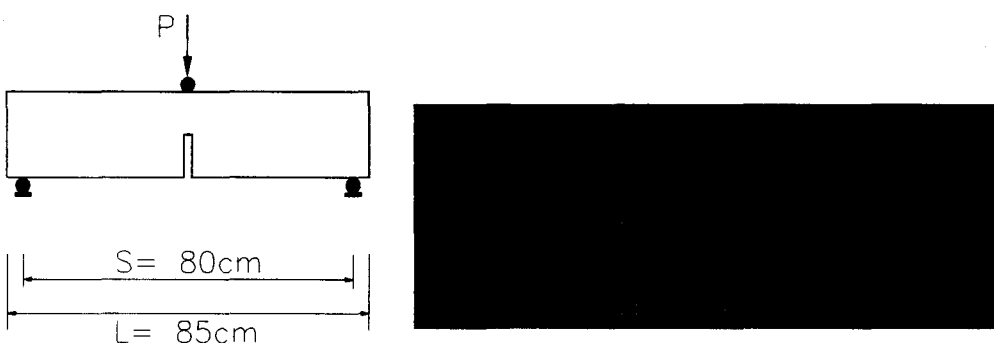


Fig. 6 Three-point bending specimen and lattice analysis modelling

To get insight in the behaviour of these different lattice types a comparison was made by simulating the behaviour of a single notched specimen under three point bending test. The area of the specimen that was used to control the deformations in the laboratory test, was modelled with three different lattices,

denoted Type A to C in Table 1.

Table 1. Overview of the lattice types compared in a bending test.

lattice type	abbreviation
Triangular lattice with particle structure (Number of node and element 672, 1283)	TYPE A
Triangular lattice with particle structure (Number of node and element 2542, 4966)	TYPE B
Triangular lattice with particle structure (Number of node and element 10004, 19772)	TYPE C

In Fig. 7(a)~(b) the cracks in the lattice part of the specimen are shown at almost the same crack mouth opening displacement(CMOD). Because of the presence of a grain overlay in the mesh, micro-cracks develop in the bond zones around the grains, apart from the continuous cracks in the cement matrix. Since this mechanism resembles the actual fracture process in concrete, TYPE B seems most suitable for modelling this material.



Fig. 7. Simulation of a bending test on a specimen using different lattice element types

The load-midspan deflection curves of the lattice element types are shown in Fig. 8 (a)~(b), respectively. The displacement given in the figure is measured at the load application point. Three curves correspond to the three types in each size group, which differ only by the random distribution of the equal amount of aggregates.

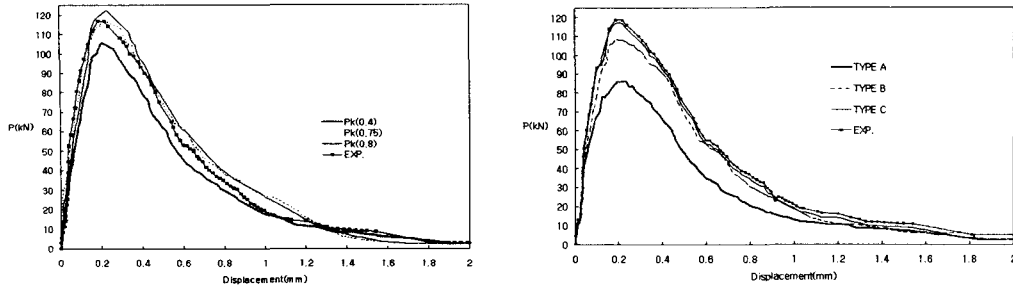


Fig. 8 Compare P-  $\delta$  of analyzed lattice types to experiment specimen

## 5. Conclusion

This paper discussed 2D lattice model for fracture simulations. In this literature, models with various types of elements can be found. The equations for the network models with these different elements are all discretizations of different continuum equations. For fracture the results that are obtained strongly depend on the chosen element type. Beam elements with three degrees of freedom per node give the best agreement with experimentally obtained crack patterns. In simulations with different  $pk$  (aggregate size) is 0.75, realistic crack patterns are also obtained. However the crack patterns that are simulated are not complicated. Bending tests are simulated in a straight crack surrounded by microcracks develops. In Fig.7 of this paper, it is shown that if the crack pattern is more complex, and the cracks are curved, elements with three degrees of freedom are necessary. The shape or orientation of the beams in a lattice also influences the simulated crack patterns, with the cracks tending to follow the mesh lines.

## ACKNOWLEDGEMENT

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