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다층간분리된 적층판의 자유진동해석

Free Vibration Analysis of Multi-delaminated Composite Plates

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ABSTRACT

In this proposed work new finite element model for multi-delaminated plates is proposed. In the current analysis procedures of multi-delaminated plates, plate element based on Mindlin plate theory is used in order to obtain accurate results of out-of-plane displacement of thick plate. And for delaminated region, plate element based on Kirchhoff plate theory is considered. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. The numerical results show that the effect of delaminations on the modal parameters of delaminated composites plates is dependent not only on the size, the location and the number of the delaminations but also on the mode number and boundary conditions. Kirchhoff based model have higher natural frequency than Mindlin based model and natural frequency of the presented model is closed to Mindlin based model.

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1. Introduction

Delamination is caused mainly because the transverse tensile and interlaminar shear strength of composite laminates are quite low in contrast to their in-plane properties. This damage may result in the degradation of strength overall stiffness of the laminate, especially in the structure of the laminated construction under compression (Pavier and Clarke, 1995). And Tracy and Pardoen (1989) show the effect of delamination on the natural frequencies of composite laminates.

During last two decades, many researches on one-dimensional delamination problems have been made. Wang et al. (1982) examined the free vibration of an isotropic through-width delaminated beam considering the coupling effect. Shen and Grady (1992) studied the delamination effect of a composite beam using a variational principle. And then many studies about one-dimensional delamination problems performed in various ways (Lee et al., 2002; Lee et al., 2003; Park et al., 2003).

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However, studies on vibration analysis of delaminated plates are very few compared with one-dimensional analysis. The first work for laminated plates with delamination was Tenek et al. (1993). In their work, a three-dimensional finite element method was used to analyze the natural frequencies of delaminated composite plates as well as the delamination dynamics over a broad range of frequencies. Such a three-dimensional analysis is accurate and instructive, but is very computationally intensive. Thus, Ju et al. (1995) presented a two-dimensional finite element approach for the analysis of free vibration of laminated composite plates with multiple delaminations.

Gim (1994) developed a plate finite element based on a lamination theory, which includes the effect of transverse shear deformation. In modeling 2-D delaminations in laminated plates, the global region is modeled by a single layer of plate elements while the delaminated region is modeled by two layers of plate elements whose interface contains the delamination. To ensure the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack tip, a multipoint constraint algorithm has been developed and incorporated into the finite element code. Parhi et al. (2000) extend the simple model proposed by Gim (1994) to the general case of a laminated composite plate having arbitrarily located multiple delaminations for analyzing its dynamic behavior.

In this proposed work, computational, finite element model for multi-delaminated plates is developed. In the current analysis procedures of multi-delaminated plates, elements used in the delaminated and undelaminated regions are based on Mindlin and Kirchhoff plate theories, respectively. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. Element mass and stiffness matrices of each region are assembled for global matrix.

2. Theoretical Formulation

Figure 1 shows a delaminated composite plate where the delaminations are presumed to be parallel to the mid-plane of the plate. The global coordinate system is located at the mid-plane of the plate with the z-axis perpendicular to it. Figure 1 shows the four elements used to model a region with three delaminations.

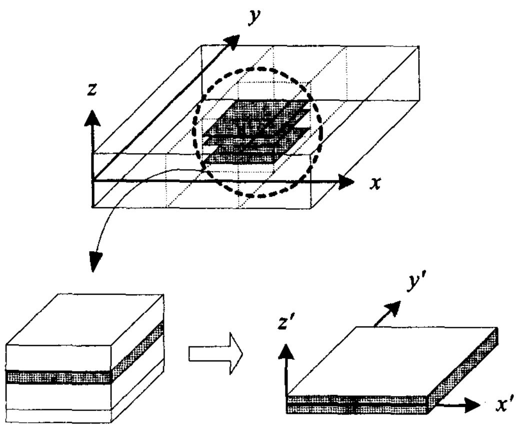

Figure 1. Laminated plate with rectangular delaminations, global (xyz) and local ($x'y'z'$) coordinates.

The displacement field of the element is assumed to be of the following form relative to its own local coordinate system.

$$u'(x', y', z') = z'\varphi_x$$
$$v'(x', y', z') = z'\varphi_y \quad (1)$$
$$w'(x', y', z') = w_0'(x', y')$$

where $\varphi_x = -\partial w_0'/\partial x'$ and $\varphi_y = -\partial w_0'/\partial y'$. $w_0'(x', y')$ is the displacement of the middle surface in the z'-direction.

Using the four nodes, twelve degree of freedoms plate element the displacement field can be interpolated as

$$w'(x', y', z') = \sum_{i=1}^{4} N_i(\xi, \eta) w_{0i}' \quad (2)$$

where φ_{xi} and φ_{yi} are the nodal rotations and w_{0i}' is the nodal translations. $N_i(\xi, \eta)$ is the shape functions.

The strain-displacement and stress-strain relations for the k-th lamina are expressed as

$$\boldsymbol{\varepsilon}' = \mathbf{B}'\mathbf{d}' \quad (3)$$

$$\boldsymbol{\sigma}' = \mathbf{Q}^{k}{}'\boldsymbol{\varepsilon}' \quad (4)$$

where $\boldsymbol{\sigma}'$ and $\mathbf{Q}^{k}{}'$ are stress vector and transformed reduced stiffness matrix of the k-th lamina, respectively. And $\boldsymbol{\varepsilon}'$, $\mathbf{d}'$ and $\mathbf{B}'$ are strain, displacement vectors and strain-displacement matrix, respectively, which are defined by

$$\boldsymbol{\varepsilon}' = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \end{bmatrix}^{\mathrm{T}} \quad (5)$$

$$\mathbf{d}' = \begin{bmatrix} \mathbf{d}_1' & \mathbf{d}_2' & \mathbf{d}_3' & \mathbf{d}_4' \end{bmatrix}^{\mathrm{T}} \quad (6)$$

$$\mathbf{B}' = \begin{bmatrix} \mathbf{B}_1' & \mathbf{B}_2' & \mathbf{B}_3' & \mathbf{B}_4' \end{bmatrix} \quad (7)$$

where, superscript T stands for transpose matrix.

The element strain energy $U^{e}{}'$ and element kinetic energy $T^{e}{}'$ are given by

$$U^{e}{}' = \mathbf{d}'^{\mathrm{T}}\,\mathbf{k}'\mathbf{d}' \quad (8)$$

$$T^{e}{}' = \dot{\mathbf{d}}'^{\mathrm{T}}\,\mathbf{m}'\dot{\mathbf{d}}' \quad (9)$$

where $\dot{\mathbf{d}}'$ is the element nodal velocity vector and dot stands for differentiation with respect to time of vectors. $\mathbf{k}'$ and $\mathbf{m}'$ are the element stiffness and mass matrixes, respectively, which are defined by

$$\mathbf{m}' = \int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{n}\int_{z_{k-1}'}^{z_k'} \mathbf{N}'^{\mathrm{T}}\,\boldsymbol{\rho}^{k}\mathbf{N}'\,\mathrm{d}z'\right)\det|J|\,\mathrm{d}\xi\,\mathrm{d}\eta \quad (10)$$

$$\mathbf{k}' = \int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{n}\int_{z_{k-1}'}^{z_k'} \mathbf{B}'^{\mathrm{T}}\,\mathbf{Q}^{k}{}'\mathbf{B}'\,\mathrm{d}z'\right)\det|J|\,\mathrm{d}\xi\,\mathrm{d}\eta \quad (11)$$

where, n, ρ^k and $\det|J|$ are the number of layers in the delaminated element, the average density of the k-th lamina and the determinant of the Jacobian transformation matrix.

When shear deformation is important, it cannot be assumed that normals to the middle surface remain normal to it. Based on the Mindlin plate theory, the displacement field is given by

$$u(x,y,z)=z\theta_x$$
$$v(x,y,z)=z\theta_y \quad (12)$$
$$w(x,y,z)=w_0(x,y)$$

where θ_x , θ_y and $w_0(x,y)$ are the rotations of a transverse normal about the y- and x-axes, and displacement of the middle surface in the z-direction, respectively.

The strain-displacement and stress-strain relation for the k-th lamina are expressed as

$$\boldsymbol{\varepsilon}=\mathbf{Bd} \quad (13)$$

$$\boldsymbol{\sigma}=\mathbf{Q}^k\boldsymbol{\varepsilon} \quad (14)$$

where $\boldsymbol{\varepsilon}$, $\mathbf{d}$, $\boldsymbol{\sigma}$, $\mathbf{B}$ and $\mathbf{Q}^k$ are strain, displacement, stress vectors, strain-displacement matrix and transformed reduced stiffness matrix of the k-th lamina based on Mindlin plate theory, respectively. Strain and stress vectors of the undelaminated region have transverse shear strain and stress terms, respectively, which are neglected in the delaminated region. Thus, reduced stiffness matrix of the undelaminated region has 5×5 size.

The element stiffness and mass matrixes based on the Mindlin plate theory can be obtained as

$$\mathbf{m}=\int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{m}\int_{z_{k-1}}^{z_k}\mathbf{N}^{\mathrm{T}}\boldsymbol{\rho}^k\mathbf{N}\,dz\right)\det|J|\,d\xi\, d\eta \quad (15)$$

$$\mathbf{k}=\int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{m}\int_{z_{k-1}}^{z_k}\mathbf{B}^{\mathrm{T}}\mathbf{Q}^k\mathbf{B}\,dz\right)\det|J|\,d\xi\, d\eta \quad (16)$$

where m is the number of layers in the undelaminated element. More detail formulation is described in Ma (2004).

3. Element for Transition Region

In the previous sections, the elements in the delaminated region and undelaminated region are derived separately. However, at the boundary connecting two regions, the continuity conditions for the displacement field must be satisfied.

Displacements of the middle surface in the z-direction of the two regions are same. Let the common nodes at the connection boundary are denoted i. For common nodes i, generalized displacement of the delaminated region is transformed as follows.

$$\mathbf{d}_{Ti}=\mathbf{T}_i\mathbf{d}_{Mi} \quad (17)$$

where subscript T denotes the transition region. $\mathbf{T}_i$ and $\mathbf{d}_{Ti}$ are the nodal transformation matrix and nodal displacement vector of the transition element, respectively. Subscript M stands for the Kirchhoff elements.

Using the displacement field of the element, transformation of the displacement vector can be written as

$$\mathbf{d}_T=\mathbf{T}\mathbf{d}_M \quad (18)$$

where, if the second and third nodes of the element are common nodes, transformation matrix $\mathbf{T}$ is expressed as follows.

$$\mathbf{T}=Diag[\mathbf{I} \quad \mathbf{T}_2 \quad \mathbf{T}_3 \quad \mathbf{I}] \quad (19)$$

where $\mathbf{I}$ and $Diag$ stand for the identity matrix and diagonal matrix, respectively. This transformation in the numerical analysis is accomplished by keyword *MPC in ABAQUS (Hibbitt, Karlsson and Sorensen, 2003) and definition of the transition domain is represented in figure 2.

Stiffness and mass matrices are expressed as follows for each region.

$$\mathbf{K} = \sum_{k=1}^{l} \begin{cases} \mathbf{T}^T \mathbf{k}_K \mathbf{T} & \text{(for trasition region)} \\ \mathbf{k}_M & \text{(for undelaminated region)} \\ \mathbf{k}_K & \text{(for delaminated region)} \end{cases} \tag{20}$$

$$\mathbf{M} = \sum_{k=1}^{l} \begin{cases} \mathbf{T}^T \mathbf{m}_K \mathbf{T} & \text{(for trasition region)} \\ \mathbf{m}_M & \text{(for undelaminated region)} \\ \mathbf{m}_K & \text{(for delaminated region)} \end{cases} \tag{21}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness matrix and mass matrix, respectively. Subscript K and l stand for the Kirchhoff elements and the total number of element in the global model.

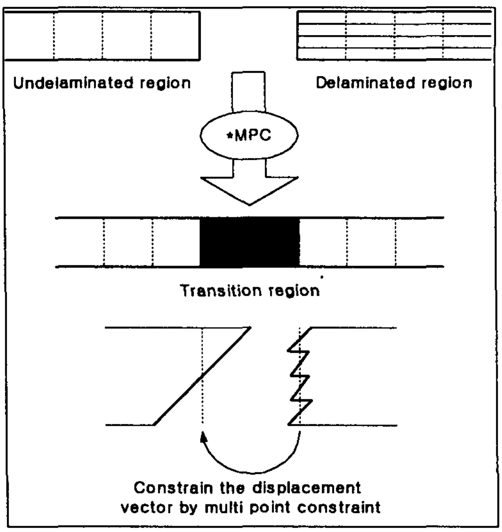

Figure 2. Definition of the transition region

4. Eigenvalue Equation

The application of Hamilton's principle to the Lagrange's equations gives

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = 0 \tag{22}$$

Since the motion is harmonic then

$$\mathbf{d}(t) = \Delta \sin \omega t \tag{23}$$

where the amplitudes Δ are independent of time and ω is the frequency of vibration. Substituting equation (23) into (22) gives

$$\left[\mathbf{K} - \omega^2 \mathbf{M}\right]\Delta = 0 \tag{24}$$

Equation (24) represents a set of n linear homogeneous equations in the unknowns Δ_1, Δ_2, ..., Δ_n. The condition that these equations should have a non-zero solution is that the determinant of coefficients should vanish, that is

$$\det\left[\mathbf{K} - \omega^2 \mathbf{M}\right] = \left|\mathbf{K} - \omega^2 \mathbf{M}\right| = 0 \tag{25}$$

Equation (25) can be expanded to give a polynomial of degree n. This polynomial equation will have n roots ω_1^2, ω_2^2, ..., ω_n^2.

5. Numerical Analysis

Using the commercial finite element program ABAQUS, eigenvalue analysis of the laminated plate is performed. For the application of the presented theory to numerical model, S4R and S4R5 element of the ABAQUS are used. S4R element is the general-purpose elements that are valid for thick problems. However, S4R5 elements formulation is based on Kirchhoff plate theory and neglects the transverse shear flexibility. To employ the present theory to numerical model, keyword *MPC in ABAQUS is used. The degrees of freedoms of the Mindlin based plate element at the connecting boundary are constrained to degree of freedoms of Kirchhoff based plate element.

5.1 Natural frequencies of the single delamination case

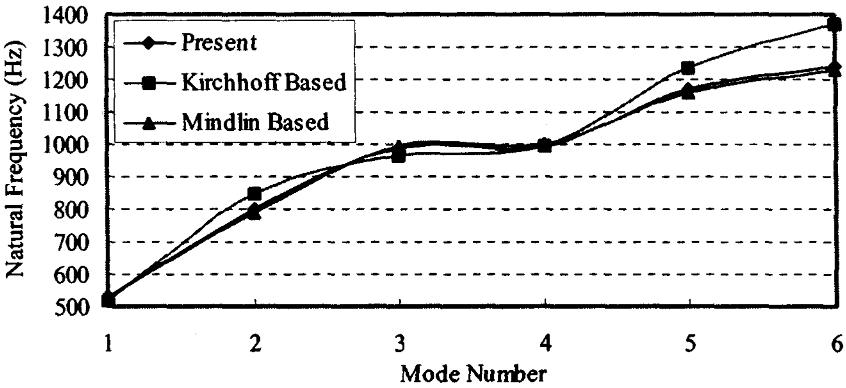

Figure 3. Natural frequencies of the presented model for single delamination case

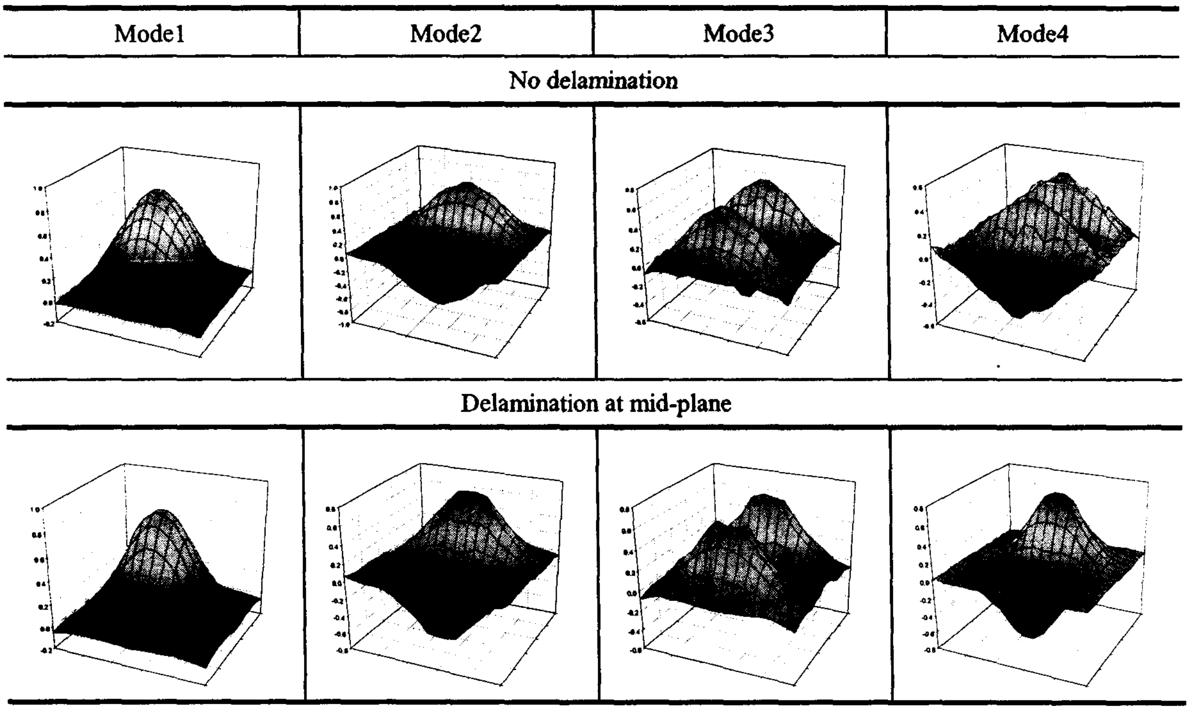

Figure 4. Mode shape of plates with and without a delamination

Natural frequencies of the presented model are closed to Mindlin based model. Length of square plate is 1,800 mm. The plate is made of 20 plies of graphite-epoxy in a $[(+45/-45)_5]_s$ lay-up. Each ply has a thickness of 18 mm; thus, the total plate thickness is 360 mm. The material properties are given in table 1. Four edges of the plate are all clamped. Delamination is located at the center of plate, and delamination size is 600×600 mm. Figure 4 shows the first foure mode shapes of plate with and without a delamination. It can be seen that the higher mode shapes are affected more by the delaminations.

Table 1. Material properties

E_1(N/m^2)	E_2	ν_{12}	G_{12}(N/m^2)	G_{13}	G_{23}	ρ
1.35×10^{11}	1.3×10^9	0.38	6.4×10^8	6.4×10^8	4.3×10^8	1590kg/m^3

5.2 Effect of position of delamination

To investigate the effect of thickness and X-axis direction position of delamination, the position of delamination is considered as the following figure.

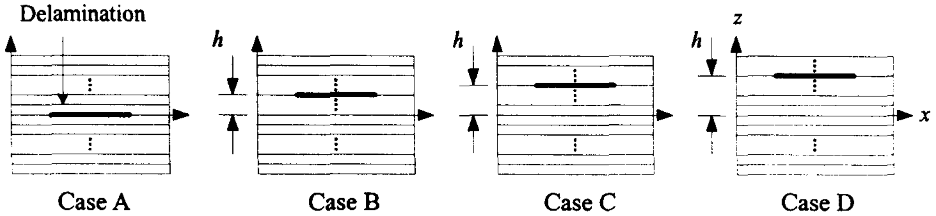

Figure 5. Thickness direction position of delamination

From the result, as the z-axis position of delamination is closed to mid-plane of the plates, natural frequency of the plates have more lower values because of the reduction of stiffness. Similarly, as the x-axis position of delamination is closed to center of the plates, natural frequency of the plates have lower values because of the reduction of stiffness. Especially, natural frequency of the plate dropped remarkably at the center of the plates. Results of free vibration analysis are presented in Ma (2004).

5.3 Effect of size and number of delaminations and boundary condition

The natural frequencies are greatly dependent on the size of delamination like as in the case of section 5.2. These differences of the frequencies caused from the reduction of stiffness of plates. If the size of delaminations is smaller, then the frequencies of the plate are more reduced. And it can be seen that the effect of delamination on the natural frequencies is greatly dependent on the boundary conditions, that is, the more strongly the plate is restrained, the greater the effect of the delaminations on the natural frequencies.

The natural frequencies are greatly dependent on the number of delaminations, that is, the more delaminations are existed in the plates then the natural frequencies of the plates are decreased because of the reduction of stiffness of plates. If multiple delaminations are existed in the laminated plates, the differences of the natural frequencies between presented model and Mindlin based model and presented model and Kirchhoff based model are not remarkable. Comparing with case of boundary condition's effect, this behavior must be caused from change of thickness of plates in the delaminated region.

6. Conclusions

In the current analysis procedures of multi-delaminated plates, different elements are used at delaminated and undelaminated region separately. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. Element mass and stiffness matrices of each region are assembled for global matrix. From the numerical analysis, Kirchhoff based model have higher natural frequency than Mindlin based model and natural frequency of the presented model is closed to Mindlin based model. Present model is more accurate than Kirchhoff based model and more efficient than Mindlin based model. The numerical results show that the effect of delaminations on the modal parameters of delaminated composites plates is dependent not only on the size, the location and the number of the delaminations but also on the mode number and boundary conditions.

REFERENCES

Gim, C.K. (1994) "Plate Finite Element Modeling of Laminated Plates," Computers & Structures, Vol. 52, No. 1, pp. 157-168.

Hibbitt, Karlsson and Sorensen, Inc. (2003) ABAQUS/Standard User's Manual, Ver. 6.3.

Ju, F., Lee, H.P. and Lee, K.H. (1995) "Finite Element Analysis of Free Vibration of Delaminated Composite Plates," Composites Engineering, Vol. 5, No. 2, pp. 195-209.

Lee, S.H., Park, T.H. and Voyiadjis, G.Z. (2002) "Free Vibration Analysis of Axially Compressed Laminated Composite Beam-Columns with Multiple Delaminations," Composites Part B: Engineering, Vol. 33, pp. 605-617.

Lee, S.H., Park, T.H. and Voyiadjis, G.Z. (2003) "Vibration Analysis of Multi-delaminated Beams," Composites Part B: Engineering, Vol. 34, No. 7, pp. 647-659.

Ma, S.O. (2004) "Finite Element Analysis for Free Vibration of Laminated Plates Containing Multi-Delamination," A Master's Thesis, Hanyang University.

Parhi, P.K., Bhattacharyya, S.K. and Sinha, P.K. (2000) "Finite Element Dynamic Analysis of Laminated Composite Plates with Multiple Delaminations," Journal of Reinforced Plastics and Composites, Vol. 19, No. 11, pp. 863-882.

Park, T.H., Lee, S.H. and Voyiadjis, G.Z. (2003) "Recurrent Single Delaminated Beam Model for Vibration Analysis of Multi-Delaminated Beams," ASCE Journal of Engineering Mechanics, Accepted for publication.

Pavier, M.J. and Clarke, M.P. (1995) "Experimental Techniques for the Investigation of the Effects of Impact Damage on Carbon-fibre Composites," Composites Science and Technology, Vol. 55, pp. 157-169.

Shen, M.H.H. and Grady, J.E. (1992) "Free Vibrations of Delaminated Beams," AIAA Journal, Vol. 30, No. 5, pp. 1361-1370.

Tenek, L.H., Henneke, E.G. II and Gunzburger, M.D. (1993) "Vibration of Delaminated Composite Plates and Some Applications to Non-Destructive Testing," Composite Structures, Vol. 23, pp. 253-262.

Tracy, J.J. and Pardoen, G.C. (1989) "Effect of Delamination on the Natural Frequencies of Composite Laminates," Journal of Composite Materials, Vol. 23, pp. 1200-1215.

Wang, J.T.S., Liu, Y.Y. and Gibby, J.A. (1982) "Vibrations of Split Beams," Journal of Sound and Vibration, Vol. 84, No. 4, pp. 491-502.