

A Study on a New Evaluation of Collision Risk and the Problems Involved

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ABSTRACT

Evaluating the risk of collision quantitatively plays a key role in developing the expert system of navigation and collision avoidance. This study suggested and developed a new approach to the evaluation by using the *sech* function as an alternative to the existing methods of appraising the collision risk. This study also investigated and built up theoretically how to determine the gradient coefficients in this approach and suggested the appropriate values as much as applicable. Finally this study analyzed thoroughly how to determine the threshold function of avoiding time and developed the appropriate equation.

1. Introduction

In general, collision risk should be evaluated in advance before avoiding action is taken to prevent collisions between ships at sea. Likewise, before avoiding action evaluating the risk quantitatively plays a key role in developing the expert system of navigation and collision avoidance. The collision risk of an approaching ship is predicted and determined directly by visual means or by radar/ARPA. Some ways of determining the risk contain the traditional method of CPA and TCPA (A.G. Bole et al. 1982), the method of PAD (predicted area of danger) (R.F. Riggs et al. 1979, A.G. Bole et al. 1992), the method by using the range of collision course and speed (H. Imazu 1978, 1979), the method of SOD (Sector of Danger) (T. Degre et al. 1981, W. Burger 1998), and the method of collision probability by estimated position error (H. Imazu, 1984).

These methods have difficulty being applicable to the expert system of navigation and collision avoidance (Jeong 2003 No.2). Therefore this study suggests a new approach to the evaluation by using *sech* function and deals with how to decide the gradient coefficients and the threshold function of avoidance time.

This study assumes that all the echoes in the radar screen are displayed normally and the movement of a target is analyzed by true motion plot regarding AIS(Automatic Identification System) adopted in many ships nowadays.

2. New Evaluation of Collision Risk by *sech* function

2.1 Factors of Collision Risk

2.1.1 CPA of a Target

A CPA or closest distance is one of important factors depicting collision risk together with TCPA. Less CPA is thought to be higher risk of collision. CPA of zero means exact collision and big CPA drops the risk. CPA is depicted by Eq. (1)

$$(1) \quad d_{cpa} = R \sin \zeta$$

where d_{cpa} is a CPA(or closest distance) and R is a distance between own ship and a target. ζ is the absolute value of the difference of a target's bearing (θ) added to 180° and the relative course(C_r) of a target, which are the following.

$$\zeta = |C_r - (\theta + 180)|$$
$$0 \leq \zeta \leq 180$$

2.1.2 Approach Time of a Target

The approach time of a target is used here to overcome the limit of TCPA, It is shown by Eq. (2) and represented by the distance to a target divided by the directional cosine of the relative speed

$$(2) \quad t_a = \frac{R}{v_r \cos \zeta}$$

Where t_a is the approach time of a target and v_r is the relative speed. This approach time is used to understand the marginal time of own ship with regard to the approach of a target.

2.1.3 Own Ship State Function

Own ship state function is to determine whether she is stand-on or give-way. This function consists of the variables of the bearing and aspect¹ of a target. This function is used to indicate the collision risk differently as to own ship's state

2.2 Representation of Collision Risk

The collision risk CR in this paper is suggested and represented by Eq. (3),

$$(3) \quad CR = p \sec h(a \cdot dcpa) + q \sec h(b \cdot t_a) + r \Phi(\theta, \alpha).$$

Coefficients p, q, r, a and b are to determine the change of collision risk appropriately. Coefficients p, q and r are to determine the amplitude and are called the amplitude coefficient. Coefficient a and b are to determine the change of $\sec h$ function and are called the gradient coefficient. $\Phi(\theta, \alpha)$ is own ship state function of determining whether own ship is in situation of keeping her own way or of keeping out of way and is assigned to zero in case own ship is in a stand-on situation and to unity in case she is in a give-way situation.

3. Determination of Gradient Coefficients

Eq. (3) has the amplitude and gradient coefficients. Here the way of determining the gradient coefficients a and b is investigated and the values, which are as appropriate as applicable, are suggested (Jeong 2003, No.4). It is assumed that the amplitude coefficients p and q are unity, respectively, and r is zero.

3.1 How to Determine Gradient Coefficients a, b

3.1.1 Approach Time

As ζ in Eq. (2) becomes 90° , the approach time t_a increases to infinity. So Eq (2) is depicted again by Eq. (4).

$$(4) \quad \begin{aligned} t_a &= \frac{2 \cdot dcpa}{v_r \sin 2\zeta} & \text{if } \zeta \neq 0(^\circ) \text{ or } \zeta \neq 180(^\circ) \\ &= \pm \frac{R}{v_r} & \text{if } \zeta = 0(^\circ) \text{ or } \zeta = 180(^\circ) \end{aligned}$$

Eq. (4) shows that t_a is a minimum or maximum if ζ becomes 45° or 135° respectively. In this case the approach time t_a is given by Eq. (5).

$$(5) \quad t_a = \pm \frac{2 \cdot dcpa}{v_r}$$

The problem that t_a reaches $\pm \infty$ when ζ goes near 90° can be solved by using Eq (6). This paper uses Eq. (6) as the approach time.

$$(6) \quad \begin{aligned} t_a &= \frac{2 \cdot dcpa}{v_r} & \text{if } 45^\circ \leq \zeta \leq 90^\circ \\ &= -\frac{2 \cdot dcpa}{v_r} & \text{if } 90^\circ < \zeta < 125^\circ \\ &= \frac{R}{v_r} \cos \zeta & \text{otherwise} \end{aligned}$$

3.1.2 Condition of Determining Gradient Coefficients a and b

It should be made such that the collision risk of avoidance time is greater than the maximum value of the risk which increases as a target approaches after avoiding action. The difference between the collision risks ψ is represented by Eq. (7),

¹) The aspect is defined as the relative bearing of own vessel taken from the target. A starboard or port bearing is indicated as Green or Red respectively. For example an aspect of Red 90 means that the target's portside is observed to be beam-on to own ship; a target head-on has zero aspect, stern-on 180° aspect.

$$\begin{aligned}
\Psi &= \{\sec h(a \cdot dcpa_1) + \sec h(b \cdot t_{a1})\} - \{\sec h(a \cdot dcpa_2) + \sec h(b \cdot t_{a2})\} \\
&= \{\sec h(a \cdot dcpa_1) - \sec h(a \cdot dcpa_2)\} - \{\sec h(b \cdot t_{a1}) - \sec h(b \cdot t_{a2})\} \\
(7) \quad &= \Psi_a + \Psi_b > 0 \\
\Psi_a &= \sec h(a \cdot dcpa_1) - \sec h(a \cdot dcpa_2) \\
\Psi_b &= \sec h(b \cdot t_{a1}) - \sec h(b \cdot t_{a2})
\end{aligned}$$

where $dcpa_1$ and t_{a1} are the closest distance and the approach time of avoidance time respectively and also, $dcpa_2$ and t_{a2} are the closest distance and the approach time, respectively, in case the approach time is a minimum value, that is, the collision risk reaches a maximum one.

To make $\Psi > 0$, $\Psi_a + \Psi_b$ should be greater than 0. However $dcpa_2$ is always greater than $dcpa_1$ after avoiding action. Therefore Ψ_a is always greater than zero. If $dcpa_1$ and $dcpa_2$ are given, Ψ_a will be the function of variable a .

The necessary condition that Ψ has a minimum or maximum value is that its derivatives with respect to variables a and b are zero simultaneously (C.H. Edwards et al 2002, G. James 2001).

Let's obtain the gradient coefficient a . The partial differentiation of Ψ with respect to the coefficient a is given by Eq. (8).

$$\begin{aligned}
(8) \quad \frac{\partial \Psi}{\partial a} &= \frac{\partial \Psi_a}{\partial a} = -dcpa_1 \sec h(a \cdot dcpa_1) \tanh(a \cdot dcpa_1) \\
&\quad + dcpa_2 \sec h(a \cdot dcpa_2) \tanh(a \cdot dcpa_2) = 0
\end{aligned}$$

In Eq (8) the gradient coefficient a can be obtained by using a graph. Because the closest distance $dcpa_1$ is greater than $dcpa_2$, Ψ_a has a maximum value at the a value which meets Eq. (8).

Meanwhile the gradient coefficient b is calculated as follows. Let $a=\xi$ be a maximum value. Eq. (7) can be rearranged by Eq. (9)

$$\begin{aligned}
(9) \quad \Psi &= \Psi_a(\xi) + \sec h(b \cdot t_{a1}) - \sec h(b \cdot t_{a2}) > 0 \\
&\text{where, } \Psi_{a \ a=\xi} = \Psi_a(\xi)
\end{aligned}$$

As shown in Eq. (9) the gradient coefficient b is determined as a range, not only one real value.

3.1.3 Range of Gradient Coefficient b and Approach Time t_{a1}

The range of the gradient coefficient b is as follows. From Eq. (9) we can get

$$(10) \quad \sec h(b \cdot t_{a1}) > \sec h(b \cdot t_{a2}) - \Psi_a(\xi).$$

Because the left side of the above inequality is always greater than zero and less than 1 and also greater than the right side, the right side is shown as the following inequality.

$$\begin{aligned}
\Psi_a(\xi) &< \sec h(b \cdot t_{a2}) < 1 \\
a \sec h[\Psi_a(\xi)] &> b \cdot t_{a2} > a \sec h(1) (= 0)
\end{aligned}$$

Therefore the range of the gradient coefficient b is given by Eq. (10).

$$(11) \quad 0 < b < \frac{a \sec h[\Psi_a(\xi)]}{t_{a2}}$$

Meanwhile from Eq. (10) the approach time t_{a1} has the range as shown in Eq. (12).

$$\begin{aligned}
(12) \quad b \cdot t_{a1} &< a \sec h[\sec h(b \cdot t_{a2}) - \Psi_a(\xi)] \\
t_{a1} &< \frac{a \sec h[\sec h(b \cdot t_{a2}) - \Psi_a(\xi)]}{b}
\end{aligned}$$

It assumed that the approach time t_{a1} is greater than t_{a2} . Using Eq. (12) the approach time t_{a1} is given by Eq. (13).

$$(13) \quad t_{a2} < t_{a1} < \frac{a \sec h[\sec h(b \cdot t_{a2}) - \Psi_a(\xi)]}{b}$$

It is noted that Eq. (13) is represented only by a given b value.

3.2 Determination of Gradient Coefficients a, b

3.2.1 Closest Distances

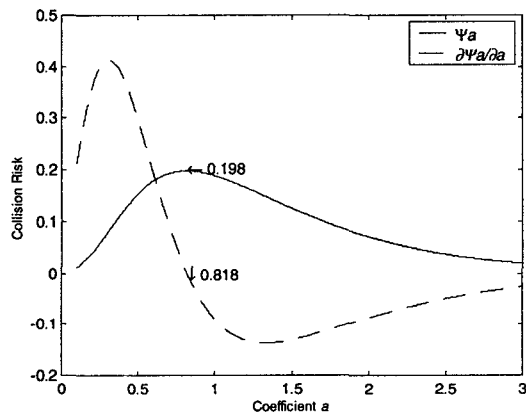
The first closest distance $dcpa_1$, at which avoiding action is taken, depends on the situation at that time. Here the closest distance of 1.5 mile is considered because it is thought to be comparatively safe. It is the upper limit. When a closest distance is less than this upper limit, Ψ from Eq. (7) is naturally greater than zero. The second closest distance $dcpa_2$ is $2.12(1.5\sqrt{2})$ miles, which is generally accepted to be safe.

3.2.2 Approach time t_{a2}

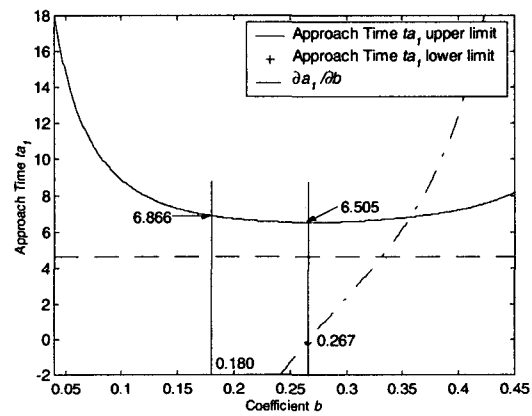
The minimum approach time after avoiding action is determined by Eq. (5). It depends on the relative speed of a target, once the second closest distance is given to 2.12 miles. It is considered that the maximum relative speed is 1.0 mile per minute while the minimum one is 0.1 mile per minute. If the maximum speed meets Eq. (9), the minimum one does so. Therefore if the maximum speed is only considered, the approach time t_{a2} will be 4.24 minutes.

3.2.3 Gradient Coefficient a, b and Approach Time t_{a1}

The gradient coefficient a is obtained by using the graphs Ψ_a and $\partial\Psi_a/\partial a$ on the ground that $dcpa_1$ and $dcpa_2$ are 1.5 mile and 2.12 miles respectively. As shown in <Fig. 1>, Ψ_a has a maximum value of 0.198 at the coefficient $a = 0.818$.



<Fig. 1> Ψ_a and $\partial\Psi_a/\partial a$ with respect to $dcpa_1$ of 1.5 mile and $dcpa_2$ of 2.12 miles



<Fig. 2> Approach Time t_{a1} and t_{a2}

The gradient coefficient b is calculated by Eq. (11). Under the condition of the above closest distances, $\Psi_a(\xi)$ equals 0.918 and $\text{asech}[\Psi_a(\xi)]$ is 2.3034. And because t_{a2} is 4.24 minutes, the range of the coefficient b will be that $0 < b < 0.54$.

The approach time t_{a1} is obtained as follows. First the approach time t_{a1} with respect to the coefficient b is drawn by using Eq. (12) and the range of approach time is obtained by Eq. (13). When the coefficient b is 0.267 as represented in <Fig. 2>, $\partial\Psi_a/\partial a$ becomes zero and at that time the approach time t_{a1} has a minimum of 6.505 minutes. If the approach time t_{a1} has the range of 4.24 and 6.505 minutes, Ψ is always greater than zero. And if the approach time t_{a1} has the range of 4.24 and 6.866 minutes under the condition of the coefficient $b=0.180$, Ψ is also greater than zero. Therefore if the approach time t_{a1} is a minimum of 6.505 and it meets that Ψ is greater than zero, other values of approach time will meet the condition.

4. Method of Obtaining Threshold Function of Avoidance Time

There are two types of threshold in this new evaluation of collision risk. One is to determine when the avoiding action has to be taken if the risk of collision exists. It is called the threshold of avoidance time. The other is to determine which sector will be safe for own ship, which is obtained by the range of own ship's choice, that is, alteration of own ship's course and/or speed. It is called the threshold of avoidance sector.

This paper first deals with how to determine the threshold of avoidance time (Jeong 2003, No.6). The approach time of Eq. (2) is can be rewritten by using closest distance as Eq. (14),

$$(14) \quad t_a = \frac{R^2}{v_r \sqrt{R^2 - dcpa^2}}$$

where R is equal to or greater than $\sqrt{2} \cdot dcpa$.

Substituting Eq. (14) into Eq. (3) yields Eq. (15).

$$(15) \quad CR = \sec h(a \cdot dcpa) + \sec h\left(b \cdot \frac{R^2}{v_r \sqrt{R^2 - dcpa^2}}\right)$$

4.1 Things to be Considered When Determining Threshold of Avoidance Time

As shown in Eq. (15), the collision risk CR is expressed by three variables of $dcpa$, R and v_r . Representing the threshold of collision risk as the above three variables is not suitable because the distance R is related to avoidance time (or the time of avoiding action). Therefore in this paper the threshold will be expressed by two variables of the relative speed and closest distance of a target.

The followings are assumed or taken into consideration to determine the threshold of avoidance time.

- (a) The gradient and amplitude coefficients in the new evaluation of collision risk depend on danger zone (or safe minimum distance), closest distance and approach time. When we assume here that the closest distance ranges from 0.09 to 1.5 mile, that the approach time ranges between 4.24 minutes and 6.9 minutes, and that the danger zone is $2.12(=1.5\sqrt{2})$ miles, we can use the gradient coefficients $a=0.818$ and $b=0.180$ as obtained before.
- (b) The maximum threshold of the new evaluation of collision risk can be obtained on the assumption that the closest distance of a target is 0 mile and the approach time is 11 minutes. If a target approaches own ship at a relative speed of 1.0 mile per minute, ie 60 knots, own ship can afford to take avoiding action 11 minutes or 11 miles off before the collision takes place. The maximum threshold, CR_{thmx} , is obtained as below.

$$(16) \quad \begin{aligned} CR_{thmx} &= \sec h(0 \cdot dcpa) + \sec h(b \cdot t_a) \\ &= \sec h(0) + \sec h(0.180 \cdot 11) = 1.271 \end{aligned}$$

- (c) Let the range of relative speed be from 0.1 to 1.0 mile per minute. Because the relative speed governs the time elapsed after avoiding action is taken and the collision risk, the size of danger zone should be governed by the relative speed. As shown in <Table 1>, we here consider that for the target with her relative speed of 0.39 mile per minute and upwards, avoiding action has to be taken to pass outside the danger zone of 2.12 miles, and that for the target with her relative speed of 0.28 mile per minute and more, but less than 0.39 mile per minute, avoiding action has to be taken to pass outside the danger zone of 1.59 miles. We can also assume that for the target with her relative speed of 0.17 mile per minute and more, but less than 0.28 mile per minute, avoiding action has to be taken to pass outside the danger zone of 1.06 miles, and that for the target with her relative speed of less than 0.17 mile per minute, avoiding action has to be taken to pass outside the danger zone of 0.71 mile.

<Table 1> Relative Speed and Danger Zone

Relative Speed (mile/min)	$vr \geq 0.39$	$0.39 > vr \geq 0.28$	$0.28 > vr \geq 0.17$	$vr < 0.17$
Danger Zone (mile)	2.12	1.59	1.06	0.71

- (d) As mentioned above in order to have a target pass outside the danger zone the minimum approach range is required, which is obtained as follows. First the distance at which avoiding action has to be taken when the closest distance is 0 mile, is determined. Also, the distance when the closest distance equals the danger zone, is determined. Next they are curve-fitted respectively by the closest distance $dcpa$. Meanwhile the range at which avoiding action has to be taken if $dcpa$ equals zero, R_i and the range if $dcpa$ equals danger zone, R_f are given respectively by

$$R_i = \frac{dz}{\sin \eta}, \quad R_f = \sqrt{2} \cdot dz$$

where dz denotes the danger zone. η is the difference between the relative courses before and after avoiding action when $dcpa$ equals zero. It is considered as 25° here. The minimum approach range, R_{mn} is expressed as below.

If $v_r \geq 0.39$

$$(17a) \quad R_{mn} = 0.0078 dcpa^4 + 0.0312 dcpa^3 + 0.0925 dcpa^2 - 1.3632 dcpa + 5.0192 .$$

If $0.39 > v_r \geq 0.28$

$$(17b) \quad R_{mn} = 0.0188 dcpa^4 + 0.0548 dcpa^3 + 0.1240 dcpa^2 - 1.3634 dcpa + 3.7644 .$$

If $0.28 > v_r \geq 0.17$

$$(17c) \quad R_{mn} = 0.0627 dcpa^4 + 0.1249 dcpa^3 + 0.1850 dcpa^2 - 1.3632 dcpa + 2.5096 .$$

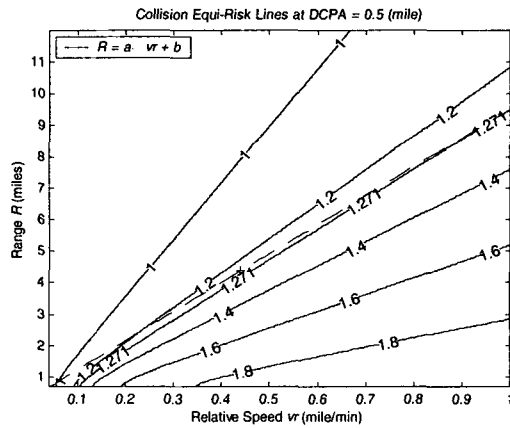
If $v_r < 0.17$

$$(17d) \quad R_{mn} = 0.2157 dcpa^4 + 0.2760 dcpa^3 + 0.2796 dcpa^2 - 1.3635 dcpa + 1.6731 .$$

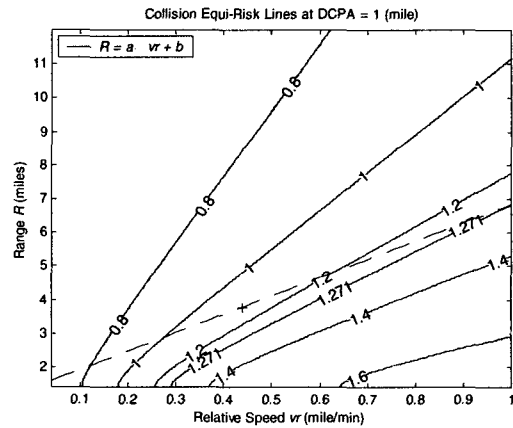
(e) In general, the threshold of avoidance time cannot be represented as only one value. It takes a very long time for a target and own ship to keep clear each other in case the relative speed is very slow. Therefore the threshold should be used together with the minimum approach range of Eq. (16).

4.2 Method of Determining Threshold of Avoidance Time

The threshold of avoidance time cannot be expressed by only one value as mentioned above. Even so, expressing all kinds of cases as thresholds one by one is highly complicated. Therefore the threshold of avoidance time is here represented as two variables of relative speed and closest distance.



<Fig. 3> Collision Equi-Risk Lines at DCPA=0.5(mile)



<Fig. 4> Collision Equi-Risk Lines at DCPA=1.0(mile)

From the collision risk of Eq. (15) we can get the collision equi-risk lines represented by the relative speed and the range of a target if the closest distance is given. <Fig. 3> and <Fig. 4> show the examples of collision equi-risk lines in case $dcpa = 0.5$ and $dcpa = 1.0$ respectively. In one of these figures (that is, at a given closest distance) we can draw an appropriate straight line and obtain the intersections between this straight line and the collision equi-risk lines. From the intersections we can get the collision risk and the relative speed and then the collision risk can be curve-fitted by each relative speed. Next because each coefficient in the polynomial of relative speed can be expressed by the closest distance, it is also curve-fitted by the closest distance.

The above-mentioned straight line can be obtained as follows. Using the collision equi-risk lines of the maximum threshold of Eq. (16), we can read the range corresponding to the maximum threshold at a relative speed of 1.0 mile per minute and then can obtain a position expressed by relative speed and range. Next using Eq. (17a) we can get the minimum approach range of about 5.02 miles. Dividing it by the approach time of 11 minutes, we can obtain the relative speed of 0.456 mile per minute. However this relative speed is critical. That

is, when the minimum approach range is applied at a bigger relative speed than this, and if the relative speed becomes smaller the threshold is apt to increase. Meanwhile when the minimum approach range is applied at a smaller relative speed than this, and if the relative speed becomes smaller the threshold is apt to decrease. Because the latter is thought to be reasonable, the relative speed of 0.44 mile per minute is used here. So we get another position given by the relative speed of 0.44 mile per minute and the minimum approach range of Eq. (17a). Therefore the straight line that we want can be obtained by connecting the above two positions. Using the method as mentioned earlier we can obtain the threshold function of avoidance time, CR_{th} as follows.

$$(18) \quad CR_{th} = \lambda_1 v_r^2 + \lambda_2 v_r + \lambda_3$$

$$\lambda_1 = -0.1187 dcpa^4 + 0.5460 dcpa^3 - 1.0934 dcpa^2 - 0.3857 dcpa - 0.0466$$

$$\lambda_2 = 0.2021 dcpa^4 - 1.0164 dcpa^3 + 2.2483 dcpa^2 - 0.8134 dcpa + 0.0977$$

$$\lambda_3 = -0.0844 dcpa^4 + 0.4759 dcpa^3 - 1.1653 dcpa^2 + 0.4312 dcpa + 1.2175$$

Eq. (18) represents a function of obtaining the threshold of avoidance time in case a target approaches at a relative speed between 0.1 and 1.0 mile per minute and at a closest distance between 0 and 1.5 mile. However if the relative speed is small, the range determined by the threshold function becomes smaller than the minimum approach range given by Eqs. (17a), (17b), (17c) and (18d). Therefore they should be used in addition.

4.3 Application of Threshold Function of Avoidance Time to Actual Avoiding Action

The application of the threshold function of avoidance time to actual avoiding action is as follows. As examples, when each target approaches within the danger zone of 2.12 miles and 1.06 mile respectively and avoiding action is taken to pass outside each danger zone, it would be examined whether the avoidance time is determined by the threshold function of Eq. (18) or the minimum approach range of Eqs. (17a) and (17c) according to the relative speeds.

<Table 2> Data of Ownship and Target

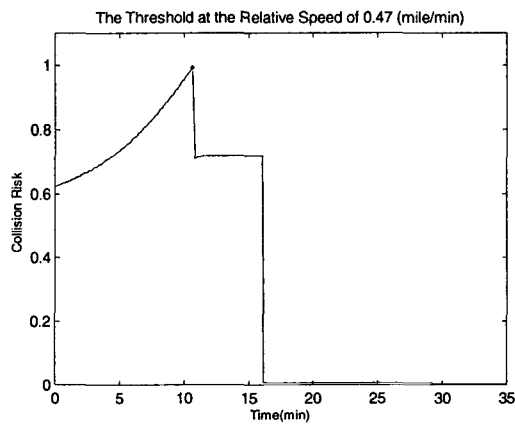
Ownship	Course(°)	000	000
	Speed(mile/min)	0.4	0.4
Tatget	Course(°)	180	000
	Speed(mile/min)	0.07	0.21
Initial Position	Bearing(°)	349.4	353.3
	Range(mile)	8.1	6.0
DCPA	before Action	1.5	0.7
	after Action	2.12	1.06
Collision Risk	Threshold	0.989	1.149
	Action	0.993	1.152
Range	Minimum	3.327	1.704
	Action	3.328	1.844
Relative Speed(mile/min)		$v_r=0.47$	$v_r=0.19$
Time of Action		Threshold	Threshold

Because the threshold function is governed only by the relative speed and closest distance of a target, there is not any difference in the relevant rules of Part B, Section II (conduct of vessels in sight of one another) of the Collision Regulations. Therefore we here assume that both own ship and a target meet head-on each other or own ship overtakes her and avoiding action is to make an alteration to starboard.

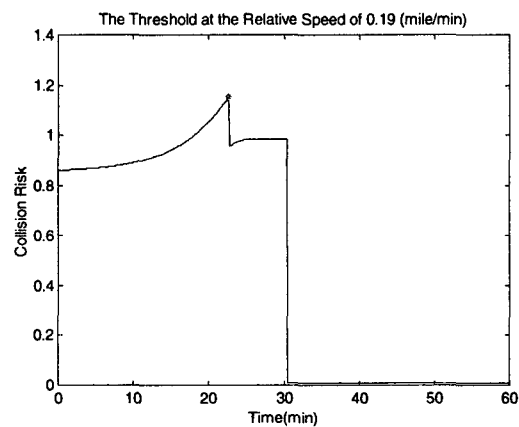
In <Table 2> own ship is steering a course of 000° at a speed of 0.4 mile per minute. First of all, let's consider that a target, which is at a distance of 8.1 miles, bearing 349.4°, approaches within a closest distance of 1.5 mile

at a relative speed of 0.39 and more mile per minute. The range corresponding to the avoidance time, at which avoiding action has to be taken so as to make the target pass outside a danger zone of 2.12 miles, is 3.328 miles. It is bigger than the minimum approach range of 3.327 miles. The collision risk at the avoidance time is 0.993, which is bigger than 0.989 generated by the threshold function. At that time the relative speed becomes 0.47 mile per minute. If the relative speed is equal to or bigger than this, avoiding action is taken by the threshold and otherwise avoiding action taken by the minimum approach range. <Fig. 5> shows the result of action taken to avoid the target approaching at a relative speed of 0.47 mile per minute. The collision risk gradually increases as time goes by. When it reaches the threshold of 0.989 avoiding action is taken. As a result the collision risk rapidly drops.

Likewise we consider that a target which is at a distance of 6.0 miles, bearing 353.3° , at a relative speed of 0.19 mile per minute and more, but less than 0.28 mile per minute within a closest distance of 0.7 mile. The distance corresponding to the avoidance time, at which avoiding action has to be taken so as to make the targets pass outside the danger zones of 1.06 mile, is 1.844 mile. It is bigger than the minimum approach range of 1.704 mile. The collision risk at the avoidance time is 1.152, which is bigger than 1.149 generated by the threshold function. At that time the relative speeds become 0.19 mile per minute. <Fig. 6> shows the result of the action taken to avoid the targets approaching at a relative speed of 0.19 mile per minute. The collision risk gradually increases as time goes by. When it reaches the thresholds of 1.149 avoiding action is taken. As a result the collision risk rapidly drop.



<Fig. 5> Action to Avoid a Target at a Relative Speed of 0.47(mile/min), taken by Collision Risk Threshold



<Fig. 6> Action to Avoid a Target at a Relative Speed of 0.19(mile/min), taken by Collision Risk Threshold

As mentioned above when the targets approach within danger zones of 2.12 miles and 1.06 mile, if their relative speeds equal to or greater than 0.47 and 0.19 mile per minute respectively each avoidance time is given by the threshold function and otherwise it is determined by each minimum approach range.

4. Conclusion

In this paper the new evaluation of collision risk using *sech* function was introduced to be used in the expert system of navigation and collision avoidance. The determination of the gradient coefficients a and b was suggested. Finally as the first stage of determining the thresholds, the method of determining the threshold function of avoidance time was analyzed and was applied to actual avoiding action in the head-on and overtaking situations. As a result, it was concluded as follows.

- (a) The new evaluation of collision risk can be represented as three variables of approach time, closest distance and relative speed so as to investigate conveniently the threshold function of avoidance time.
- (b) Using the gradient coefficients $a=0.818$ and $b=0.180$, we can obtain the threshold function of avoidance time, ie. Eq. (18). If the relative speed and closest distance are given, the threshold is determined at once. In case the relative speed is large enough at each danger zone the range corresponding to the avoidance time, which is thought to be safe, can be obtained by the threshold function only.
- (c) If the relative speed is small, avoiding action would be taken at a distance less than the minimum approach range. Therefore the minimum approach range of Eq. (17) should be used in addition.

However, the maximum threshold of Eq. (16) and the minimum approach range of Eq. (17) should be investigated again through various experiments aboard ships. The threshold of avoidance sector should be investigated. Finally, when the amplitude coefficients p and q and the function of own ship's state $\Phi(\theta, \alpha)$ are applied to the collision risk, the thresholds should be corrected. All of these will be dealt with in the future study.

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