# A study on the column subtraction method applied to ship scheduling problem

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**ABSTRACT** : *Column subtraction, originally proposed by Harche and Thompson(1994), is an exact method for solving large set covering, packing and partitioning problems. Since the constraint set of ship scheduling problem(SSP) have a special structure, most instances of SSP can be solved by LP relaxation. This paper aims at applying the column subtraction method to solve SSP which can not be solved by LP relaxation. For remained instances of unsolvable ones, we subtract columns from the finale simplex table to get another integer solution in an iterative manner. Computational results having up to 10,000 0-1 variables show better performance of the column subtraction method solving the remained instances of SSP than complex branch-and-bound algorithm by LINDO.*

**KEY WORDS** : *Column Subtraction, Ship Scheduling Problem, Set Packing, Set Partitioning, Set Covering, Branch-and-Bound*

## 1. Introduction

Set packing, set partitioning and set covering problem are often called set problems. They are alike in 0-1 integer formulation and have wide applications.

Let $A = (a_{ij})$ be $m \times n$ 0-1 matrix, $\boldsymbol{e}$ a column vector of $m$ ones, and $\boldsymbol{c}$ an $1 \times n$ row vector of positive integer weights. Set problems are defined as

Set Covering problem

Min { $cx$ | $Ax \geq e$, $x \in [0, 1]$ } ------- 1)

Set Packing problem

Max { $cx$ | $Ax \leq e$, $x \in [0, 1]$ } ------- 2)

Set ParTitioning problem

Min { $cx$ | $Ax = e$, $x \in [0, 1]$ } ------- 3)

By way of interpretation, the set packing (covering, partitioning) problem requires the selection of a maximum (minimum) weight subset W of columns of A, where each column j has weight $c_j$, with the property that each row of A is covered by at most (at least, exactly) one element of W. And we can solve them by using LP relaxation, Lagrangean relaxation, Network relaxation, Genetic algorithm.

The importance of these problems is reflected by the wide range of their common applications which include crew scheduling, truck deliveries, tanker routing, ship scheduling, information retrieval, switching circuit design, stock cutting, assembly line balancing, capital equipment decision, location of emergency units, political districting, and so on.

Among those applications, tanker routing and ship scheduling problem are closely related to the marine transport service and play an important role in its competitiveness.

When a ship costs thousands of dollars per day, significant savings can be achieved by proper fleet routing and scheduling.

In contrast to vehicle scheduling, VRP(vehicle routing problem) or VSP(vehicle scheduling problem), relatively little work has been done in ship routing and scheduling.

Ronen(1993) said that several explanations follow for the low attention drawn by ship scheduling problem, i.e. low visibility, less structured than standard vehicle scheduling problems, much more uncertainty, shipping market which is volatile, international, capital intensive and relatively free.

Some decision-making supporting system, in which those qualities of marine economy are reflected and by which scientific and logical decision-making could be done, have been made.

They have made all efforts to make optimization model for ship scheduling and construct the system, without verifying the efficiency of each algorithm that we will discuss later.

That is the reason that we applied column subtraction algorithm for ship scheduling optimization model one of set problems and programmed decision-making system that can test the efficiency of column subtraction algorithm versus branch-and-bound algorithm. We presented computational experience based on real data from randomly generated numbers.

## 2. Ship Scheduling Problem

Generally speaking, in optimization models we are dealing with two types of problems, i.e. easy problems, that is problems for which a polynomially bounded algorithm exists and hard problems, no polynomially bounded algorithm has yet been found. Hard problems can be partitioned into two sets, NP-complete problems and others. The set of NP-complete problems have the property that they are equivalent in the sense that if a polynomially bounded algorithm is ever found for just one of the problems in the set, then polynomially bounded agorithms exist for all the problems in the set.

Most of IPs are hard problems and we dealt with another one, set problems. We defined well Set Packing, Set Partitioning and Set Covering problems, so called Set Problems above. A lot of algorithm has appeared to solve set problems, e.g. implicit enumeration, LP relaxation, Lagrangean relaxation, Network relaxation, Genetic algorithm.

Kim(1999) asserted that LP relaxation algorithms are the most attractive way to find solution for set problems by which ship scheduling problems have been represented.

Cutting plane algorithm, branch-and-bound or tree search algorithm and column subtraction algorithm can be categorized in LP relaxation.

In this chapter we deal with not only the development of ship scheduling problem on which focused both tramp operation and industrial operation, but also column subtraction algorithm.

### 2.1 Previous Literatures

Dantzig and Fulkerson(1954) showed that the problem of determining the minimum number of tankers required to meet a fixed schedule of transportation of Navy fuel oil can be made into a linear programming problem of transportation type and applied simplex algorithm to solve it.

Laderman et al(1966) set a linear programming formulation of the transport of various cargoes between ports on Great Lakes is given.

Whiton(1967) added limitations on the tonnage handling and port capacities of both origin and destination ports in Laderman's formulation, and Laderman rounded off the solution, if not integer.

Bellmore(1968) developed a utility method to analyze the problem of optimizing delivery schedules with a limited number of delivery vehicles. Bellmore's reads, "a utility is associated with each vehicle delivery and the solution is found in a feasible delivery schedule that maximize the total utility of deliveries. The method uses a directed linear graph, each chain representing a feasible vehicle schedule. A set of costs is found by applying a longest chain algorithm".

Appelgren(1969, 1971) formulated ship scheduling problem, as set problem, either Set Partitioning or Set Packing problem, obtained from Swedish shipping company.

***Appelgren(1969, 1971)'s model***

**[data]**

*if cargo k is in the jth sequens for ship i.*
*otherwise.*

$v_{ij}=$ *value of the jth sequence for ship i.*

**[decision variable]**

$$x_{ij}=\begin{cases}1, & \text{if the jth sequence for ship } i \text{ is selected.}\\ 0, & \text{otherwise}\end{cases}$$

**[formulation]**

$$Max \sum_i \sum_{j\in N(i)} v_{ij}x_{ij}$$

*s. t.*

$$\sum_{j\in N(i)} x_{ij} = 1, \text{ for each ship } i.$$

$$\sum_i \sum_{j\in N(i)} a_{ijk}x_{ij} \le 1 \left( or \sum_i \sum_{j\in N(i)} a_{ijk}x_{ij} = 1 \right),$$

*for each cargo j.*

$$x_{ij} = \{0,\ 1\}, \text{ for each feasible sequence.}$$

Nowadays, Appelgren's formulation is adopted as a typical form in most of ship scheduling problem.

McKay and Hartley(1974) generated a set of acceptable routes that meet various time and capacity limitations for each tanker, and modeled a mixed integer programming problem to determine the cargoes for each proposed route.

Brown et al(1987) presented a crude oil tanker scheduling problem and solved it using an elastic set partitioning model.

Fisher and Rosenwein(1989) chose an optimal schedule for the fleet using a set packing problem and solved with a dual algorithm. All of formulations above are similar to that of Appelgran's.

Kim(1999) proposed generalized ship scheduling formulation used in tramp operation, which maximizes the profit that revenue of lifting cargos minus operating cost of each ship $i$ of fleet scheduled.

***Kim(1999)'s Model***

**[data]**

$$q_{ijk} = \begin{cases} 1, & \text{if ship } i \text{ on schedule } j \text{ lifts cargo } k. \\ 0, & \text{otherwise.} \end{cases}$$

$p_k =$ *revenue from lifting cargo k.*

$h_{ij} =$ *operating cost weight of ship i on schedule j.*

$J_i =$ *set of feasible schedules generated for ship i.*

**[decision variable]**

$$x_{ij} = \begin{cases} 1, & \text{if ship } i \text{ uses schedule } j. \\ 0, & \text{otherwise.} \end{cases}$$

**[formulation]**

$$Max \sum_i \sum_{j \in J_i} \left( \sum_k q_{ijk} P_k \right) x_{ij} - \sum_i \sum_{j \in J_i} h_{ij} x_{ij}$$

s. t.

$$\sum_{j \in J_i} x_{ij} \le 1 \quad , \text{ for all ship } i.$$

$$\sum_j \sum_{j \in J_i} q_{ijk} x_{ij} \le 1 \quad , \text{ for all cargo } k.$$

$$x_{ij} = \{0, 1\}, \; j \in J_i \text{ , for all ship } i.$$

We use Kim's model in this study.

### 2.2 Column Subtraction Algorithm

Column subtraction algorithm can be applied to SSP in case LP relaxation could not generate an integer solution. The brief description of column subtraction algorithm is as follows.

Let the marix $T(i, j)$ be a final simplex table of LP relaxed problem. The matrix $T(i, j)$ has $m+1$ rows and $n+1$ columns and $n+1$ columns have values of base variables and $m+1$ rows have values of reduced costs. $T(m+1, n+1)$ are objective values. Let $Z_{UB}$ be an integer which has a value less than one of $T(m+1, n+1)$'s. To find lower bound, let the solution of heuristic method be $Z_{LB}$. If there is no heuristic solution, let $Z_{LB} = -\infty$. To record nodes, We use a matrix $E$, which has $(m+1) \times n$, $E(i, j)$. and we use LIFO(last in, first out) or Depth First Search algorithm. The whole algorithm is shown in setp by step as follows.

**[Step 0]** (Heuristic) Let a feasible solution by heuristic method be $Z_{LB}$. If $Z_{UB} - Z_{LB} = 0$ then go to step 6. If there is no heuristic solution, Let $Z_{LB} = -\infty$. Eliminate columns those have greater value of reduced costs than the value of "$Z_{UB} - Z_{LB}$", among columns of $T$

**[*Step 1*]** (Initialization) For $i = 1, \cdots, m+1$, let $E(i, j) = T(i, n+1)$. Let $scol = 1$, $index = 1$, $acol = point\ 1$) and go to step 2.

**[*Step 2*]** (Forward Search) Let $scol = scol + 1$. For $i = 1, \cdots, m+1$, Let $E(i, scol) = E(i, scol-1) - T(i, scol)$. Let $save\_index(scol) = index$, $index = index + 1$ then go to step 3.

**[*Step 3*]** (Examine integer solution) For $i = 1, \cdots, m+1$, if $E(i, j)$ is non negative integer, revise optimized solution $Z_{LB}$ with these values, then go to step 5, otherwise, go to step 4.

**[*Step 4*]** (Revision) If $index > n$, then go to step 5. Let $acol = point(index)$. If $E(m+1, scol) - T(m+1, scol) \le Z_{LB}$, then go to step 5. Otherwise, go to step 2.

**[*Step 5*]** (Backward Search) Let $index = save\_index(scol) + 1$, $scol = scol - 1$. If $scol > 0$, then goto step 4. Otherwaise, go to step 6.

**[*Step 6*]** (Solution) Output solution

## 3. Computational Experiment

### 3.1 Methodology

Ship and cargo data, distance table can be managed by GUI(Graphic User Interface) such as spreadsheet. And some values can be changed by random number generator. SSP is modeled with these data then solved by LP relaxed method using LINDO. In case that the results of LP relaxation has not integer solution, the column subtraction method is the very thing to solve SSP. To get a final simplex table, we change LP relaxation problem to an integer problem and solve it again. To verify the validity of solution method, we compared computational time with branch-and-bound method. We use data of 10 to 30 ships and 10 to 50 cargoes to test.

## 3.2 Computational Results

The computational results shows that the ratio of fractional solutions is 1~3% while Appelgren's[1,2] one is 1~2%. The density of integer in SSP is 6.6% in average, max 43%, min 13.1% and the maximum number of variables is upto 15,401.

Table 1. shows calculation time of branch-and-bound and column subtraction method.

Table 1. Computational Time by solution methods

| Ship× Cargo | Brach-and-Bound(sec) | | | Column Subtraction(초) | | |
|---|---|---|---|---|---|---|
| | min | max | ave | min | max | ave |
| 10x20 | 0.210 | 1.650 | 0.925 | 0.001 | 0.280 | 0.076 |
| 10x30 | 0.940 | 2.030 | 1.429 | 0.050 | 0.380 | 0.187 |
| 10x40 | 1.370 | 4.230 | 2.861 | 0.110 | 0.880 | 0.405 |
| 10x50 | 3.020 | 6.430 | 5.061 | 0.380 | 3.570 | 1.176 |
| 20x10 | 0.440 | 0.440 | 0.440 | 0.050 | 0.050 | 0.050 |
| 20x30 | 2.250 | 11.870 | 5.932 | 0.270 | 2.360 | 0.901 |
| 20x40 | 5.880 | 27.910 | 11.675 | 0.770 | 20.210 | 4.978 |
| 20x50 | 9.280 | 160.870 | 28.461 | 1.260 | 25.320 | 6.600 |
| 30x20 | 1.320 | 1.750 | 1.535 | 0.170 | 0.220 | 0.195 |
| 30x30 | 1.980 | 5.390 | 4.196 | 0.160 | 1.480 | 0.694 |
| 30x40 | 7.850 | 14.280 | 10.954 | 1.430 | 3.130 | 2.501 |
| 30x50 | 7.910 | 15.160 | 10.840 | 1.600 | 6.430 | 3.203 |

Table 2. shows the number of variables, constraints and interger density by th type of model, including calculation times. An average performance of column subtraction is 9.4%~34.6% of branch-and-bound methods. The advantage of using column subtraction method is

*1. If we select non basic variables that have same reduced costs or 0 value in the final simplex table, we could find the alternative solutions which have same objective valuse with optimal solution.*

*2 Column subtraction method with an interactive GUI can offer the opportunity to select the second best solution by adjusting lower bound in the solution process.*

Table 2. Computational Results by model

| Ship× Cargo | Variables | Const. | Dens. | LINDO (sec) | Column (sec) | Ratio |
|---|---|---|---|---|---|---|
| 10 × 10 | All Problems have Integer Solution | | | | | |
| 10 × 20 | 338 | 25 | 0.11219 | 1.2 | 0.05 | 0.041 |
| | 549 | 28 | 0.02687 | 1.1 | 0.01 | 0.001 |
| | 367 | 26 | 0.11171 | 0.88 | 0.06 | 0.069 |
| 10 × 30 | 636 | 32 | 0.08820 | 0.99 | 0.05 | 0.051 |
| | 1086 | 37 | 0.08071 | 1.76 | 0.22 | 0.125 |
| | 989 | 33 | 0.09232 | 1.59 | 0.17 | 0.107 |
| 10 × 40 | 2253 | 47 | 0.06949 | 3.73 | 0.50 | 0.134 |
| | 2116 | 45 | 0.07310 | 4.23 | 0.55 | 0.130 |
| | 1605 | 50 | 0.06004 | 2.64 | 0.27 | 0.102 |
| 10 × 50 | 3279 | 53 | 0.06257 | 6.43 | 3.57 | 0.555 |
| | 2321 | 58 | 0.05486 | 5.77 | 1.64 | 0.284 |
| | 2069 | 57 | 0.05452 | 5.33 | 0.44 | 0.083 |
| 20 × 10 | 246 | 28 | 0.09858 | 0.44 | 0.05 | 0.114 |
| 20 × 20 | All Problems have Integer Solution | | | | | |
| 20 × 30 | 1308 | 49 | 0.05604 | 3.95 | 0.50 | 0.127 |
| | 3050 | 47 | 0.07053 | 9.12 | 1.53 | 0.168 |
| | 1835 | 43 | 0.07119 | 2.25 | 0.27 | 0.120 |
| 20 × 40 | 2877 | 55 | 0.05506 | 8.84 | 2.36 | 0.267 |
| | 5794 | 57 | 0.05850 | 27.91 | 20.21 | 0.724 |
| | 2877 | 55 | 0.05506 | 8.84 | 2.36 | 0.267 |
| 20 × 50 | 3022 | 61 | 0.04907 | 10.77 | 3.46 | 0.043 |
| | 4855 | 64 | 0.04863 | 14.55 | 4.89 | 0.336 |
| | 5966 | 67 | 0.04765 | 27.85 | 3.57 | 0.128 |
| 30 × 10 | All Problems have Integer Solution | | | | | |
| 30 × 20 | 790 | 48 | 0.05367 | 1.32 | 0.22 | 0.167 |
| | 1054 | 47 | 0.05664 | 1.75 | 0.17 | 0.097 |
| 30 × 30 | 2529 | 59 | 0.04912 | 4.60 | 0.50 | 0.102 |
| | 2761 | 54 | 0.05702 | 4.39 | 0.61 | 0.139 |
| | 3168 | 60 | 0.05125 | 5.16 | 0.72 | 0.140 |
| 30 × 40 | 3797 | 66 | 0.04478 | 11.15 | 3.07 | 0.275 |
| | 3555 | 66 | 0.04479 | 14.28 | 3.13 | 0.219 |
| | 3094 | 66 | 0.04362 | 8.96 | 1.75 | 0.195 |
| 30 × 50 | 9112 | 71 | 0.04824 | 15.16 | 6.43 | 0.424 |
| | 6149 | 75 | 0.04122 | 10.11 | 2.80 | 0.277 |
| | 6896 | 76 | 0.04256 | 11.7 | 3.13 | 0.268 |

* Ratio = Column(sec) / LINDO(sec)

## 4. Conclusion

In this paper, we applied column subtraction method to SSP of which the LP relaxation has non integer solution. With an interactive GUI, we found that the column subtraction method is efficient method to solve SSP. The results of this research can be summarized as follows;

*1. It shows the excellence of column subtraction method comparing with the branch-and-bound method for SSP.*

*2, The column subtraction method with interactive GUI, we can find alternative solutions and the second best solutions. it means that the user can make a flexible decision in strategic situations.*

To apply the real aspect of cost estimation in SSP formulation and use this model in practical ship routing, the topics as follows should be considered in the next study.

*1. The more interactive and intelligent GUI which can leads the user to the optimal decision should be developed using Object Oriented Programming.*

*2. The more method to solve SSP should be investigated to find the best method to solve SSP with real & practical cargo and ship data.*

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