비선형 최소화 방법을 이용한 수신신호의 파라미터 추정알고리즘에 관한 연구

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Signal Parameters Estimation in Array Sensors via Nonlinear Minimization.

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요 약 : 레이더, 소나 및 육해상 통신시스템에 있어서, 어레이 센서에 입사하는 수신신호가 가지는 도래방향 및 신호전력과 같은 파라미터 추정문제는 수십년 동안 많은 연구관심이 되어 왔으며, 현재에도 다양한 응용영역에서 중요한 문제 중의 하나이다. 수신신호의 파라미터 추정방법으로서 종래의 MUSIC 또는 ESPRIT와 같은 방법은 역행렬 연산과 고유치 계산이 필요하므로 계산적 복잡성이 부가된다. 본 논문에서는 어레이 안테나의 기본 신호모델로부터 파라미터 추정의 문제를 비선형 최소화 문제로 정식화 하여 역행렬과 고유치 연산을 요구하지 않는 수신신호의 파라미터 동시추정 알고리즘을 제안하고 평가한다.

핵심용어 : 파라미터추정, 도래방향, 레이더신호처리, 어레이 센서, 비선형최소화

ABSTRACT : *The problem for parameters estimation of the received signals impinging on array sensors has long been of great research interest in a great variety of applications, such as radar, sonar, and land mobile communications systems. Conventional subspace-based algorithms, such as MUSIC and ESPRIT, require an extensive computation of inverse matrix and eigen-decomposition. In this paper, we propose a new parameters estimation algorithm via nonlinear minimization, which is simplified computationally and estimates signal parameters simultaneously.*

KEY WORDS : *parameter estimation, Angle of Arrival, radar signal processing, array sensors, nonlinear minimization*

1. Introduction

The extraction of signal parameters such as the number of the signals, AOA(Angle of Arrival)s, power levels of the signals impinging on array sensors is of interest in radar, sonar, and mobile communication systems. The problem of estimating AOA has been applied to a variety of research fields (Lal. C. Godara, 1997), (Titus K.Y. et al., 1994). In mobile communication systems, the AOA estimates in downlink have been used to estimate an uplink channel response (Lal. C. Godara, 1997). It is well known that MUltiple SIgnal Classification algorithm (MUSIC) and Estimation Signal Invariance Translation (ESPRIT) algorithms estimate the AOAs by exploiting a specific eigenstructure of the sample covariance matrix observed from array outputs (Ralph O. Schmidt, 1986), (R.Roy and T.Kailath, 1989). With the estimated AOAs, MUSIC and ESPRIT estimate signal powers and noise power by using a linear abgebraical method including inverse matrix computation. The disadvantage of MUSIC and ESPRIT is that they need an extensive computation of inverse matrix and eigen-decomposition. In particular, they are computationally ineffective for large-sized array antenna systems.

This paper represents a new algorithm to simultaneously estimate the parameters of signals impinging on array sensors via nonlinear minimization (Y. Bard, 1974). The proposed algorithm formulates an quadratic function from an array signal model and minimize the function by using complementary pivoting algorithm (Cottle, R.W and

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G.B.Dantzig, 1968). Our approach does not require eigen-decomposition and seeks a feasible solution within a few number of iterations in calculation. Computer simulations are made to estimate signal parameters, such as AOA, signal power, and noise power. It follows that the AOA estimation ability of the proposed algorithm is compatible to MUSIC. Moreover, our approach provides the estimates of signal power and noise power.

In Section 2, we will briefly mention a basic data model in array sensors. In Section 3, a quadratic function from the data model is reformulated and the new algorithm for parameters estimation is presented. The results of computer simulations followed by conclusions are given in Sections 4 and 5, respectively. Throughout the paper, the following notation will be used.

Glossary of Notation

For an arbitrary matrix F, G,

$C^{m\times n}$: the space of $m\times n$ complex-valued matrices

$R^{m\times n}$: the space of $m\times n$ real-valued matrices

$G^{(+)}=(G^{\dagger}G)^{-1}G^{\dagger}$: the pseudo inverse of G when G is a column full rank.

$G^{\dagger}$: the complex conjugate transpose of G

G^{*} : the complex conjugate of G

G^{T} : the transpose of G

$F\otimes G$: the Hadamard product of F and G, which is defined by $[F\otimes G]_{ij}=F_{ij}G_{ij}$

$Tr[G]$: the trace of matrix G,

I : an identity matrix

$Diag[G]$: a column vector composed of diagonal elements of the matrix G

$E[\cdot]$: the expected value for a random variable.

$Op(\cdot)$: operation counts in computation.

2. Array Data Model

Consider an array composed of m sensors with arbitrary locations and arbitrary directional characteristics. Assume that q narrowband signals impinge on the array from unknown angles, $\theta_1, \theta_2, \cdots \theta_q$. The array output, $x(t)\in C^{m\times 1}$ at time, $t=1,2,\cdots,K$, is modeled by Eq. (1).

$$x(t)=As(t)+n(t) \quad (1)$$

where the unknown matrix $A\in C^{m\times q}$ is given by Eq.(2).

$$A=[a(\theta_1), a(\theta_2), \cdots a(\theta_q)], \quad (2)$$

where $a(\theta_i)\in C^{m\times 1}$ denotes the ith array mode vector for the AOA θ_i. The $s(t)\in C^{q\times 1}$ is the received signals given by Eq. (3).

$$s(t)=[s_1(t), s_2(t), \cdots s_q(t)]^T. \quad (3)$$

The signal waveforms are stationary Gaussian random process with zero mean and uncorrelated. The additive noise vector, $n(t)$ is also a complex, stationary, and Gaussian random process with zero mean. The noises are independent of the received signals, and their covariance matrix is given by νI where ν denotes the variance as an unknown scalar. Based on the above assumptions, the true covariance matrix of the observation vector $x(t)$ is expressed by Eq. (4).

$$R=E[x(t)x^{\dagger}(t)]=AR_{ss}A^{\dagger}+\nu I \quad (4)$$

where R_{ss} denotes signal covariance matrix given by $R_{ss}=E[s(t)s^{\dagger}(t)]$. Taking K snapshots by observation, the sample covariance matrix is given by Eq. (5).

$$\hat{R}=\frac{1}{K}\sum_{t=1}^{K}x(t)x^{\dagger}(t) \quad (5).$$

Given $\hat{R}$, the problem of estimating AOAs is to search the combination of $2q+1$ unknown parameters set, $\{\theta_1, \theta_2, \cdots \theta_q, \nu, \sigma_1, \sigma_2, \cdots \sigma_q\}$, which minimizes the cost function defined by Eq. (6).

$$J=\| AR_{ss}A^{\dagger}+\nu I-\hat{R}\|_F^2 \quad (6)$$

where σ_i denotes the power of the ith signal. The Eq. (6) is a criterion that searches a good approximation to the true covariance matrix, Eq. (4). The minimization of J causes a complex nonlinear optimization problem. Since all but $\hat{R}$ in Eq. (5) are unknown parameters, this problem is not tractable. We herein introduce a new approach for finding a set of unknown parameters.

3. Signal Parameters Estimation

We consider p dimensional azimuth space as a collection of possible AOAs with equi-spaced discrete angles, $\{\phi_1, \phi_2, \cdots \phi_p\}$. For this case, a array mode matrix, $\tilde{A} \in C^{m \times p}$ can be obtained by Eq. (7).

$$\tilde{A} = [a(\phi_1), a(\phi_2), \cdots a(\phi_q)] \quad (7)$$

where $a(\phi_i) \in C^{m \times 1}$ denotes the ith assigned array mode vector for equi-spaced discrete angle, ϕ_i. Here, it should be noted that $\tilde{A}$ is known. With the matrix $\tilde{A}$, the expected covariance matrix can be given as $\tilde{A}\tilde{R}_{ss}\tilde{A}^{\dagger} \in C^{m \times n}$ where $\tilde{R}_{ss} \in C^{p \times p}$ denotes the signal covariance matrix to be estimated, and it is the diagonal matrix which has p diagonal elements, i.e., $\hat{\sigma}_1, \hat{\sigma}_2, \cdots \hat{\sigma}_p$. Considering the additive noise in array sensors, one possible criterion is to minimize Frobenius distance between $\hat{R}$ in Eq. (5) and the matrix $(\tilde{A}\tilde{R}_{ss}\tilde{A}^{\dagger} + \hat{\nu} I)$ in the sense of least-square fit. It follows that the problem for parameters estimation can be solved by minimizing the cost function Eq. (8).

$$\psi(\hat{\sigma}_i, \hat{\nu}) = | \tilde{A}\tilde{R}_{ss}\tilde{A}^{\dagger} + \hat{\nu} I - \hat{R} |_F^2, \quad (8)$$
$$subj.\ to\ \hat{\sigma}_i \geq 0, i = 1, 2, \cdots p,\ \hat{\nu} \geq 0.$$

The minimization of ψ is a nonlinear optimization problem. One way to solve this problem is to use steep descent algorithm as shown in (H. A. D'assumpcao, 1980), However, the function ψ has many local minima that prevent convergence to a global minima. It also consumes much time to converge a minimal point. As a result, it is necessary to select an initial set of parameters sufficiently near a global minimum in order to apply the steep descent algorithm to Eq. (8). This becomes a serious obstracle in minizing ψ. To solve this problem, we introduce a new approach, which is known as complementary pivoting method (Cottle, R.W and G.B.Dantzig, 1968). Firstly, the Eq. (8) can be reformulated as the quadratic function of Eq. (9) for $\tilde{s} \in R^{(p+1) \times 1}$, since $\tilde{R}_{ss}$ is a diagonal matrix.

$$\psi(\tilde{s}) = \tilde{s}^t H \tilde{s} - 2 d^t \tilde{s} + c,\ subj.\ to\ \tilde{s} \geq 0, \quad (9)$$

where $H \in R^{(p+1) \times (p+1)} = \begin{bmatrix} G & h \\ h^t & m \end{bmatrix}$,

$G \in R^{p \times p}$, $G_{ij} = | a^{\dagger}(\phi_i) a(\phi_j) |^2$ for $i, j = 1, 2, \cdots p$,

$h \in R^{p \times 1}$, $h_i = a^{\dagger}(\phi_i) a(\phi_j)$ for $i = 1, 2, \cdots p$,

$\tilde{s} \in R^{(p+1) \times 1}$, $\tilde{s} = [\hat{\sigma}_1, \hat{\sigma}_2, \cdots \hat{\sigma}_p, \hat{\nu}]^t$,

$d \in R^{(p+1) \times 1}$, $d_i = a^{\dagger}(\phi_i) \hat{R} a(\phi_i)$ for $i = 1, 2, \cdots p$.

$d = [d_1, d_2, \cdots d_p, e]^t$, $e = Tr[\hat{R}]$, $c = Tr[\hat{R}^t \hat{R}]$.

The problem of finding the feasible solution of Eq. (9) subject to constraints may be solved by a quadratic programming. The number of unknown parameters to be minimized, $p + 1 \gg m$ is very large. In general, it is not effective to find an unique minimal solution by applying a steep descent technique to Eq. (9), as mentioned above. Here, notice that H is a semidefinite positive matrix with rank $m+1$. For this case, the complementary pivoting method, can be applied to Eq. (9). When Kuhn-Tucker condition for optimality of the equation is satisfied (Y. Bard, 1974), Eq. (9) can be reformulated as Eq. (10).

$$w = Mz + q \quad subj.\ to\ w \geq 0,\ z \geq 0,\ w^t z = 0 \quad (10)$$

where $M \in R^{(p+1) \times (p+1)}$, $w, z \in R^{p+1}$ and $M = 2H$, $q = 2d$. Observing Eq. (10), we know that the nonlinear quadratic programming given by Eq. (9) can be simplified to an linear programming. Thus, this linear programming seeks a pair of w and z as a feasible solution by applying linear complementary pivoting method to Eq. (10). While seeking a feasible solution, a pair of w and z should always satisfy the constraints added to Eq. (10). It is known that this method is a fast algorithm which seeks feasible solutions with a few number of iterations in calculation. As the result, a minimal solution, $\tilde{s}$ are obtained. From $\tilde{s}$, we find the estimated signal powers, $\hat{\sigma}_1, \hat{\sigma}_2, \cdots \hat{\sigma}_p$, and the estimated noise power, $\hat{\nu}$. The estimated AOAs can be directly calculated from $\tilde{s}$, since the index i of $\hat{\sigma}_i$, $i = 1, 2, \cdots p$, correspond to the discretized angles.

4. Computer Simulations

Numerical results were demonstrated to evaluate the performance of the new algorithm. The AOA estimates of the proposed method were compared with ones by MUSIC. The proposed minimization method provides not only the AOA estimates but also the estimates of signal powers and noise power. This is to show a robustness of the proposed algorithm. In all simulations, we considered an uniform linear array(ULA) with equi-spaced sensors of half a wavelength, $\lambda/2$. Two uncorrelated BPSK signals impinge on array sensors. Assumed that $E[|s_1(t)|] = 1$. It is assumed that at each sensor the additive white noise is present and is uncorrelated from sensor to sensor. alternatively. Firstly, the total number of elements in array is $m=8$, SNR(Signal-to-Noise Ratio) is 10 dB, and 100 snapshots are taken. The discretized angles are assigned at intervals of 0.25 in the range of $-40 \sim 40$ for the antenna broadside. The angular separation of the two signals is one beamwidth. Fig. 1 and Fig. 2 show the AOA estimates of MUSIC and the proposed method, respectively. The abscissa represents an angular separation in a standard beamwidth between two received signals, i.e., AOA in the beamwidth. The standard beamwidth is defined by $2\pi/m$ in radian for the ULA. It shows that both MUSIC and the proposed method resolve the two signal accurately. Note that the ordinate in Fig. 1 represents the spectrum of the cost function in MUSIC, while the ordinate in Fig. 2 corresponds to the received signal power. The true signal powers and noise power is $E[|s_1(t)|]=1$, $E[|s_2(t)|]=1$, $E[|n(t)|]=0.1$. For this case, the estimated signal powers were found to be 0.9419 and 0.9558 and the estimated noise power was 0.0901. These results are very close to true values. Next, several simulation parameters are changed, while the other simulation conditions is the same as in Fig. 1 Fig, 2. The total number of antenna elements to be $m=6$. Two signals with SNR 3 dB are received. Note that $E[|s_1(t)|]=1$, $E[|s_2(t)|]=1$, $E[|n(t)|]=0.5012$. The angular separation is assumed to be 0.2 beamwidth. Fig. 3 and Fig. 4 show AOA estimates of MUSIC and the proposed method, respectively. The propose method provides accurate AOAs estimates, while MUSIC fails to resolve the two signals. The two estimated signal powers were found to be 0.6736 and 0.7912, and the estimated noise power was 0.4859. Comparing with the results in Fig. 1 and Fig. 2, the estimation performance is degraded due to low SNR and a small number of antenna elements.

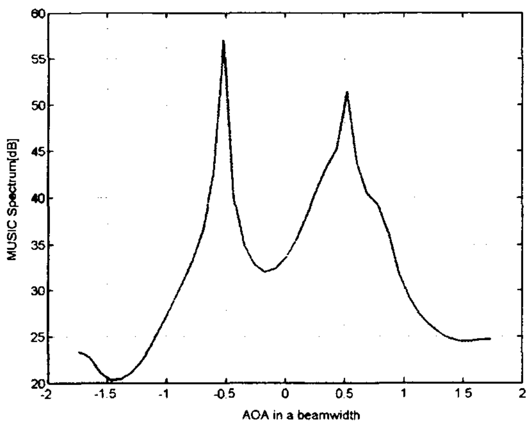

Fig. 1 AOA estimates of MUSIC at SNR 10dB.

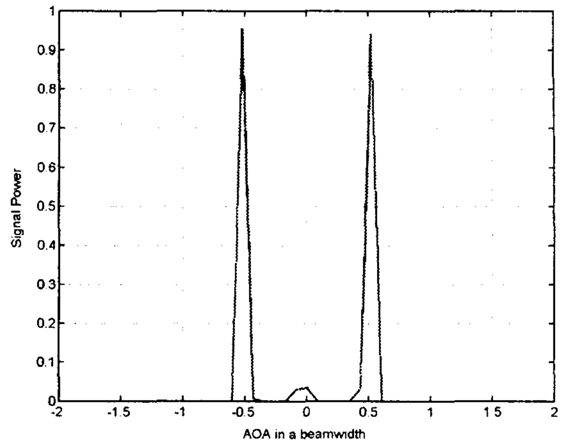

Fig. 2 AOA estimates of the proposed method at SNR 10dB.

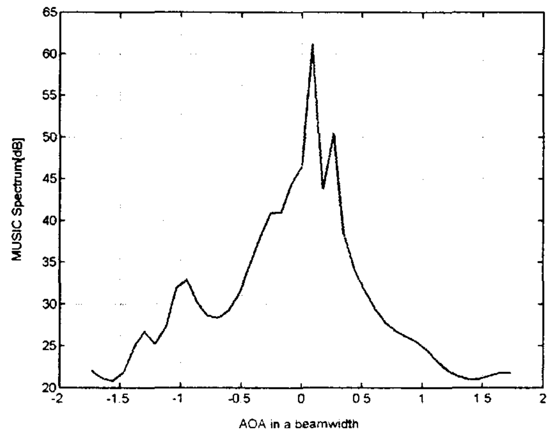

Fig. 3 AOA estimates of MUSIC at SNR 3dB.

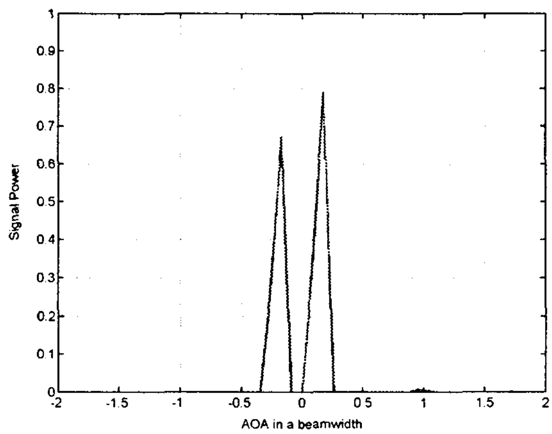

Fig. 4 AOA estimates of the proposed method at SNR 3dB.

5. Conculations

In this paper, the problem of estimating simultaneously signal parameters via nonlinear minimization was discussed.

The new algorithm which does not need eigen-decomposition and inverse matrix computation was proposed. Through computer simulations, the performance evaluation of the new algorithm was made by comparing with the results of MUSIC. It is found that the estimation performance of the proposed method is better than that of MUSIC. The other advantage of our approach is to provide the simultaneous estimates of signal parameters, i.e., signal powers and noise power. Finally, we shall stress that our approach can be effectively used to enhance the performance of wireless communication systems and radar systems. In the future, the evaluation of beamforming performance will be one of useful topics.

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