Intelligent Kalman Filter for Tracking an Anti-Ship Missile

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ABSTRACT

An intelligent Kalman filter (IKF) is proposed for tracking an incoming anti-ship missile. In the proposed IKF, the unknown target acceleration is regarded as an additive process noise. When the target maneuver is occurred, the residual of the Kalman filter increases in proportion to its magnitude. From this fact, the overall process noise variance can be approximated from the filter residual and its variation at every sampling time. A fuzzy system is utilized to approximate this variance, and the genetic algorithm (GA) is applied to optimize the fuzzy system. In computer simulations, the tracking performance of the proposed IKF is compared with those of conventional maneuvering target tracking methods.

Key words: intelligent Kalman filter, anti-ship missile, target maneuver, fuzzy system, genetic algorithm

I. Introduction

The Kalman filter (KF) has been widely used as a tracking filter to estimate the states of a target moving with nearly constant velocity. However, when the maneuver for turn or evasion is considered, the standard KF cannot be applied, because the unknown target acceleration for the maneuver works as a large process noise on the target model, and the original process noise variance can not cover it. As the first attempt to solve this problem, Singer proposed a target tracking model in which maneuver was assumed as a first order Markov process with time correlation [1]. Recent researches have been roughly divided into two main approaches. One approach is to detect a maneuver and cope with it effectively. Examples of this approach include the input estimation (IE) techniques, the variable dimension (VD) filter, the two-stage Kalman estimator, and so on. In addition to basic filtering computation, these techniques required additional efforts such as the estimation and detection of acceleration, and the compensation of state estimate or the transition between the non-maneuvering filter and the maneuvering filter in order to deal with the unknown target maneuver. The other approach is to describe the motion of a target using multiple sub-filters. The generalized pseudo-Bayesian (GPB) method, the interacting multiple model (IMM) algorithm, and the adaptive interacting multiple model (AIMM) algorithm are included in this approach. Also, these techniques needed the extra efforts such as the predefinition of multiple sub-filters and the update of model transition probability, and large computational load by using multiple sub-filters. Until now, no simple method to deal with the target maneuver has been proposed. It still remains a theoretically challenging issue in the maneuvering target tracking.

In this paper, we propose a new intelligent KF (IKF) to relax the additional efforts for conventional methods, improve the tracking performance, and establish the simple tracking procedure for a maneuvering target. In the proposed IKF, the unknown target acceleration for the maneuver is regarded as an additive process noise. When the target maneuver is occurred, the residual of the KF increases in proportion to its magnitude. From this fact, the overall process noise variance can be determined from the residual and its variation at every sampling time, and thus we can treat the target maneuver by adjusting this process noise variance. However, because analyzing their mathematical relation is very difficult, we utilize a fuzzy system. The basic idea arises from the fact that the fuzzy system can approximate an unknown or highly nonlinear system arbitrarily well [2]. In addition, to optimize the employed fuzzy system within the range of practicable target acceleration, we utilize the genetic algorithm (GA) [3, 4]. To show the effectiveness of the IKF, a maneuvering target scenario for an incoming anti-ship missile is examined and simulated. The proposed IKF is compared with the existing IMM and AIMM algorithms from the viewpoints of the tracking performance and the computational load.

II. Maneuvering Target Model

The linear discrete-time model for a maneuvering target is represented for each axis as

$$X(k+1) = FX(k) + G[u(k) + w(k)] \qquad (1)$$

$$Z(k) = HX(k) + v(k) \qquad (2)$$

where $X(k) = [p(k) \quad \dot{p}(k)] = [p(k) \quad v(k)]$ is the state vector, and $u(k)$ is the unknown deterministic target acceleration. The process noise $w(k)$ and the measurement noise $v(k)$ are zero-mean white Gaussian and have variances q and r, respectively. The transition matrix F, the excitation matrix G, and the measurement matrix H are specified as

$$F=\begin{bmatrix}1 & T\\0 & 1\end{bmatrix},\quad G=\begin{bmatrix}T^2/2\\T\end{bmatrix},\quad H=[1\quad 0]$$

where T is the sampling period.

When the target maneuver is occurred in (1), the standard KF cannot track a maneuvering target because the original process noise variance q cannot cover the target acceleration $u(k)$. To treat $u(k)$ easily, we consider it as an additive process noise. Hence, Eq. (1) can be rewritten as

$$X(k+1)=FX(k)+G[\overline{w}(k)] \qquad (3)$$

where $\overline{w}(k)$ is the overall process noise with time-varying variance $\overline{q}(k)$. From the fact that in the presence of a target maneuver, the residual of the KF increases in proportion to its magnitude, the variance $\overline{q}(k)$ can be determined from the residual and its variation at every sampling instant, and thus we can treat the target maneuver by adjusting this process noise variance.

III. Intelligent Kalman Filter

A. IKF

The time-varying variance $\overline{q}(k)$ for the IKF is inferred by a double-input single-output (DISO) fuzzy system, of which the j th fuzzy IF-THEN rule is represented by

$$R_j\text{: IF } \chi_1 \text{ is } A_{1j} \text{ and } \chi_2 \text{ is } A_{2j}\text{, THEN } y \text{ is } q_j \qquad (4)$$

where two premise variables χ_1 and χ_2 are the filter residual $\upsilon(k)$ and its variation $\Delta\upsilon(k)$, respectively, and a consequence variable y is the process noise variance q_j. A_{ij}, $i\in\{1,2\}$ and $j\in\{1,\cdots,M\}$, are fuzzy sets, and throughout this paper, it has the Gaussian membership function with the center c_{ij} and the standard deviation σ_{ij} as follows:

$$\theta_{A_{ij}}(\chi_i)=\exp\left(-\frac{1}{2}\left(\frac{\chi_i-c_{ij}}{\sigma_{ij}}\right)^2\right) \qquad (5)$$

By using singleton fuzzifier, product inference, and center-average defuzzifier, the unknown time-varying variance $\overline{q}(k)$ is approximated in the following form:

$$\overline{q}(k)=\frac{\sum_{j=1}^{M} q_j\left(\prod_{i=1}^{2}\theta_{A_{ij}}(\chi_i(k))\right)}{\sum_{j=1}^{M}\left(\prod_{i=1}^{2}\theta_{A_{ij}}(\chi_i(k))\right)} \qquad (6)$$

The IKF algorithm using the fuzzy system is as follows:

1. State and measurement prediction

$$\hat{X}(k\mid k-1)=F\hat{X}(k-1\mid k-1) \qquad (7)$$

$$\hat{Z}(k\mid k-1)=H\hat{X}(k\mid k-1) \qquad (8)$$

2. Inputs for the fuzzy system (measurement residual and its variation)

$$\chi_1(k)=\upsilon(k)=Z(k)-\hat{Z}(k\mid k-1) \qquad (9)$$

$$\chi_2(k)=\Delta\upsilon(k)=\upsilon(k)-\upsilon(k-1) \qquad (10)$$

3. Time-varying process noise variance calculation

$$\overline{q}(k)=\frac{\sum_{j=1}^{M} q_j\left(\prod_{i=1}^{2}\theta_{A_{ij}}(\chi_i(k))\right)}{\sum_{j=1}^{M}\left(\prod_{i=1}^{2}\theta_{A_{ij}}(\chi_i(k))\right)} \qquad (11)$$

4. State prediction covariance

$$P(k\mid k-1)=FP(k-1\mid k-1)F^T+G\overline{q}(k)G^T \qquad (12)$$

5. Innovation covariance

$$S(k)=HP(k\mid k-1)H^T+r \qquad (13)$$

6. Kalman gain

$$K(k)=P(k\mid k-1)H^TS^{-1}(k) \qquad (14)$$

7. Updated state estimate and its covariance

$$\hat{X}(k\mid k)=\hat{X}(k\mid k-1)+K(k)\upsilon(k) \qquad (15)$$

$$P(k\mid k)=P(k\mid k-1)-K(k)S(k)K^T(k) \qquad (16)$$

B. Optimization of fuzzy system

The fuzzy system to infer $\overline{q}(k)$ is constructed by domain experts at maneuvering target tracking. However, designing the proper fuzzy system with the satisfactory performance is not easy, and the tuning of its parameters is generally a time-consuming procedure, due to the unknown, highly nonlinear, and multi-parametric nature of the fuzzy system. In this paper, the GA is applied to simultaneously optimize the parameters and the structure of the fuzzy system for the IKF.

Obviously the fuzzy system should be designed such that the following objective function is minimized.

$$J=\sqrt{(\text{sum of position error})^2+(\text{sum of velocity error})^2} \qquad (17)$$

The GA represents the searching variables of the given optimization problem as a chromosome containing one or more sub-strings. In this case, the searching variables are the center c_{ij} and the standard deviation σ_{ij} for a Gaussian membership function of the fuzzy set A_{ij}, and the singleton output q_j. A convenient way to convey the searching variables into the chromosome is to gather all searching variables associated with the j th fuzzy rule into a string and to concatenate the strings as

$$S_j=\{c_{1j},\sigma_{1j},c_{2j},\sigma_{2j},q_j\} \qquad (18)$$

$$S=\{S_1,S_2,\cdots,S_M\} \qquad (19)$$

where S_j is the real coded parameter sub-string for the j th fuzzy rule in an individual S. At the same time, to identify the number of fuzzy rules, we utilize the binary coded rule number string, which assigns a 1 or a 0 for a valid or an invalid rule for the GA.

In chromosome, the premise string of each initial individual is determined at random within the given search space, i.e., the range of residual $\upsilon(k)$ and the range of variation of residual $\Delta\upsilon(k)$. The corresponding consequent string is determined at random using the possible range of the standard deviation of the process noise [5]:

$$0.5(a_{\max}+\sqrt{q})\le\sigma_j\le(a_{\max}+\sqrt{q}) \qquad (20)$$

where σ_j is the standard deviation of the process noise variance and $a_{\max}$ is the maximum value of the acceleration input.

Each individual is evaluated by a fitness function. Since the GA originally searches the optimal solution so that the fitness function value is maximized, mapping the objective function Eq. (17) to the fitness function is necessary. Furthermore, since reducing the number of the fuzzy IF-THEN rules from the viewpoints of the hardware implementation and the computation resource is strongly desired, we use the fitness function of the form

$$f(J) = \frac{\lambda}{J+1} + \frac{1-\lambda}{M+1} \tag{21}$$

where λ is a positive scalar to adjust the weight between the objective function and the rule number.

A population is evolved as a new one by reproduction, crossover, and mutation. During the evolution, an individual with the maximum fitness is preserved by the elitist reproduction. The evolution is continued until the generation number reaches.

IV. Simulation Results

To show the effectiveness of the proposed IKF, a maneuvering target scenario for an incoming anti-ship missile is examined and the fuzzy system is optimized off-line. For comparison purposes, we also simulate the conventional IMM and AIMM algorithms [6, 7]. The comparisons are achieved from the viewpoints of the tracking performance and the computational load.

Table 1 The initial parameters of the GA

Parameters	Values
Maximum generation (G_n)	200
Maximum rule number (P_r)	50
Population size (P_s)	500
Crossover rate (P_c)	0.9
Mutation rate (P_m)	0.01
λ	0.75

The initial parameters of the GA to optimize the fuzzy system are presented in Table 1. The functional relationship of the optimized fuzzy system is shown in Fig. 1.

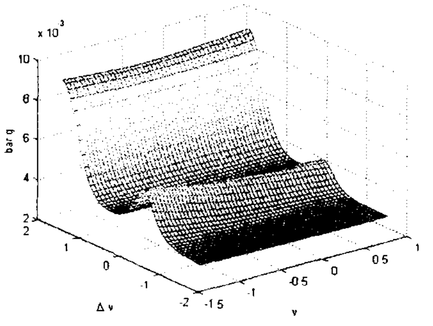


Fig. 1 The functional relationship of the fuzzy system

The target is assumed as an incoming anti-ship missile on the $x-y$ plane [8]. Initial position of the target is at $(72.9, 21.5)km$, and it moves with the constant velocity of $0.3km/s$ along the -150 degree line to the x-axis. The target has the lateral accelerations as shown in Fig. 2, and the corresponding target motion is illustrated in Fig. 3. For both x and y axes, the standard deviation of the zero mean white Gaussian measurement noise is $0.5km$ and that of a random acceleration noise is $0.001km/s^2$.

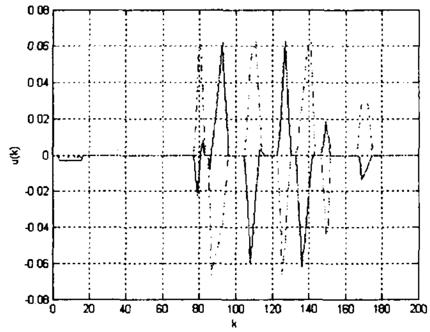


Fig. 2 The target accelerations $u(k)$ on x-axis (dashed) and y-axis (dash-dotted)

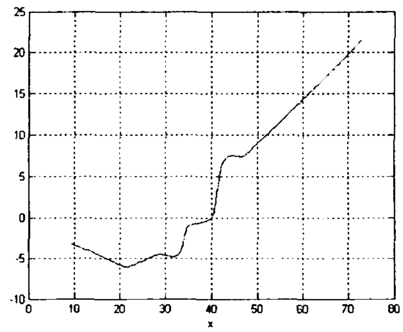


Fig. 3 The ideal target motion

The normalized position and velocity errors to compare the performances of three techniques are defined as

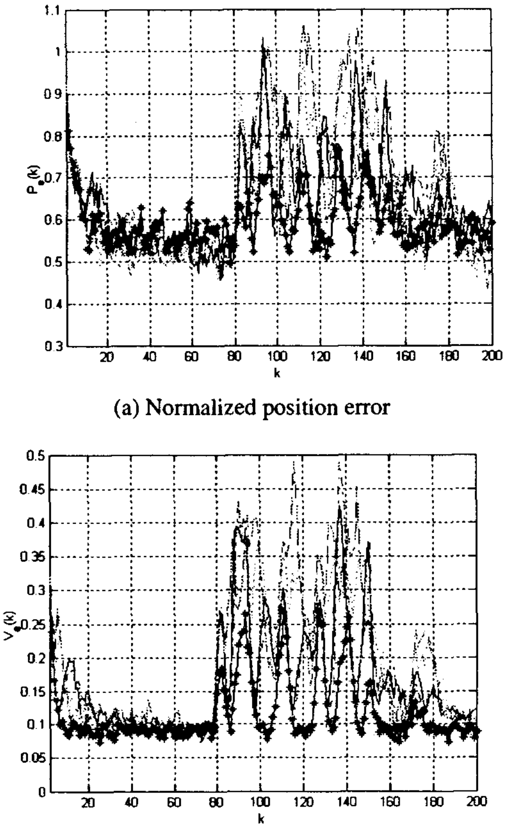
$$P_e(k) = \frac{\sqrt{\sum_{s=1}^{N_s}\left[\left(p_x^s(k)-\hat{p}_x^s(k)\right)^2+\left(p_y^s(k)-\hat{p}_y^s(k)\right)^2\right]}}{\sqrt{\sum_{s=1}^{N_s}\left[\left(p_x^s(k)-z_x^s(k)\right)^2+\left(p_y^s(k)-z_y^s(k)\right)^2\right]}} \tag{22}$$

$$V_e(k) = \frac{\sqrt{\sum_{s=1}^{N_s}\left[\left(v_x^s(k)-\hat{v}_x^s(k)\right)^2+\left(v_y^s(k)-\hat{v}_y^s(k)\right)^2\right]}}{\sqrt{\sum_{s=1}^{N_s}\left[\left(v_x^s(k)-\bar{v}_x^s(k)\right)^2+\left(v_y^s(k)-\bar{v}_y^s(k)\right)^2\right]}} \tag{23}$$

where $p_x^s(k)$, $p_y^s(k)$ and $\hat{p}_x^s(k)$, $\hat{p}_y^s(k)$ denote the true and the estimated positions of the target for each axis, respectively, $z_x^s(k)$ and $z_y^s(k)$ are the measured x and y positions of the target, and N_s is the total number of runs. $v_x^s(k)$, $v_y^s(k)$ and $\hat{v}_x^s(k)$, $\hat{v}_y^s(k)$ denote the true and the estimated velocities of the target for each axis, and $\bar{v}_x^s(k)$ and $\bar{v}_y^s(k)$ are the x and y velocities corresponding to the measured positions of the target. For the quantitative comparison between the performances of two algorithms, we utilize the averages of $P_e(k)$ and $V_e(k)$ over the total sampling times S.

$$\zeta_p = \frac{\sum_{k=1}^{S} P_e(k)}{S}, \quad \zeta_v = \frac{\sum_{k=1}^{S} V_e(k)}{S}$$

The simulation results of the IKF are compared with those of the IMM3, IMM5, AIMM3, and the AIMM5 algorithms in Fig. 4, and the numerical results are shown in Table 2.



(a) Normalized position error

(b) Normalized velocity error

Fig. 4 The simulation results: IKF (star-dashed), IMM3 (double-dashed), IMM5 (dotted), AIMM3 (dashed), and AIMM5 (dash-dotted)

Table 2 The numerical simulation results

Confs.	No. of sub-filters	ζ_p	ζ_v	CPU time
IMM3	3	0.6563	0.2005	57 sec
IMM5	5	0.6556	0.1936	114 sec
AIMM3	3	0.6503	0.1716	67 sec
AIMM5	5	0.6496	0.1723	124 sec
IKF	(1)	0.5997	0.1236	30 sec

V. Conclusions

The IKF has been proposed for tracking an incoming anti-ship missile and its feasibility has been examined. To deal with the unknown maneuver, the unknown acceleration input has been considered as an additive process noise, and the complex time-varying variance of overall process noise has been inferred by a fuzzy system. The GA has been applied to optimize the fuzzy system. Compared with the conventional maneuvering target tracking methods, the proposed method has the following three advantages: (i) Unlike the IE technique and the similar types of methods, the proposed IKF requires no additional efforts such as the estimation and detection of target maneuver, and the compensation of the state estimate or the transition between the non-maneuvering filter and the maneuvering filter, to treat the target maneuver; (ii) Unlike the multiple model methods, no extra efforts such as the predefinition of multiple sub-filters and the update of the model transition probability are required; (iii) Since the proposed filter can effectively track a maneuvering target only with a filter and a fuzzy system, it is possible to reduce the computational load in comparison with the conventional methods. Since the simulation results have shown that the proposed IKF has the better tracking performances, especially during maneuvering interval, and the fewer computational loads than the IMM and the AIMM algorithms, it can be claimed that IKF has the strong potential for the real target tracking applications.

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