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지능형 디지털 재설계: 전건부 변수가 측정 불가능한 경우

Intelligent Digital Redesign: Unmeasurable Premise Variable Case

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Abstract

An intelligent digital redesign technique (IDR) for the observer-based output feedback Takagi-Sugeno (T-S) fuzzy control system with unmeasurable premise variables is developed. The considered IDR condition is cubically parameterized as convex minimization problems of the norm distances between linear operators to be matched.

Key Words : 지능형 디지털 재설계, TS 퍼지 시스템. 관측기. 안정도

1. Introduction

It has been noted that the digital redesign schemes basically work only for a class of linear systems [1]. For that reason, it has been highly demanded to develop some intelligent digital redesign methodology for complex nonlinear systems, in which the first attempt was made by Joo *et al.* [2]. They synergistically merged both the Takagi-Sugeno (T-S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang *et al.* extended the intelligent digital redesign to uncertain T-S fuzzy systems[3] and elaborated it [4]. However, until now, no tractable method for IDR tackling on the observer-based output-feedback T-S fuzzy system with fuzzy outputs and the unmeasurable premise variables has been proposed. They remain yet to be theoretically challenging issues in IDR and thereby must be fully tackled.

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Motivated by the above observations, this paper aims at developing IDR for the T-S fuzzy observer-based output-feedback control (FOBOFC) system with fuzzy outputs and unmeasurable premise variable. To resolve the problems above stated, we propose an alternative way-numerical optimization-based IDR. The main contribution of this paper is to derive sufficient conditions of IDR in terms of matrix inequalities. The stability condition is naturally incorporated with ease.

The rest of this paper is organized as follows: Section 2 briefly reviews T-S fuzzy systems both continuous and discrete-time cases. In Section 3, a new IDR method is proposed for observer-based output-feedback T-S fuzzy control systems. This paper concludes with Section 4.

2. Preliminaries

Consider a T-S fuzzy system in which the ith rule is formulated in the following form:

R^i: IF $z_1(t)$ is about Γ_1^i ... $z_n(t)$ is about Γ_1^n

$$\text{THEN} \quad \begin{aligned} \dot{x}_c(t) &= A_i x_c(t) + B_i u_c(t) \\ y_c(t) &= C_i x_c(t) \end{aligned} \tag{1}$$

where $x_c(t) \in R^n$ is the state vector, $u_c(t) \in R^m$ is the control input vector. The subscript 'c' means the analog control, while the subscript 'd' will denotes the digital control in the sequel. R^i denotes the ith fuzzy inference rule, $z_h(t)$ is the premise variable, Γ^i_h, $i \in I_Q$, $h \in I_N$ is the fuzzy set of the hth premise variable in the ith fuzzy inference rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this T-S fuzzy system (1) is described by

$$\dot{x}_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t))$$

$$y_c(t) = \sum_i C_i x_c(t)$$

in which $\omega_i(z(t)) = \prod_{h=1}^{n} \Gamma^i_h(z_h(t))$,

$\theta_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{q} \omega_i(z(t))$ and $\Gamma^i_h(z_h(t))$ is the membership value of the hth premise variable $z_h(t) \in \Gamma^i_h$..

Throughout this paper, a well-constructed continuous-time observer-based fuzzy-model-based control law is assumed to be pre-designed, which will be used in redesigning the digital control law. In real control problems, we cannot always observe all the states of a system. Furthermore, when there exists an injective mapping from $x_c(t)$, rather than from $y_c(t)$, to $z(t)$, the premise variable from the observed state, denoted by $\hat{z}(t)$. Hence a fuzzy-model-based observer is introduced as follows:

R^i: IF $\hat{z}_1(t)$ is about Γ^i_1... $\hat{z}_n(t)$ is about Γ^n_1

$$\text{THEN} \quad \begin{aligned} \dot{\hat{x}}_c(t) = A_i \hat{x}_c(t) + B_i u_c(t) \\ + L^i_c(y_c(t) - \hat{y}_c(t)) \end{aligned}$$

The defuzzified output of the observer rules is represented by

$$\begin{aligned} \dot{\hat{x}}_c(t) = \sum_{i=1}^{q} \theta_i(\hat{z}(t))(A_i \hat{x}_c(t) + B_i u_c(t) \\ + L^i_c(y_c(t) - \hat{y}_c(t))) \end{aligned} \tag{2}$$

The controller rule is of the following form:

R^i: IF $\hat{z}_1(t)$ is about Γ^i_1... $\hat{z}_n(t)$ is about Γ^n_1

THEN $u_c(t) = K^i_c \hat{x}_c(t)$

The defuzzified output of the controller rules is given by

$$u_c(t) = \sum_{i=1}^{q} \theta_i(\hat{z}(t)) K^i_c \hat{x}_c(t) \tag{3}$$

Let the estimation error $e_c(t) = x_c(t) - \hat{x}_c(t)$ then we obtain the augmented continuous-time closed-loop T-S fuzzy system is

$$\begin{aligned} \dot{\chi}_c(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(\hat{z}(t)) \theta_h(\hat{z}(t)) \\ \times \begin{bmatrix} A_i + B_i K^j_c & -B_i K^h_c \\ A_i - A_j + (B_i - B_j) K^h_c - L^j_c(C_i - C_h) & A_j - L^j_c C_h - (B_i - B_j) K^h_c \end{bmatrix} \chi_c(t) \end{aligned} \tag{4}$$

where $\chi_c(t) = [\hat{x}_c(t)^T, e_c(t)^T]^T$. The pointwise dynamical behavior is expressed by

$$\begin{aligned} \chi_c(kT+T) = \sum_{i=1}^{q} \sum_{j=1}^{q} \theta_i(z(t)) \theta_j(\hat{z}(t)) \theta_h(\hat{z}(t)) \\ \times \Phi_{ijh} \chi_c(kT) \end{aligned} \tag{5}$$

where

$$\Phi_{ij} = \exp\left(\begin{bmatrix} A_i + B_i K^j_c & -B_i K^h_c \\ A_i - A_j + (B_i - B_j) K^h_c - L^j_c(C_i - C_h) & A_j - L^j_c C_h - (B_i - B_j) K^h_c \end{bmatrix} T \right).$$

Now we discusses the discretization of the hybrid T-S fuzzy system. Consider the digitally controlled T-S fuzzy systems and the discrete T-S fuzzy observer governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \tag{6}$$

$$\begin{aligned} \hat{x}_d(kT+T) = \sum_{i=1}^{q} \theta_i(z(kT))(G_i \hat{x}_d(kT) \\ + H_i u_d(t) + L^i_d(y_d(kT) - \hat{y}_d(kT))) \end{aligned} \tag{7}$$

where $G_i = \exp(A_i T)$ and $H_i = (G_i - I) A_i^{-1} B_i$, and $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined, in the time interval $[kT, kT+T)$, and $T > 0$ is a sampling period. For the digital control of the continuous-time T-S fuzzy system, the digital fuzzy-model-based controller is employed. Let the fuzzy rule of the digital control law for the system (6) take the following form:

R^i: IF $\hat{z}_1(t)$ is about Γ^i_1... $\hat{z}_n(t)$ is about Γ^n_1

THEN $u_d(t) = K^i_d \hat{x}_d(kT) + {}^i F y_d(kT)$

for $t \in [kT, kT+T)$, where K_d^i and F_d^i is the digital control gain matrix to be redesigned for the ith rule, and the overall control law is given by

$$u_d(t) = \sum_{i=1}^{q} \theta_i(\hat{z}(kT))(K_d^i \hat{x}_d(kT) + F_d^i \hat{y}_d(kT)) \tag{8}$$

for $t \in [kT, kT+T)$.

3. Main Result

The objective is to find gain matrices for digital controller and observers in (7) and (8) from the analog gain matrices in (2) and (3), so that the closed-loop state $x_d(t)$ in (6) with (7), (8) can closely match the closed-loop state $x_c(t)$ in (4) at all sampling time instanT-S $t = kT$, $k \in Z^+$. Thus it is more convenient to convert the T-S fuzzy system into discrete-time version for derivation of the state matching condition.

The pointwise dynamical behavior of the closed-loop system is reconstructed as

$$\begin{aligned} \chi_d(kT+T) = & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{h=1}^{r} \theta_i(z(kT))\theta_j(\hat{z}(kT)) \\ & \times \theta_h(\hat{z}(kT)) \\ & \times \begin{bmatrix} G_i + H_i K_d^h + H_i F_d^h C_g & -H_i K_d^h \\ 0 & G_j - L_d^j C_h \end{bmatrix} \chi_d(kT) + \zeta(\chi_d(kT)) \end{aligned} \tag{9}$$

where $\chi_d(kT)^T = [\hat{x}_d(kT)^T, e_d(kT)^T]^T$ and

$$\begin{aligned} \zeta(\chi_d(kT)) = & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{h=1}^{r}\sum_{g=1}^{r} \\ & \times \theta_i(z(kT))\theta_j(\hat{z}(kT)) \\ & \times \theta_h(\hat{z}(kT))\theta_g(z(kT)) \\ & \times \begin{bmatrix} 0 & 0 \\ \begin{pmatrix} G_i - G_j + (H_i - H_j)K_d^h \\ + (H_i - H_j)F_d^h C_g \\ - L_d^j (C_g - C_h) \end{pmatrix} & -(H_i - H_j)K_d^h \end{bmatrix} \chi_d(kT) \end{aligned}$$

Lemma 1: The term $\zeta(\chi_d(kT))$ is uniformly in order of $O(\|\chi_d(kT)\|)$, that is,

$$\lim_{\|\chi_d(kT)\| \to 0} \frac{\|\zeta(\chi_d(kT))\|}{\|\chi_d(kT)\|} = 0$$

Theorem 1 [Estimated Premise Variable Case] Suppose there exists an injective mapping from $x_c(t)$, not from $y_c(t)$, to $z(t)$. If there exist symmetric positive definite matrices P_1, P_2, Z_1, Z_2, symmetric matrices $X_{1_{ij}} = X_{1_{ji}}, X_{2_{ij}} = X_{2_{ji}}$, and matrices K_d^i, F_d^i, L_d^i with compatible dimensions, and possibly small $\gamma_i \in R_+, i \in Z_{[1,4]}$ such that the following minimization problem has solutions:

Minimize $trace\{diag\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}\}$

over

$P_1, P_2, Z_1, Z_2, X_{1_{ij}}, X_{2_{ij}}, K_d^i, F_d^i, L_d^i$

subject to

$$\begin{bmatrix} -\gamma_1 I & (\cdot)^T \\ \phi_{ijh}^{11} - G_i - H_i K_d^h - H_i F_d^h C_g & -\gamma_1 I \end{bmatrix} < 0$$

$$(i, j, h, g) \in I_R \times I_R \times I_R \times I_R$$

$$\begin{bmatrix} -\gamma_2 I & (\cdot)^T \\ \phi_{ijh}^{12} + H_i K_d^h & -\gamma_2 I \end{bmatrix} < 0$$

$$(i, j, h) \in I_R \times I_R \times I_R$$

$$\begin{bmatrix} -P_1 + X_{1_{ig}} & (\cdot)^T \\ \left(\dfrac{\begin{pmatrix} G_i + H_i K_d^h + H_i F_d^h C_g \\ + G_g + H_g K_d^h + H_g F_d^h C_i \end{pmatrix}}{2} \right) & -Z_1 \end{bmatrix} < 0$$

$[X_{1_{ig}}] > 0 \qquad (i, h, g) \in I_G \times I_R \times I_R$

$$\begin{bmatrix} -Z_1 & (\cdot)^T \\ P_1 Z_1 & -P_1 \end{bmatrix} < 0$$

$$\begin{bmatrix} -\gamma_3 I & (\cdot)^T \\ \begin{pmatrix} \phi_{ijh}^{21} - G_i + G_j \\ -(H_i - H_j)(K_d^h + F_d^h C_g) \\ + L_d^j (C_g - C_h) \end{pmatrix} & -\gamma_3 I \end{bmatrix} < 0$$

$$(i, j, h, g) \in I_R \times I_R \times I_R \times I_R$$

$$\begin{bmatrix} -\gamma_4 I & (\cdot)^T \\ \phi_{ijh}^{22} - G_j + L_d^j C_h + (H_i - H_j)K_d^h & -\gamma_4 I \end{bmatrix} < 0$$

$$(i, j, h) \in I_R \times I_R \times I_R$$

$$\begin{bmatrix} -P_2 + X_{2_{jh}} & (\cdot)^T \\ \left(\dfrac{G_j - L_d^j C_h + G_h - L_d^h C_j}{2} \right) & -Z_2 \end{bmatrix} < 0$$

$[X_{2_{jh}}] > 0 \quad (j, h) \in I_H \times I_R$

$$\begin{bmatrix} -Z_2 & (\cdot)^T \\ P_2 Z_2 & -P_2 \end{bmatrix} < 0$$

then, $x_d(kT)$ of the discrete representation (9) of (6) controlled by the intelligently redesigned digital FOBOFC (8) in cascade with (7) closely matches $x_c(kT)$ of the discrete representation of the analog FOBOFC system (5), and (9) is locally exponentially stable in the sense of Lyapunov stability criterion.

Remark 1: The above formulation leads to a BMI optimization problem in P_1 and Z_1, and P_2 and Z_2. Therefore, if Z_1 and Z_2 are fixed, finding P_1 and P_2 becomes a linear matrix inequality (LMI) problem and vice versa. In what follows, we solve the problem in an iterative fashion by treating the bilinear matrix inequality (BMI) as a double LMI. Notice that the double LMI problem is feasible only if the BMI problem is feasible.

4. Conclusions

In this paper, a new IDR has been proposed for the FOBOFC. The developed technique formulated the given IDR problem as numerical optimization problems so that the powerful and flexible numerical algorithms can be utilized.

5. References

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