# The Design Methodology of Fuzzy Controller by Means of Evolutionary Computing and Fuzzy-Set based Neural Networks

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E-mail: ohsk@wonkwang.ac.kr Abstract

In this study, we introduce a noble neurogenetic approach to the design of fuzzy controller. The design procedure dwells on the use of Computational Intelligence (CI), namely genetic algorithms and Fuzzy-Set based Neural Networks (FSNN). The crux of the design methodology is based on the selection and determination of optimal values of the scaling factors of the fuzzy controllers, which are essential to the entire optimization process. First, the tuning of the scaling factors of the fuzzy controller is carried out by using GAs, and then the development of a nonlinear mapping for the scaling factors is realized by using GA based FSNN. The developed approach is applied to a nonlinear system such as an inverted pendulum where we show the results of comprehensive numerical studies and carry out a detailed comparative analysis.

### 1. Introduction

The intent of this study is to develop, optimize and experiment with the controllers when developing a general design scheme of Computational Intelligence. One of the difficulties in the construction of the fuzzy controller is to derive a set of optimal control parameters of the controller such as linguistic control rules, scaling factors, and membership functions of the fuzzy controller[7,8]. Genetic algorithms (GAs) can be used to find the optimal control parameters. However, computationally evolutionary computing is intensive and this may be a point of concern when dealing with amount of time available to such search. For instance, when controlling a nonlinear plant such as an inverted pendulum of which initial states vary in each case, the search time required by GAs could be prohibitively high when dealing with dynamic systems. As a consequence, the parameters of the fuzzy controller cannot be easily adapted

to the changing initial states of this system such as an angular position and an angular velocity of the pendulum. To alleviate this shortcoming, we introduce a nonlinear mapping such as HCM-LMS tandem based polynomial. The development process consists of two main phases. First, using genetic optimization we determine optimal parameters of the fuzzy controller for various initial states (conditions). Second, we build up a nonlinear model that captures a relationship between the initial states of the system and the corresponding genetically optimized control parameters.

### 2. Fuzzy PID Controller

The block diagram of fuzzy PID controller is shown in Figure 1

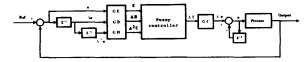


Figure 1. An overall architecture of the fuzzy PID controller

The above fuzzy PID controller consists of rules of the form [9][10]

 $R_j$ : if E is  $A_{ij}$  and  $\Delta E$  is  $A_{ij}$  and  $\Delta E$  is  $A_{ij}$  then  $U_j$  is  $D_j$ . The capital letters standing in the rule  $(R_j)$  denote fuzzy variables (linguistic terms) whereas D is a numeric value (singleton) of the control action. In each control rule, a level of its activation is computed in a standard fashion (1). The inferred value of consequence part is converted into numeric values with the aid of (2-1)[11].

$$w_i = \min\{\mu_{A_i}(E), \mu_{B_i}(\Delta E), \mu_{C_i}(\Delta^2 E)\}$$
 (1)

$$\Delta U^* = \sum_{i=1}^{n} w_i D_i / \sum_{i=1}^{n} w_i$$
 (2-1)

$$u(k) = U*(k) \times GC \tag{2-2}$$

We use triangular membership functions defined in the input and output spaces.

# 3 Auto-tuning of the fuzzy controller by GAs

Genetic algorithms (GAs) are the search algorithms inspired by Nature in the sense that we exploit a fundamental concept of a survival of the fittest as being encountered in selection mechanisms among species. In GAs, the search variables are encoded in bit strings called chromosomes. They deal with a population of chromosomes with each representing a possible solution for a given problem. A chromosome has a fitness value that indicates how good a solution represented by it is. In control applications, the chromosome represents the controllers adjustable parameters and fitness value is a quantitative measure of the performance of the controller. In general, the population size, a number of bits used for binary coding, crossover rate, and mutation rate are specified in advance. The genetic search is guided by a reproduction, mutation, and crossover. Each of these phases comes with a set of specific numeric parameters characterizing the phase. In this study, the number of generations is set to 100, crossover rate is equal to 0.6, while the mutation rate is taken as 0.1. The number of bits used in the coding is equal to 10. Let us recall that this involves tuning of the scaling factors and a construction of the control rules. These are genetically optimized. We set the initial individuals of GAs using three types of parameter estimation modes such as a basic mode, contraction mode and expansion mode. In the case of a basic mode (BM), we use scaling parameters that normalize error between reference and output, one level error difference and two level error difference by [-1, 1] for the initial individuals in the GA. In a contraction mode (CM), we use scaling parameters reduced by 25% in relation to the basic mode. While in the expansion mode (EM), we use scaling parameters enlarged by 25% from a basic mode. The standard ITAE expressed for the reference and the output of the system under control is treated as a fitness function [2].

The design procedure consists of the following steps

[step 1] Select the general structure of the fuzzy controller according to the purpose of control and dynamics of the process. In particular, we consider architectural options (PID, FPD/Fuzzy PD), and FPID (Fuzzy PD) controller)

[step 2] Define the number of fuzzy sets for each variable and set up initial control rules, refer to Figure 2 and 3.

[step 3] Form a collection of initial individuals of GAs. This involves the following

1. set the initial individuals of GAs for the scaling factor of fuzzy controller. The scaling factors can be described as normalized coefficients. Each scaling factor is expressed by (3).

Figure 2 illustrates three types of estimation modes of the scaling factor being used in setting the initial individuals of GAs describing the fuzzy controller.

$$E(kT) = error(kT) \times GE \qquad (3.a)$$

$$\Delta E(kT) = [error(kT) - error(k-1)T] \times GD \qquad (3.b)$$

$$\Delta^2 E(kT) = [error(kT) - 2error(k-1)T + error(k-2)T] \times GH \qquad (3.c)$$

$$U(kT) = U(k-1)T + \Delta U(kT) \times GC \qquad (3.d)$$

$$Expansion Mode$$

$$Expansion Mode$$

$$Contraction Mode$$

Figure 2. Three types of estimation modes for the scaling factors: basic, expansion, and contraction

[step 4] Here, all the control parameters such as the scaling factors GE, GD, GH and GC are tuned at the same time.

# 4. HCM-LMS tandem based polynomial model

In this algorithm, we use HCM clustering algorithm to classify the data and identify the divided data on each cluster by means of LMS method. We use a type of such polynomial as (4), and estimate coefficients of the polynomial.

$$\hat{y}(i) = C_0 + C_1 \theta(i) + C_2 \theta(i)^2 + \dots + C_n \theta(i)^n \tag{4}$$

Given a set of data  $X = \{x_1, x_2, \dots, x_n\}$ , where  $x_k = \{x_{k1}, x_{k2}, \dots, x_{kn}\}$ , n is the number of data and m is the number of variables[3]. Let P(X) be the power set of X, that is, the set of all the subsets of X. A hard c-partition of X is the family  $\{A_i \in P(X): 1 \le i \le c\}$  such that

 $\bigcup_{i=1}^{c} A_i = X$  and  $A_i \cap A_j = \emptyset$  for  $1 \le i \ne j \le c$ . Each  $A_i$  is viewed as a cluster, so  $\{A_1, ..., A_c\}$  partitions X into c clusters. The hard c-partition can be reformulated through the characteristic (membership) function of the element  $\mathbf{x}_k$  in  $A_i$ . Specifically, define

$$u_{ik} = \begin{pmatrix} 1, \mathbf{x}_k \in A_i \\ 0, \mathbf{x}_k \notin A_i \end{pmatrix}$$
 (5)

where  $\mathbf{x}_k \in X$ ,  $A_i \in P(X)$ ,  $i=1,2,\cdot,n$ . Clearly,  $u_{ik}=1$  means that  $\mathbf{x}_k$  belongs to cluster Ai. Given the value of  $u_{ik}$ , we can uniquely determine a hard c-partition of X, and vice versa. The  $u_{ik}$ 's should satisfy the following three conditions:

$$u_{ik} \in 0, 1, 1 \le i \le c, 1 \le k \le n \tag{6}$$

$$\sum_{i=1}^{c} u_{ik} = 1, \ \forall k \in \{1, 2, \dots, n\}$$
 (7)

$$0 \langle \sum_{i=1}^{n} u_{ik} \langle n, \forall i \in \{1, 2, \dots, c\}$$
 (8)

(6) and (7) together mean that each  $x_k \in X$  hould belong to one and only one cluster. (8) requires that each cluster Ai must contain at least one and at most n-1 data point. Collecting  $u_{ik}$  with  $1 \le i \le c, 1 \le k \le n$  into a c by n matrix  $U = [u^{ik}]$ . We obtain the matrix representation for hard c-partition, defined as follows.

$$M_c = \left\{ U \mid u_{ik} \in [0, 1], \sum_{k=1}^{c} u_{ik} = [1, 0] < \sum_{k=1}^{n} u_{ik} < [n] \right\}$$
 (9)

Step 1: Fix the number of clusters  $c(2 \le c \le n)$  and initialize the partition matrix  $U^0 \in M_c$ 

Step 2 Calculate the center vectors  $v_i$  of each cluster:

$$\mathbf{v}_{i}^{(r)} = \{v_{i1}, v_{i2}, \cdots, v_{ij}, v_{im}\}$$
 (10)

$$v_{ij}^{(r)} = \sum_{k=1}^{n} u_{ik}^{(r)} \cdot x_{kj} / \sum_{k=1}^{n} u_{ik}^{(r)}$$
 (11)

where,  $[u_{ik}] = \mathbf{U}^{(r)}$ ,  $i = 1, 2, \cdot, c$ ,  $j=1, 2, \cdot, m$ . Step 3: Update the partition matrix  $\mathbf{U}^{(r)}$ ; these modifications are based on the standard Euclidean distance function between the data points and the prototypes

$$d_{ik} = d(\mathbf{x}_k - \mathbf{v}_i) = \left| |\mathbf{x}_k - \mathbf{v}_i| \right| = \left[ \sum_{j=1}^m (\mathbf{x}_{kj - \mathbf{v}_{ij}})^2 \right]^{1/2}$$
(12)

$$u_{ik}^{(r+1)} = \begin{pmatrix} 1 & d_{ik}^{(r)} = \min\{d_{jk}^{(r)}\} \text{ for } all \ j \in c \\ 0 & otherwise \end{pmatrix}$$
(13)

Step 4: Check a termination criterion.

If 
$$||U^{(r+1)} - U^{(r)}|| \le \varepsilon$$
 (14)

stop; otherwise set r = r + 1 and return to step 2.

# 5. Simulation Study

The inverted pendulum system is composed of a rigid pole and a cart on which the pole is hinged [4][5]. The cart moves on the rail tracks to its right or left, depending on the force exerted on the cart. The pole is hinged to the car through a frictionless free joint such that it has only one degree of freedom. The control goal is to balance the pole starting from nonzero conditions by supplying appropriate force to the cart. In this study, the dynamics of the inverted pendulum system are characterized by two state variables:  $\Theta$ (angle of the pole with respect to the vertical axis),  $\theta$ (angular velocity of the pole). The behavior of these two state variables is governed by the following second-order equation. The dynamic equation of the inverted pendulum is shown as the following.

$$\theta = \frac{g sin\theta + \cos\theta \left(\frac{-F - ml\theta^2 sin\theta}{m_c + m}\right)}{\sqrt{\frac{4}{3} - \frac{m \cos^2\theta}{m_c + m}}}$$
(15)

Where g (acceleration due to gravity) is 9.8m/s2, mc (mass of cart) is 1.0kg, m (mass of pole) is 0.5m, and F is the applied force in newtons. Figure 3 shows auto-tuned scaling factors according to the change of initial angle and angular velocity of the inverted pendulum.

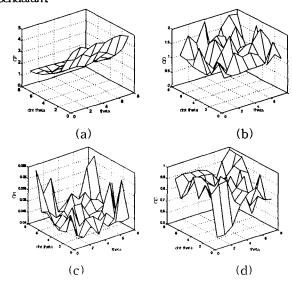


Figure 3. Auto-tuned scaling factors according to the change of initial angles (a) GE, (b) GD, (c) GH and (d) GC

Figure 4 demonstrates (a)pole angle (b)pole angular velocity (c)state space of fuzzy PID controller for initial angle=0.78(rad) and initial angular velocity=0.78(rad/sec) for each estimation algorithm respectively.

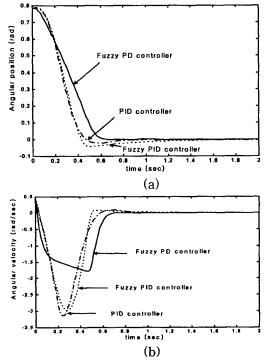


Figure 4 (a)pole angle (b)pole angular velocity of fuzzy PID controll er for initial angle-0.78(rad) and initial angular velocity-0.78(rad/sec) for each estimation algorithm respectively.

# 6. Conclusions

In this paper, we have proposed a two-phase optimization scheme of the fuzzy PID and PD controllers. The parameters under optimization concern scaling factors of the input and output variables of the controller that are known to exhibit an immense impact on its quality. The first phase of the design of the controller uses genetic computing that aims at the global optimization of its scaling factors where they are optimized with regard to a finite collection of initial conditions of the system under control. In the second phase, we construct a nonlinear mapping between the initial conditions of the system and the corresponding values of the scaling factors. From the simulation studies, using genetic optimization by scaling factor estimation modes and the estimation algorithm of the HCM-LMS tandem based polynomial model, we showed that the

fuzzy PI/PID controller controls effectively the inverted pendulum system. While the study showed the development of the controller in the experimental framework of control of a specific dynamic system (inverted pendulum), this methodology is general and can be directly utilized to any other system. Similarly, one can envision a number of modifications that are worth investigating. For instance, a design of systems exhibiting a significant level of variability could benefit from the approach pursued in this study.

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