Proceedings of KFIS Spring Conference 2004 Volume 14, Number 1

퍼지 엔트로피 함수를 이용한 데이터추출

Selection of data set with fuzzy entropy function

이상혁, 천성표, 김성신
부산대학교 전기공학과

Sang-Hyuk Lee, Seong-Pyo Cheon and Sung-Shin Kim
School of Electrical Engineering, Pusan National University
30 Jangjeon-dong, Geumjeong-gu, Busan 609-735, Korea
E-mail : leehyuk@pusan.ac.kr

Abstract

In this literature, the selection of data set among the universe set is carried out with the fuzzy entropy function. By the definition of fuzzy entropy, we have proposed the fuzzy entropy function and the proposed fuzzy entropy function is proved through the definition. The proposed fuzzy entropy function calculate the certainty or uncertainty value of data set, hence we can choose the data set that satisfying certain bound or reference. Therefore the reliable data set can be obtained by the proposed fuzzy entropy function. With the simple example we verify that the proposed fuzzy entropy function select reliable data set.
Keywords: Fuzzy entropy, distance measure, reliable data set,

I. Introduction

Generally for the linear system, reliable input invokes reliable output. Hence the reliable data selection is necessary in the view of reliable result. Previous results concerning this area is related to the pattern recognition and information theory. However pattern recognition is generally used for classifying patterns[1]. Also it is well known that the entropy represents the uncertainty of the fact. Hence entropy has been studied in the field of information theory, thermodynamics, or system theory etc.. The results that entropy of a fuzzy set is a measure of fuzziness of the fuzzy set are known by the numerous researchers[2-9]. the axiomatic definitions of entropy was proposed by Liu. The relation between distance measure and fuzzy entropy was viewed by Kosko. Bhandari and Pal gave a fuzzy information measure for discrimination of a fuzzy set A relative to some other fuzzy set B. Pal and Pal analyzed the classical Shannon information entropy. Also Ghosh used this entropy to neural network. However, these studies are focussed at the design of entropy function and analysis of fuzzy entropy measure, distance measure and similarity measure. Hence we carried out the application of fuzzy entropy to the selection of reliable data set among the universe set.

In this paper, we derived the fuzzy entropy with distance measure. The proposed fuzzy entropy is constructed by the Hamming distance measure, and which has the simple structure compared to the previous proposed entropy. With the proposed entropy, we verify the usefulness through the application of fuzziness measure to the universe data set. We also carried out calculate fuzziness of sample data set.

In the next section, definitions of entropy, distance measure and similarity measure of fuzzy sets are introduced and the proposed entropy proof is discussed. In section III, Construction of fuzzy membership function is proposed by the Extension Principle[10]. Also in section IV, sample data are measured by the proposed entropy measure. Conclusions are followed in section V.

Notations: Through out this paper, $R^+ = [0, \infty)$, $F(X)$ and $P(X)$ represent the set of all fuzzy sets and crisp sets on the universal set X respectively. $\mu_A(x)$ is the membership function of $A \in F(X)$, and the fuzzy set A , we use A^C to express the complement of A i.e., $\mu_{A^c}(x) = 1-\mu_A(x)$. For fuzzy sets A and B, $A \cup B$, the union of A and B is defined as $\mu_{A\cup B}(x) = max(\mu_A(x), \mu_B(x))$, $A \cap B$, the intersection of A and B is defined as $\mu_{A\cap B}(x) = min(\mu_A(x), \mu_B(x))$. A fuzzy set A^* is called a sharpening of A, if $\mu_{A^*}(x) \geq \mu_A(x)$ when $\mu_A(x) \geq 1/2$ and $\mu_{A^*}(x) \leq \mu_A(x)$ when $\mu_A(x) < 1/2$. For any crisp sets D, A_{near} and A_{far} of fuzzy set A are defined as

$$\mu_{\frac{1}{2}D}(x) = \begin{cases} \frac{1}{2} & x \in D \\ 0 & x \notin D \end{cases}, \quad \mu_{A_{near}}(x) = \begin{cases} 1 & \mu_A(x) \geq \frac{1}{2} \\ 0 & \mu_A(x) < \frac{1}{2} \end{cases},$$

and $\mu_{A_{far}}(x) = \begin{cases} 0 & \mu_A(x) \geq \frac{1}{2} \\ 1 & \mu_A(x) < \frac{1}{2} \end{cases}$.

II. Fuzzy entropy

It is often required that the reliable data set selection is necessary among many data set. In this section, we introduce the relation of fuzzy membership function and the fuzzy entropy. Let X be a space of objects and x be a generic element of X. A classical set A, $A \subseteq X$, is defined as a collection of elements or objects $x \in X$, such that each x can either belong or not belong to the set A. Whereas a fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

where $\mu_A(x)$ is called the membership function for the fuzzy set A. The membership function maps each element of X to a membership grade between 0 and 1.

Liu suggested axiomatic definition of fuzzy entropy as follows[5]. By this definition, we explain the fuzzy entropy of the membership function.

Definition 2.1 [4] A real function e: $F(X) \to R^+$ is called an entropy on $F(X)$, if e has the following properties:

(E1) $e(D) = 0$, $\forall D \in P(X)$

(E2) $e([1/2]) = max_{A \in F(X)} e(A)$

(E3) $e(A^*) \leq e(A)$, for any sharpening A^* of A

(E4) $e(A) = e(A^c)$, $\forall A \in F(X)$.

Let $S(x)$ be $S(x) = -x ln x - (1-x) ln(1-x)$, $0 \leq x \leq 1$. For fuzzy set A one of entropies can be represented by

$$e(A) = -\sum_{i=1}^{n} (S(\mu_A(x_i)), \ \forall A \in F(X), \qquad (1)$$

where $X = \{x_1, x_2, \cdots x_n\}$.

Then (1) satisfies the properties of (E1) - (E4), and it can be easily proved.

We propose entropy that is induced by the distance measure. Among distance measures, Hamming distance is commonly used distance measure between fuzzy sets A and B,

$$d(A, B) = \frac{1}{n}\sum_{i=1}^{n} | \mu_A(x_i) - \mu_B(x_i) | \qquad (2)$$

where $X = \{x_1, x_2, \cdots x_n\}$.

Fuzzy entropy means the uncertainty of the fuzzy set, hence it represents the shaded area of Fig. 1. In Fig. 1, A_{near} denotes the crisp set of fuzzy set A.

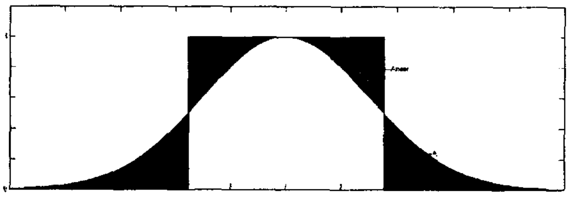

Fig. 1 Representation of entropy

Previous results about fuzzy entropy explain the shaded area of the Fig. 2[6,7]. Now we propose another fuzzy entropy induced by distance measure which is different from Theorem 3.1 of Fan, Ma and Xie[7]. Proposed entropy needs only A_{near} crisp set, and it has the advantage in computation of entropy.

Theorem 2.1 Let d be a σ-distance measure on $F(X)$; if d satisfies

$d(A^C, B^C) = d(A, B)$, $A, B \in F(X)$,

then

$$e(A) = 2d((A \cap A_{near}), [1]) + 2d((A \cup A_{near}), [0]) - 2 \qquad (3)$$

is a fuzzy entropy.

Proof. The proposed equation (3) become entropy for the fuzzy set A if it satisfies Definition 2.1. Hence we prove from (E1),

For (E1), $\forall D \in P(X)$, $D_{near} = D$, so

$$e(D) = 2d((D \cap D_{near}), [1]) + 2d((D \cup D_{near}), [0]) - 2$$
$$= 2d(D, [1]) + 2d(D, [0]) - 2 = 0$$

is satisfied. (E2) represent that crisp set $[1/2]$ has the maximum entropy 1, so the entropy $e[1/2]$

satisfies

$$\begin{aligned} e([1/2]) &= 2d(([1/2] \cap [1/2]_{near}), [1]) \\ &\quad + 2d(([1/2] \cup [1/2]_{near}), [0]) - 2 \\ &= 2d(([1/2] \cap [1]), [1]) + 2d(([1/2], [0]) - 2 \\ &= 1. \end{aligned}$$

In the above, $[1/2]_{near} = [1]$ is satisfied.

(E3) means that entropy of the sharpened version of fuzzy set A, $e(A^*)$ is less than or equal $e(A)$. In the proof, $A^*_{near} = A_{near}$ is also used.

$$\begin{aligned} e(A^*) &= 2d((A^* \cap A^*_{near}), [1]) + 2d((A^* \cup A^*_{near}), [0]) - 2 \\ &= 2d((A^* \cap A_{near}), [1]) + 2d((A^* \cup A_{near}), [0]) - 2 \\ &\leqq 2d((A \cap A_{near}), [1]) + 2d((A \cup A_{near}), [0]) - 2 \\ &= e(A) \end{aligned}$$

Inequality is satisfied because of $d(A, A_{near}) \geq d(A^*, A_{near})$[8].

Finally, (E4) is proved using the assumption $d(A^C, B^C) = d(A, B)$[7,8], hence we have

$$\begin{aligned} e(A) &= 2d((A \cap A_{near}), [1]) + 2d((A \cup A_{near}), [0]) - 2 \\ &= 2d((A \cap A_{near})^C, [1]^C) \\ &\quad + 2d((A \cup A_{near})^C, [0]^C) - 2 \\ &= 2d((A^C \cup A^C_{near}), [0]) \\ &\quad + 2d((A^C \cap A^C_{near}), [1]) - 2 \\ &= e(A^C). \end{aligned}$$

Q.E.D.

Theorem 2.1 uses only A_{near} crisp set, hence we can consider another entropy. Which considers only A_{far}, and it has more compact form than Theorem 2.1.

Theorem 2.2 Let d be a σ-distance measure on $F(X)$; if d satisfies

$d(A^c, B^c) = d(A, B),\ A, B \in F(X)$,

then

$$e(A) = 2d((A \cap A_{far}), [0]) + 2d((A \cup A_{far}), [1]) \quad (4)$$

is a fuzzy entropy.

Proofs are similar to those of Theorem 2.1

Proposed entropies Theorem 2.1 and 2.2 have some advantages to the Liu's, they don't need half part of assumption of Theorem [8] to prove (3) and (4). Furthermore (3) and (4) use only one crisp sets A_{near} and A_{far}, respectively.

III. Fuzzy membership function design

In this subsection, we propose fuzzy membership function construction with the Extension Principle[10]. Zadeh had proposed Extension principle for extending nonfuzzy mathematical concepts to fuzzy sets.

Extension Principle

Let $X_1, X_2, \cdots, X_n$, and Y be nonempty(crisp sets, $X = X_1 \times X_2 \times \cdots \times X_n$ be the product set of $X_1, X_2, \cdots, X_n$, and f be mapping from X to Y. Then, for any given n fuzzy sets $A_i \in \mathcal{F}(X_i)$, $i = 1, 2, \cdots, n$, we can induce a fuzzy set $B \in \mathcal{F}(Y)$ through f such that

$$\mu_B(y) = sup_{y = f(x_1, x_2, \ldots, x_n)} min[\mu_{A_1}(x_1), \mu_{A_2}(x_2), \ldots \mu_{A_n}(x_n)],$$

where we use the convention

$sup_{x \in \varnothing}\{x | x \in [0, \infty]\} = 0$ if $f^{-1}(y) = \varnothing$.

As an example, we can obtain a binary operator $*$ on fuzzy sets in $\mathcal{F}(X)$

$$\mu_{A*B}(z) = sup_{x*y=z}[\mu_A(x) \wedge \mu_B(y)],\ \forall z \in X,$$

where $A, B \in \mathcal{F}(X)$.

IV. Illustrative Example

We illustrate the example of reliable data set selection from the universe sets. However statistical mean and variance do not propose the fuzziness or reliability "how much". Hence with the help of the definition of entropy, fuzzy measure has introduced in Section II. Assume that one class consist 65 students. Studying level can be classified by the two viewpoints, the one is heuristic representation and the other is grade. Mean of 65 students reveals 53.73, and the average level student membership function is shown in Fig. 2. The average level students have the grade of B and C, points between 37 and 71. In this case, Selecting average level students is obtained by measuring the entropy function of Fig. 2. Each time we choose 4 students randomly. Next calculating (3) or (4), then we can calculate student group entropy. As obtained value reaches to zero, it means that student group has higher tendency containing in B, C grade statistically. First we choose 4 students randomly. Students points are illustrated by $s_1 - s_4$ in Fig. 3. And its fuzzy entropy value can be calculated with eqs. (3) or (4). Actual values, membership function value and fuzzy entropy values are shown in Table

1. Hence the fuzzy entropy value of the group is 0.344. Next, repeating this procedure we obtained following results. Additional experiments are carried, and the 5 students are chosen as Fig. 4 and 5. Table 2 and 3 shows the entropy values of the samples.

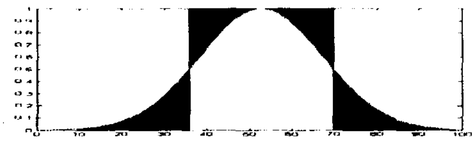

Fig. 2 Average level student membership function and B, C grade

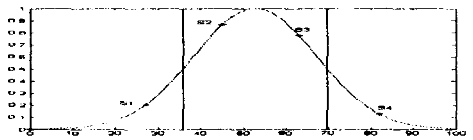

Fig. 3 Selection of 4 students

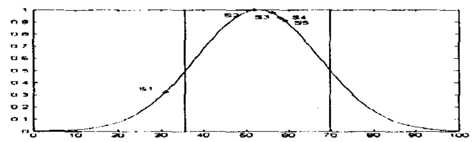

Fig. 4 Selection of 5 students

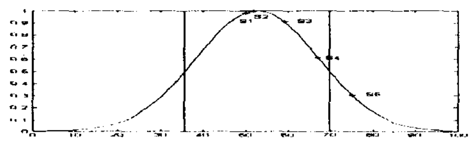

Fig. 5 Selection of 5 students

By the Table 2 and 3, the fuzzy entropy values of the groups are 0.2 and 0.3199. Therefore 2nd trial is the most reliable.

V. Conclusions

In this literature, we derive the fuzzy entropy with distance measure. The proposed fuzzy entropy is constructed by the Hamming distance measure, and which has the simple structure compared to the previous proposed entropy. With the proposed entropy, the usefulness is verified through the application of measure the fuzziness to the sampled set among universe data set.

Table 1 Point, membership value and entropy value of samples(Fig.3 case)

Sample	Point	Membership value	Fuzzy entropy value
S1	27	0.2025	0.4050
S2	45	0.8656	0.2689
S3	63	0.7758	0.4484
S4	82	0.1269	0.2538

Table 2 Point, membership value and entropy value of samples(Fig. 4 case)

Sample	Point	Membership value	Fuzzy entropy value
S1	31	0.3200	0.6401
S2	52	0.9987	0.0026
S3	56	0.9747	0.0506
S4	58	0.9354	0.1291
S5	59	0.9098	0.1804

Table 3 Point, membership value and entropy value of samples(Fig. 5 case)

Sample	Point	Membership value	Fuzzy entropy value
S1	50	0.9821	0.0358
S2	52	0.9987	0.0026
S3	59	0.9098	0.1804
S4	67	0.6124	0.7752
S5	75	0.3028	0.6055

VI. References

[1] J. C. Bezdek and S.K. Pal, Fuzzy models for pat ten recognition, IEEE, 1992.

[2] D. Bhandari, N. R. Pal, Some new information measure of fuzzy sets, Inform. Sci. 67, 209-228, 199 3.

[3] A. Ghosh, Use of fuzziness measure in layered n etworks for object extraction: a generalization, Fuzz y Sets and Systems 72, 331-348, 1995.

[4] B. Kosko, Neural Networks and Fuzzy Systems, Prentice-Hall, Englewood Cliffs, NJ, 1992.

[5] Liu Xuecheng, Entropy, distance measure and si milarity measure of fuzzy sets and their relations, F uzzy Sets and Systems, 52, 305-318, 1992.

[6] N.R. Pal, S.K. Pal, Object-background segmentat ion using new definitions of entropy, IEEE Proc. 36, 284-295, 1989.

[7] J. L. Fan, W. X. Xie, Distance measure and indu ced fuzzy entropy, Fuzzy Set and Systems, 104, 305 -314, 1999.

[8] J. L. Fan, Y. L. Ma, and W. X. Xie, On some p roperties of distance measures, Fuzzy Set and Syste ms, 117, 355-361, 2001.

[9] S.H. Lee and S.S. Kim, On some properties of d istance measures and fuzzy entropy, Proceedings of KFIS Fall Conference 2002, 9-12.

[10] Z. Wang and G. T. Klir, Fuzzy measure theory, New York : Plenum Press, 1992.