Proceedings of KFIS Spring Conference 2004 Volume 14, Number 1

계층적 적응 퍼지제어기법을 사용한 역진자시스템의 안정화 및 위치제어

Balancing and Position Control of Inverted Pendulum System Using Hierarchical Adaptive Fuzzy Controller

김용태*, 이희진*, 김동연**
*한경대학교 정보제어공학과, **한경대학교 전자공학과, 전자기술종합연구소

Yong-Tae Kim*, Hee-Jin Lee*, Dong-Yon Kim**
*Dept. of Information & Control Engineering, Hankyong National University
**Dept. of Electronic Engineering, Hankyong National University
E-mail : ytkim@hnu.hankyong.ac.kr

요 약

In the paper is proposed a hierarchical adaptive fuzzy controller for balancing and position control of the inverted pendulum system. Because balancing control rules of the pendulum and position control rules of the cart can be opposite, it is difficult to design an adaptive fuzzy controller that satisfy both objectives. To stabilize the pendulum at a specified position, the hierarchical adaptive fuzzy controller consists of a robust indirect adaptive fuzzy controller for balancing, a forced disturbance generator which emulates heuristic control strategy, and a supervisory decision maker for the arbitration of two control objectives. It is proved that all the signals in the overall system are bounded. Simulation results are given to verify the proposed adaptive fuzzy control method.

1. Introduction

The inverted pendulum system is a typical example of an unstable nonlinear control system which is difficult to control. In fact, the inverted pendulum problem has been widely used as an example to test new control concepts as well as demonstrate the effectiveness of modern control theory. Control objectives of the inverted pendulum system can be swinging-up the pendulum, balancing of the pendulum at the upright position, and regulation of the cart at an arbitrary specified position. There have been a large amount of research efforts: stabilization of double inverted pendulum on an inclined rail[1], swing up and stabilization of the pendulum with scheduled control input[2], attitude control of a triple-inverted pendulum[3], control of the pendulum with a angular motion type cart[4], swing up control of the pendulum with tree search technique[5], and linear control of the pendulum with a random search[6].

Also, many researchers have used stabilizing problem of the inverted pendulum for demonstrating the success of their adaptive and learning control methods. After Barto *et. al.*[7] proposed the neural network-based balancing controller, a number of neural network-based learning controllers have been developed[8, 9, 10]. Also, Ha [11] proposed a fuzzy control scheme with multiple rule bases and rule supervisors for swing-up, balancing, and position control. However, while taking nonlinear nature of the pendulum system into account, it is difficult to design rule supervisors and arbitrating rule bases which are implemented to achieve above three objectives.

In this paper, balancing and position control of an inverted pendulum system are considered. To stabilize the pendulum at an arbitrary specified position, a fuzzy control system with a robust adaptation algorithm, a forced disturbance generator which emulates heuristic control strategy. It is proved that all the signals in the overall system are bounded. In Section 2, we describe the inverted pendulum problem. In Section 3, to solve the problem, an indirect adaptive fuzzy control system is proposed. In Section 4, simulation results are given to verify the effectiveness of the proposed method. Section 5 concludes this paper.

2. Problem Description

Let x_1 be the angle of the pendulum with respect to the vertical line, x_2 be the angular velocity, x_3 be the position of the cart and x_4 be the position velocity. Then the dynamic equations of the inverted pendulum system are [12]

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{g sin x_1 - \frac{ml x_2^2 cos x_1 sin x_1}{M+m} + \frac{cos x_1}{M+m} u}{l\left(\frac{4}{3} - \frac{m cos^2 x_1}{M+m}\right)}, \qquad (1)$$

$$\dot{x}_3 = x_4,$$

$$\dot{x}_4 = \frac{ml x_2^2 \sin x_1 - \dot{x}_2 cos x_1 - b x_4 + u}{M+m}$$

(2)

where $g = 9.8m/s^2$ is the acceleration due to gravity, M is the mass of the cart, m is the mass of the pole, l is the length of the pole, u is the applied force. We choose $M = 1Kg$, $m = 0.1\,Kg$, $b = 0.1$ and $l = 0.5\,m$.

In this paper, balancing of the pendulum and position control of the cart are considered. Swing up controllers of pendulum were proposed by some researchers [2, 6, 11]. It can be easily designed by using fuzzy rule tables which are constructed by examining mathematical model and understanding its dynamic behavior. Also, fuzzy controller for balancing of the pendulum and a fuzzy controller for regulation of the cart can be easily designed. However, it is difficult to design fuzzy controller that satisfies two control objectives, that is, balancing of the pendulum and regulation of the cart at arbitrary position. To solve the problem, Ha [11] uses another rule base to arbitrate both objectives. Because control rules for balancing of the pendulum and control rules for position control of the cart can be opposite, it is very difficult to obtain the arbitration rules. In addition, it is more difficult to design an adaptive control algorithm that satisfy both objectives. In the paper, to solve this problem, a hierarchical adaptive fuzzy controller is developed.

3. Design of Hierarchical Adaptive Fuzzy Controller

We consider two control objectives of the inverted pendulum system. The first objective is to control the cart at its target position from an arbitrary initial position. The second objective is to balance the pendulum at the upright position. To achieve this two objectives, a hierarchical adaptive fuzzy control system and a robust adaptation algorithm is developed considering its dynamic behavior as well as heuristic control strategy to stabilize the pendulum at a specified position. Heuristic control strategy is as follows(Fig. 1).

Step 1. Balance the pendulum at an arbitrary position.

Step 2. If cart is not at the specified position, move cart toward the opposite direction of the specified position until angle error of pendulum reaches to certain value that is determined according to the distance from the desired position.

Step 3. Stabilize the pendulum according to the appropriate performance indices that are selected based on the balancing experiences.

Step 4. If the cart is at the specified position, stop. Otherwise, continue with Step 2.

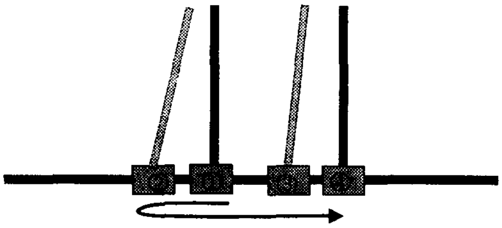

Fig 1. Graphical description of heuristic control strategy of the inverted pendulum.

Fuzzy control can be used as an effective means to capture the human expertized knowledge and achieve multiple control objectives of nonlinear systems. Therefore, the proposed controller is designed based on the above heuristic control strategy by using the fuzzy control theory. Overall structure of the proposed controller is shown in Fig.2.

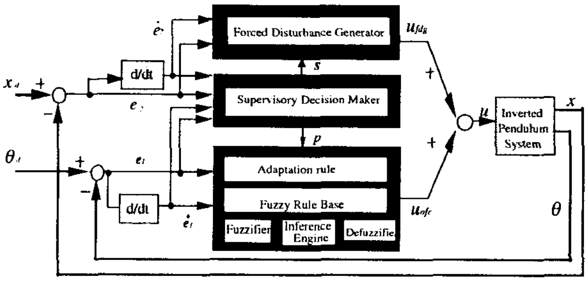

Fig 2. Overall structure of the proposed hierarchical adaptive fuzzy control system.

It includes a supervisory decision maker, two fuzzy rule tables for balancing of pendulum, position control of cart. Supervisory decision maker decides whether the system is in the balancing phase or forced disturbance phase by monitoring the plant states. Rule table in the forced disturbance phase can be obtained from human experience and insight(Table 1). Balancing rules can be learned using indirect adaptive fuzzy algorithms, independently. However, the control action in the forced disturbance phase generates similar situation produced by external disturbance and the balancing rules will be modified according to the performance decision(Fig. 3)[13]. It should be noted that the balancing rules will be continuously modified according to the performance evaluation as long as the states are disturbed by the control action in the forced disturbance phase, even though the states may move satisfactorily toward the band. This implies that when there exist frequent control actions by the forced disturbance, the control performance can be eventually deteriorated by the repeated modification of balancing rules [13][14].

Table 1. Fuzzy rule table of the proposed forced disturbance generator.

$p=FDG(e_3,e_4)$		e_3				
		NB	NS	ZO	PS	PB
e_4	PB	ZO	ZO	ZO	PB	PB
	PS	ZO	ZO	ZO	PS	PB
	ZO	NS	NS	ZO	PS	PS
	NS	NB	NS	ZO	ZO	ZO
	NB	NB	NB	ZO	ZO	ZO

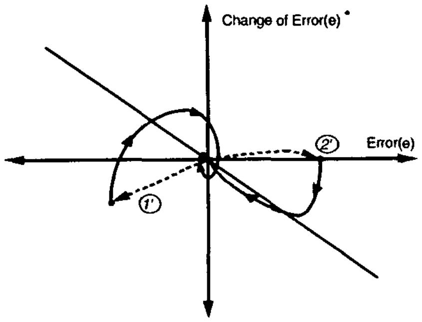

Fig 3. State trajectories in the forced disturbance phase.

To overcome the problem, the robust indirect adaptive fuzzy control method[14] is used to learn the balancing rules of the pendulum. The system (1) can be described by a class of single-input nonlinear systems as

$$\ddot{x}_1 = f(x_1, \dot{x}_1) + g(x_1, \dot{x}_1)\,(u_{afc} + u_{fdg}(x_3, x_4)), \qquad (2)$$
$$y = x_1,$$

where f and g are bounded continuous functions, u_{fdg} denotes the output of forced disturbance generator and is bounded by d. Let $x=(x_1, x_2)^T$. Let us denote the output tracking error $e=y_m-y$ where y_m is a given reference signal. The error state vector $e=(e, \dot{e})^T$. Also, let us define the tracking error metric as $s=c^Te$ where $c=(c_1, c_2)^T$ such that all roots of $h(p)=c_2p+c_1=0$ are in the left-half plane. We replace $f(x)$ and $g(x)$ by the fuzzy logic systems $u_f(x/\theta_f)$ and $u_g(x/\theta_g)$, i.e.,

$$u_f(x/\theta_f) = \hat{f}(x/\theta_f) = \theta_f^T\xi_f(x) = \xi_f^T(x)\theta_f, \qquad (3)$$

$$u_g(x/\theta_g) = \hat{g}(x/\theta_g) = \theta_g^T\xi_g(x) = \xi_g^T(x)\theta_g, \qquad (4)$$

where $\xi_f(x)$ and $\xi_g(x)$ are fuzzy basis vectors, θ_f and θ_g are corresponding parameter vectors of each fuzzy system. Consequently, the following certainty equivalent controller is applied.

$$u_{afc} = \frac{1}{u_g(x/\theta_g)}\left[-u_f(x/\theta_f) + y_m^{(n)} + k_d s + \sum_{i=1}^{n-1} c_i e_{i+1}\right]. \qquad (5)$$

Also, to overcome continual improper modification of the parameters of fuzzy system, we consider an adaptive law as follows.

$$\begin{aligned} \dot{\theta}_f &= -\eta_1 \cdot (\dot{s}+\lambda s) \cdot \xi_f(x), \\ \dot{\theta}_g &= -\eta_2 \cdot (\dot{s}+\lambda s) \cdot \xi_g(x) \cdot u_c, \end{aligned} \qquad (6)$$

where λ is a positive constant.

Then we can obtain the following *Theorem 1*.

Theorem 1: Consider the nonlinear system (2) with the control law (5) and the adaptive law(6). Then, the tracking error is uniformly bounded. If the extended fuzzy basis vector is persistently exciting, the parameter error vectors ϕ_f, ϕ_g are also bounded.

4. Simulation Results

We apply the proposed hierarchical adaptive fuzzy controller to the balancing and the position control of the inverted pendulum system. Let the initial state $x(0)=(8°, 0, -20, 0)^T$ and the parameter vectors $\theta_f(0)=0$, $\theta_g(0)=1.2$. We choose $\eta_1=1.0$, $\eta_2=0.01$ and $\lambda=1$. We directly integrate the differential equations of the closed-loop system and the adaptive law with step size 0.01. The simulation results are shown in Fig. 4. These results show that the proposed adaptive fuzzy controller can be successfully applied to the balancing and the regulation of the inverted pendulum. Fig. 5 shows the position state trajectory in the phase plane. Fig. 6 shows the angle state trajectory in the phase plane. The results show that the proposed adaptive controller can balance the pendulum near the specified position.

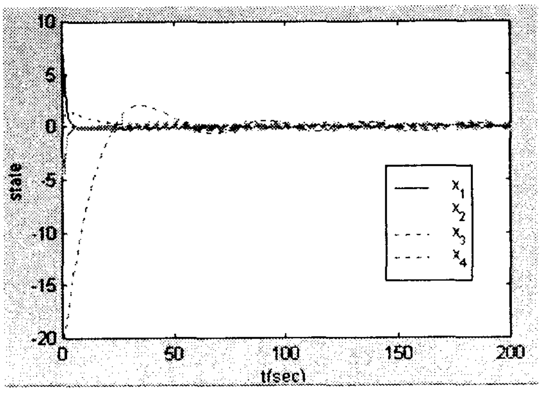

Fig 4. Simulation results for pendulum balancing and cart regulation.

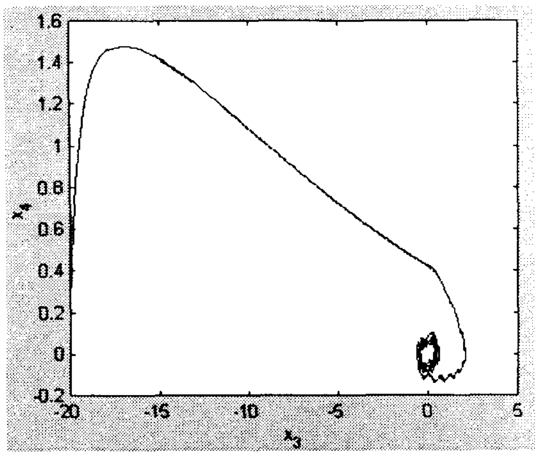

Fig 5. Position state trajectory in the phase plane.

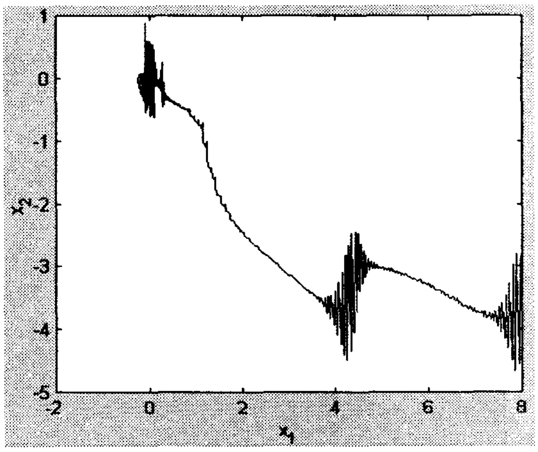

Fig 6. Angle state trajectory in the phase plane.

5. Conclusions

In the paper, a hierarchical adaptive fuzzy controller for balancing and position control of the inverted pendulum system is proposed. To satisfy two control objectives, the hierarchical fuzzy controller consists of an indirect adaptive fuzzy controller for balancing, a forced disturbance generator which emulates heuristic control strategy. Also, it is proved that all the signals in the inverted pendulum system are bounded. Simulation example shows the effectiveness of the proposed method.

References

[1] K. Furuta, K. Hiroyuki and K. Kosuge, ``Digital Control of a Double Inverted Pendulum on an Inclined Rail,'' *Int. Journal of Control*, Vol. 32, No. 5, pp. 907-924, 1980.

[2] S. Mori, H. Nishihara and K. Furuta, `` Control of Unstable Mechanical System - Control of Pendulum,'' *Int. Journal of Control}* Vol. 23, No. 5, pp. 673-692, 1976.

[3] K. Furuta, T. Ochiai and N. Ono, ``Attitude Control of a Triple Inverted Pendulum,'' *Int. Journal of Control*, Vol. 39, No. 6, pp. 1351-1365, 1984.

[4] K. Furuta, M. Yamakita and S. Kobayashi, ``Swing Up Control of Inverted Pendulum ,'' *Proc. of IEEE Int. Conf. on Industrial Electronics*, pp. 2193-2198, 1991.

[5] Y. Xia and H. Inooka, ``Application of Tree Search to the Switching Control of a Pendulum,'' *IEEE Tr. on Systems, Man, and Cybernetics*, Vol. 22, No. 5, pp. 1193-1198, 1992.

[6] S. Geva and J. Sitte, ``A Cartpole Experiment Benchmark for Trainable Controllers,'' *IEEE Control System Magazine*, pp. 40-51, 1993.

[7] A.G. Barto, R.S. Sutton and C.H. Anderson, ``Neuronlike Adaptive Elements That Can Solve Difficult Learning Control Problems,'' *IEEE Trans. Sys. Man and Cyb.*, Vol. 13, No. 5, pp. 834-846, 1983.

[8] B. Widrow, ``The Original Adaptive Neural Net Broom-Balancer,'' *Proc. IEEE Int. Symp. Circuits and Systems*, pp. 351-357, 1987.

[9] R. Hecht-Nielsen, *Neurocomputing*, Reading, MA: Addison-Wesley, pp. 342-345, 1990.

[10] C.C. Lee and H.R. Berenji, ``An Intelligent Controller Based on Approximate Reasoning and Reinforcement Learning,'' *Proc. of IEEE Int. Sysmp. on Intelligent Control*, pp. 200-205, 1989.

[11] Y.K. Ha and A. Tomizuka, ``Fuzzy Global and Local Motion Control of an Inverted Pendulum Using Multiple Rule Bases,'' *Proc. of Asian Control Conf.*, July, pp. 27-30, 1994.

[12] L.X. Wang, ``Stable Adaptive Fuzzy Control of Nonlinear Systems,'' *IEEE Tr. on Fuzzy Systems*, Vol. 1, No. 2, pp. 146-155, 1993.

[13] Y.T. Kim and Z. Bien, ``Robust self-leaning fuzzy logic controller design for a class of nonlinear MIMO systems,'' Fuzzy Sets and Systems, Vol. 111, pp. 117-135, 2000.

[14] Y.T. Kim and Z. Bien, ``Robust Adaptive Fuzzy Control in the Presence of External Disturbance and Approximation Error,'' will be published in Fuzzy Sets and Systems, 2004.