연속시간 TS 퍼지 시스템의 카오스화

Anticontrol of Chaos for a Continuous-Time TS Fuzzy System

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Abstract

In this paper, a systematic design approach based on parallel distributed compensation techniques is proposed for anticontrol of chaos in a general continuous-time Takagi-Sugeno (TS) fuzzy system. The verification of chaos in the controlled continuous-time TS fuzzy system is done by the following procedure. First, we establish an asymptotically approximate relationship between a continuous-time TS fuzzy system with time-delay and a discrete-time TS fuzzy system. Then Marotto theorem is applied. The boundedness in the controlled continuous-time TS fuzzy system is also proven via its associated discrete-time TS fuzzy system.

Key Words : Fuzzy system, Anticontrol of chaos, Chaotification.

1. Introduction

The "anticontrol of chaos" or "chaotification" means making a nonchaotic system chaotic or keeping existing chaos of a chaotic system. We propose the chaotification method for a general continuous-time TS fuzzy system by using the method proposed for the linear system in [1]. Specifically, the TS fuzzy system is composed of the fuzzy rules, which characterize local relations of the system in the state space. The local relations are sometimes called the subsystem of the TS fuzzy system. We apply the method proposed in [1] to make each subsystem chaotic. Later we will see that this, also, makes the overall TS fuzzy system chaotic in the sense of Li and Yorke.

The fuzzy parallel distributed compensation controller (FPDCC) is designed with the feedback gain and the local time-delay feedback for efficient anticontrol of chaos in a general continuous-time TS fuzzy system. The feedback gain and the local time-delay feedback are used in making each subsystem stable and chaotic, respectively. In effect, this controller makes the overall TS fuzzy system bounded and chaotic. The verification of chaos in the closed loop TS fuzzy system is done by the following procedure. We, first, establish an asymptotically approximate relationship between the continuous-time TS fuzzy system with time-delay and discrete-time TS fuzzy system. Then Marotto theorem is applied. Therefore the generated chaos is in the sense of Li and Yorke. The boundedness in the closed loop TS fuzzy system is also proven via its associated discrete-time TS fuzzy system.

This paper is organized as follows. Section 2 briefly reviews the continuous-time TS fuzzy system and an asymptotically approximate relationship between the time-delay differential equation and the difference equation established in [1]. The FPDCC design for anticontrol of chaos in a general continuous-time TS fuzzy system and verification of chaos and boundedness in the controlled TS fuzzy system is presented in Sections 3, 4 respectively. Some conclusions are finally given in Section 5.

2. Preliminary

2.1 The continuous-time TS fuzzy system

A general single input TS fuzzy system is described as follows:

Plant Rule i:

IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in},

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$ (1)

The final output of the fuzzy system is inferred by

$$\dot{x}(t)=\sum_{i=1}^{r}\mu_i\{A_i x(t)+B_i u(t)\} \qquad (2)$$

in which μ_i can be regarded as the firing strength of the IF-THEN rules.

Throughout this paper, we denote x^j is the j *th* element of the vector x, T^{ij} is the ij *th* element of the matrix T, and T^{*j} is the j *th* column of the matrix T.

2.2 Approximate relationship between the delayed-differential equation and the difference equation.

Let us consider an n *th* order single-input single-output (SISO) stable linear time-invariant (LTI) system described by phase variable form.

$$\dot{x}(t)=Ex(t)+D\nu(t), \qquad \nu(t)=h(x(t-\tau)^1) \qquad (3)$$

where

$$E=\begin{bmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1}\end{bmatrix}, \qquad D=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}$$

and $\nu(t)$ is the time-delay feedback input, $x(t)^1$ is a system output, and $h(\cdot)$ is a continuous scalar function with bounded magnitude.

Since the system (3) is assumed to be a stable system, the system matrix E is a Hurwitz stable matrix. This LTI system can be computed iteratively on each τ-time interval $(m\tau,(m+1)\tau]$ for $m=0,1,\cdots$. Denote $x(t)\equiv x(m\tau+\bar{t})\equiv x(m,\bar{t})$ for $t=m\tau+\bar{t},\bar{t}\in(0,\tau]$. It follows that

$$x(m,\bar{t})=e^{E\bar{t}}x(m-1,\tau) + \int_0^{\bar{t}} e^{E(\bar{t}-t')}Dh(x(m-1,t')^1)dt' \qquad (4)$$

Lemma 1 [1] *For a sufficiently large* τ *and a large* $\bar{t}\in(t_0,\tau]$, (4) *can be approximate to the following.*

$$x(m,\bar{t})^1\approx a_0^{-1}h(x(m-1,\bar{t})^1), \quad x(m,\bar{t})^d\approx 0$$
$$\text{for } m=0,1,\cdots \text{ and } d=2,\cdots,n$$

3. FPDCC Design for Chaotifying Continuous-Time TS Fuzzy System

For a general continuous-time TS fuzzy system, FPDCC is designed to make it chaotic. Each control rule of FPDCC is constructed from the corresponding rule of the TS fuzzy system.

Control Rule i:

IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in},
THEN $u(t)=-Kx(t)+\nu_i(t)$ (5)

where K is the feedback gain matrix and $\nu_i(t)$ is the local time-delay feedback,

$$\nu_i(t)=h((T_i^{-1}x(t-\tau))^1) \qquad (6)$$

The overall fuzzy controller is inferred by

$$u(t)=\sum_{i=1}^{r}\mu_i\{-Kx(t)+\nu_i(t)\} \qquad (7)$$

Substituting (7) into (2) we obtain the closed loop system

$$\dot{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{G_i x(t)+B_i\nu_j(t)\} \qquad (8)$$

where $G_i=A_i-B_iK$.

The equation (8) can be seen the final output of the following continuous-time TS fuzzy system, which has r^2 subsystems.

Rule ii:

IF $z_1(t)$ is M_{i1} and $M_{j1}\cdots$and $z_n(t)$ is M_{in}, and M_{jn},
THEN $\dot{x}(t)=G_i x(t)+B_i\nu_j(t)$ (9)

where G_i is the system matrix of the subsystem of (9).

3.1 The design of the feedback gain matrix

The feedback gain is designed to make that the system matrices of each subsystem of (9), G_i, are Hurwitz stable matrices. To this purpose we use a linear matrix inequality (LMI) method.

Theorem 1 [5] *The feedback gain matrix* K *is designed from the following LMI to make that* G_i *are Hurwitz stable matrices.*

$QA_i^T-M^TB_i^T+A_iQ-B_iM<0, \quad i=1,\cdots,r$

$Q>0$

where $Q=P^{-1}$, $M=KQ$,

where P *and* Q *are positive symmetric matrices.* K *is solved by* $K=MP$.

3.2 The design of the local time-delay feedback

Here, we design the local time-delay feedback $\nu_i(t)$ that makes the each subsystem chaotic, respectively. Later in Section 4, we will see that this, $\nu_i(t)$, also makes the closed loop system (8), (9) chaotic in the sense of Li and Yorke.

Since G_i are Hurwitz stable matrices and (G_i, B_i) is controllable, each subsystem

$$\dot{x}(t) = G_i x(t) + B_i \nu_j(t), \quad i,j=1,2,\cdots,r \tag{10}$$

can be converted to phase variable form.

Define a new state vector $\hat{x}_{ij}$ by

$$x = T_i \hat{x}_{ij}$$

Then,

$$\begin{aligned}\dot{\hat{x}}_{ij}(t) &= T_i^{-1} G_i T_i \hat{x}_{ij}(t) + T_i^{-1} B_i \nu_j(t) \\ &= E_i \hat{x}_{ij}(t) + D_i \nu_j(t), \quad i,j=1,2,\cdots,r \end{aligned} \tag{11}$$

where

$$E_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{i0} & -a_{i1} & -a_{i2} & \cdots & -a_{i(n-1)} \end{bmatrix}, \quad D_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Theorem 2 *There is an asymptotically approximate relationship between time-delay differential equations* (11) *and the following difference equations.*

$$\begin{aligned} i=j \quad & (\hat{x}_{ii}(k+1))^1 \approx \frac{1}{a_{i0}} h((\hat{x}_{ii}(k))^1) \\ i\neq j \quad & (\hat{x}_{ij}(k+1))^1 \approx \frac{1}{a_{i0}} h((T_j^{-1} T_i)^{11} (\hat{x}_{ij}(k))^1) \end{aligned} \tag{12}$$

Proof: The proof is omitted due to lack of space. ■

Theorem 3 *If the difference equations* (12) *are bounded chaotic maps by* $h(\cdot)$ *and time-delay* τ *is sufficiently large, each subsystem* (10) *is chaotic with time-delay feedback controller* (6).

Proof: The proof is omitted due to lack of space. ■

Clearly, $h(\cdot)$ which makes all difference equations (12) chaotic is not unique. But in this paper we select $h(\cdot)$ a simple sinusoidal function which was used as a chaotic map in [1], [2] to proof the overall continuous-time TS fuzzy system (8), (9) is chaotic. The proofs for other $h(\cdot)$ need to be further study.

$$\nu_i(t) = h((T_i^{-1} x(t-\tau))^1) = \sigma \sin\left(\frac{\beta\pi}{\sigma} (T_i^{-1} x(t-\tau))^1\right) \tag{13}$$

4. Verification of choas in the controlled TS fuzzy system

First, we establish an asymptotically approximate relationship between the continuous-time TS fuzzy system with time-delay and the discrete-time TS fuzzy system. Then we show the continuous-time TS fuzzy system (8) or (9) is chaotic as proving that its associate discrete-time TS fuzzy system is chaotic by Marotto theorem. The boundedness of (8) or (9) is also shown via its associated discrete -time TS fuzzy system.

Theorem 4 *There is an asymptotically approximate relationship between the continuous-time TS fuzzy system with time-delay* (11) *and the following discrete -time TS fuzzy system if the time-delay* τ *is a sufficiently large.*

Rule ii $(i=j)$:

IF $z_1(k)$ is M_{i1} and M_{j1} ⋯and $z_n(k)$ is M_{in}, and M_{jn},

THEN $$x(k+1) \approx \begin{bmatrix} \frac{(T_i)^{11}}{a_{i0}} h\left(\frac{1}{(T_i)^{11}} x_1(k)\right) \\ \frac{(T_i)^{21}}{a_{i0}} h\left(\frac{1}{(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{(T_i)^{n1}}{a_{i0}} h\left(\frac{1}{(T_i)^{n1}} x_n(k)\right) \end{bmatrix}.$$

Rule ij $(i\neq j)$:

IF $z_1(k)$ is M_{i1} and M_{j1} ⋯and $z_n(k)$ is M_{in}, and M_{jn},

THEN $$x(k+1) \approx \begin{bmatrix} \frac{(T_i)^{11}}{a_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{11}} x_1(k)\right) \\ \frac{(T_i)^{21}}{a_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{(T_i)^{n1}}{a_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{n1}} x_n(k)\right) \end{bmatrix}$$

The final output of this discrete-time TS fuzzy system is inferred by

$$x(k+1) \approx \sum_{i=1}^{r} \mu_i^2 \begin{bmatrix} \frac{(T_i)^{11}}{\alpha_{i0}} h\left(\frac{1}{(T_i)^{11}} x_1(k)\right) \\ \frac{(T_i)^{21}}{\alpha_{i0}} h\left(\frac{1}{(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{(T_i)^{n1}}{\alpha_{i0}} h\left(\frac{1}{(T_i)^{n1}} x_n(k)\right) \end{bmatrix}$$

$$+ \sum_{i=1}^{r} \sum_{j=1, j \neq i}^{r} \mu_i \mu_j \begin{bmatrix} \frac{(T_i)^{11}}{\alpha_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{11}} x_1(k)\right) \\ \frac{(T_i)^{21}}{\alpha_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{(T_i)^{n1}}{\alpha_{i0}} h\left(\frac{(T_j^{-1}T_i)^{11}}{(T_i)^{n1}} x_n(k)\right) \end{bmatrix} \quad (14)$$

Proof: The proof is omitted due to lack of space. ■

Theorem 5 (boundedness) *If* $|h(\cdot)| \le \sigma$, *the final output of discrete-time TS fuzzy system* (13) *is bounded by the following constant.*

$$x(k) \le \begin{bmatrix} \max_{1\le i\le n}\left\{\left|\frac{(T_i)^{11}}{\alpha_{i0}}\right|\right\}\sigma \\ \max_{1\le i\le n}\left\{\left|\frac{(T_i)^{21}}{\alpha_{i0}}\right|\right\}\sigma \\ \vdots \\ \max_{1\le i\le n}\left\{\left|\frac{(T_i)^{n1}}{\alpha_{i0}}\right|\right\}\sigma \end{bmatrix} < \infty \quad , \quad k=1,2,\cdots$$

Proof: The proof is omitted due to lack of space. ■

Now, we present that (14) is chaotic in the sense of Li and Yorke when $h(\cdot)$ is (13). Substituting (13) to (14), we obtain

$$x(k+1) \approx \sum_{i=1}^{r} \mu_i^2 \begin{bmatrix} \frac{\sigma(T_i)^{11}}{\alpha_{i0}} \sin\left(\frac{\beta\pi}{\sigma(T_i)^{11}} x_1(k)\right) \\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}} \sin\left(\frac{\beta\pi}{\sigma(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}} \sin\left(\frac{\beta\pi}{\sigma(T_i)^{n1}} x_n(k)\right) \end{bmatrix}$$

$$+ \sum_{i=1}^{r} \sum_{j=1, j \neq i}^{r} \mu_i \mu_j \begin{bmatrix} \frac{\sigma(T_i)^{11}}{\alpha_{i0}} \sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{11}} x_1(k)\right) \\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}} \sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{21}} x_2(k)\right) \\ \vdots \\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}} \sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{n1}} x_n(k)\right) \end{bmatrix} \quad (15)$$

Theorem 6 *Suppose that* $\mu_i(k)\mu_j(k)$, $i,j=1,2,\cdots,r$ *are continuously differentiable in the neighborhood of the fixed point,* $x^*=0$, *of the controlled system* (15). *Then there exists a positive constant* $\bar{\beta}$ *such that if* $\beta > \bar{\beta}$, *then the controlled discrete-time TS fuzzy system* (15) *is chaotic in the sense of Li and Yorke.*

Proof: The proof is omitted due to lack of space. ■

5. Conclusion

A systematic approach for anticontrol of chaos in a general continuous-time TS fuzzy system was developed in this paper. The FPDCC is very simple. It is composed of two parts. One is the feedback gain to make each subsystem of TS fuzzy system stable. The other is the local time-delay feedback to make it chaotic. As a result, the overall TS fuzzy system becomes to be bounded and chaotic. We only prove it when bounded maps are sinusoidal maps. For the other bounded chaotic map, more study is needed.

Reference

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