Fundamental thermodynamic concepts for the constitutive modeling of damaged concrete

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ABSTRACT

Many damage models has been developed to express the degradation of materials. However, only minor damage model for concrete has been developed because of the heterogeneity of it unlike metals. To model the damaged behavior of concrete, this peculiarity as well as a load-induced anisotropic feature must be considered. In this paper, basic concepts of the thermodynamic theory is investigated to model the behavior of the damaged concrete in the phenomenological viewpoint. And the general constitutive relations and damage evolution equations are investigated too.

1. Introduction

Concrete is a strongly heterogeneous material which exhibits several mutually interacting inelastic mechanisms such as microcrack growth and plastic flow. This peculiarity is attributed that concrete is a composite material comprising two main constituents such as cement paste and aggregate. Concrete is characterized by elastic behavior coupled with a nonlinear stress-strain relation because of the presence of interfaces between cement paste and aggregate and the development of bond microcracks at those interfaces. Moreover, it was revealed that the extension of microcracks brings about the remarkable inelasticity and interacts with the plasticity of the material.

As result, the damaged concrete under service conditions is strongly induced anisotropic behaviour. A few papers was published to account for induced anisotropy[1,2]. Generally, the damage of concrete is modelled as elastic-brittle[3,4] to represent the nucleation and growth of microscopic cracks caused by elastic deformations. Many researchers have studied about the constitutive modelling of the damaged quasi-brittle materials such as rock and concrete using thermodynamic theories of anisotropic damage[3,5], energy-based-elastic-plastic damage model[6] and the coupling between elastic-plastic deformation and anisotropic damage[7]. And the degradation of effective stiffness and compliance of the damaged concrete is described with damage evolution.

In the present paper, some fundamental concepts of anisotropic damage behavior of concrete was developed within the framework of thermodynamics. It was assumed that concrete is elastic-brittle material, small deformations and isothermal conditions.

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2. General thermodynamics of the damaged material

In the case of infinitesimal deformation, the Clausius-Duhem inequality is derived from the second principle of thermodynamics as follows:

$$\sigma : \dot{\varepsilon} - \rho(\dot{\Psi} + s\dot{T}) - \frac{q}{T} \cdot \nabla T \geq 0 \quad (1)$$

where σ and $\dot{\varepsilon}$ are the Cauchy stress and strain tensor and ρ, Ψ, s, T and q are mass density, Helmholtz free energy, specific entropy, absolute temperature and heat flux, respectively. The symbol : is a tensorial product contracted on two indices.

Generally, the state variables, also called thermodynamic or independent variables, are defined to describe physical phenomena of material in the phenomenological viewpoint: observable variables T, ε and internal variable ε^e and ε^{ie}, defined the elastic and inelastic strain tensor, respectively, and V_k. $V_k\{k=1,2,3...\}$ is the variable that defined by experience and physical feeling and the type of application when needed.

The total strain tensor ε can be decomposed into elastic and inelastic strain tensor ε^e, ε^{ie} as:

$$\varepsilon = \varepsilon^e + \varepsilon^{ie} \quad (2)$$

The Helmholtz free energy is defined as follows:

$$\Psi = \Psi(\varepsilon^e, V_k, T) \quad (3)$$

Substituting (2) and (3) into the Clausius-Duhem inequality, the state laws are derived as:

$$\sigma = \rho\frac{\partial\Psi}{\partial\varepsilon^e}, \quad s = -\frac{\partial\Psi}{\partial T}, \quad A_k = \rho\frac{\partial\Psi}{\partial V_k} \quad (4a, 4b, 4c)$$

where A_k is the thermodynamic conjugate forces corresponding to V_k.

In addition to the dissipation potentials must be defined to express dissipation processes by the evolution of the internal damage of material. Let $g = \nabla T$. Taking into account the state laws, it is derived from the Clausius-Duhem inequality as:

$$\Phi = \sigma : \dot{\varepsilon}^{ie} - A_k \cdot \dot{V}_k - \frac{g}{T} \cdot q \geq 0 \quad (5)$$

where Φ is a sum of Φ_1 and Φ_2, called the mechanical dissipation and the thermal dissipation, and defined respectively as follows:

$$\Phi_1 = \sigma : \dot{\varepsilon}^p - A_k \cdot \dot{V}_k, \quad \Phi_2 = -\frac{g}{T} \cdot q \quad (6a, 6b)$$

3. Application to concrete

Helmholtz free energy is based on a function of the elastic strain tensor ε^e, the second order damage tensor $\mathbf{D}$ and another damage scalar β.

The damage variable $\mathbf{D}$ is defined by the second order symmetric damage tensor to express the anisotropic damage process of concrete such as[8,9]:

$$D = \sum_{i=1}^{3} D_i n_i \otimes n_i \tag{7}$$

where D_i and n_i are principal values and the unit vector of principal directions of the damage tensor **D**.

Then, Helmholtz free energy is postulated as follows[10]:

$$\rho\Psi(\varepsilon^e, \mathrm{D}, \beta) = \rho\Psi^e(\varepsilon^e, \mathrm{D}) + \rho\Psi^d(\beta) \tag{8}$$

$$\rho\Psi^e(\varepsilon^e, \mathrm{D}) = \frac{1}{2}\lambda(tr\varepsilon^e)^2 + \mu\, tr(\varepsilon^e)^2 + \eta_1\, tr\mathrm{D}\,(tr\varepsilon^e)^2 + \eta_2\, tr\mathrm{D}\, tr(\varepsilon^e)^2 + \eta_3\, tr\varepsilon^e\, tr(\varepsilon^e : \mathrm{D}) + \eta_4\, tr[(\varepsilon^{e*})^2 : \mathrm{D}] \tag{9}$$

$$\rho\Psi^d(\beta) = \frac{1}{2}K_d\beta^2 \tag{10}$$

where $\lambda = E\nu/(1+\nu)(1-2\nu)$ and $\mu = E/2(1+\nu)$ are Lamè coefficients for undamaged materials, η_1, η_2, η_3, η_4 and K_d are material constants and ε^{e*} is a modified elastic strain tensor used to represent the unilateral damage response. It is defined as follows[11]:

$$\varepsilon^{e*} = \langle\varepsilon^e\rangle - \zeta\langle-\varepsilon^e\rangle \tag{11}$$

$$[\langle\varepsilon^e\rangle] = \begin{bmatrix} \langle\varepsilon_1\rangle & 0 & 0 \\ 0 & \langle\varepsilon_2\rangle & 0 \\ 0 & 0 & \langle\varepsilon_3\rangle \end{bmatrix}, \quad [\langle-\varepsilon^e\rangle] = \begin{bmatrix} \langle-\varepsilon_1^e\rangle & 0 & 0 \\ 0 & \langle-\varepsilon_2^e\rangle & 0 \\ 0 & 0 & \langle-\varepsilon_3^e\rangle \end{bmatrix} \tag{12a, 12b}$$

4. Constitutive and damage evolution equations

Using the state laws (4a) and (12), the constitutive equations of anisotropic elasticity coupled with damage is derived as follows:

$$\begin{aligned}\sigma &= \frac{\partial(\rho\Psi)}{\partial\varepsilon^e} = \frac{\partial(\rho\Psi^e)}{\partial\varepsilon^e} = \mathbf{C} : \varepsilon^e \\ &= [\lambda\, tr\varepsilon^e + 2\eta_1\, tr\mathrm{D}\, tr\varepsilon^e + \eta_3\, tr(\varepsilon^e : \mathrm{D})]\mathbf{1} + 2(\mu + \eta_2\, tr\,\mathrm{D})\varepsilon^e \\ &\quad + \eta_3(tr\varepsilon^e)\mathrm{D} + \eta_4\frac{\partial\varepsilon^{e*}}{\partial\varepsilon^e}(\varepsilon^{e*} : \mathrm{D} + \mathrm{D} : \varepsilon^{e*})\end{aligned} \tag{13}$$

$$\mathbf{Y} = -\frac{\partial(\rho\Psi)}{\partial\mathrm{D}} = -\frac{\partial(\rho\Psi^e)}{\partial\mathrm{D}} = -\left[\eta_1(tr\varepsilon^e)^2 + \eta_2\, tr(\varepsilon^e)^2\right]\mathbf{1} - \eta_3(tr\varepsilon^e)\varepsilon^e - \eta_4\varepsilon^{e*} : \varepsilon^{e*} \tag{14}$$

$$B = \frac{\partial(\rho\Psi)}{\partial\beta} = \frac{\partial(\rho\Psi^d)}{\partial\beta} = K_d\beta \tag{15}$$

where **C** is the fourth order symmetric tensor as a function of the second order damage tensor **D** and thermodynamic conjugate force **Y**, the derivative of Helmholtz free energy with respect to the damage variable **D**, is known as the damage strain energy release rate.

The existence of a damage criterion in the space of the thermodynamic conjugate forces $\{\mathbf{Y}, -B\}$ is also assumed such as:

$$F(\mathbf{Y}, B) = Y_{eq} - (B_0 + B) = 0 \tag{16}$$

$$Y_{eq} = \left(\frac{1}{2}\mathbf{Y} : \mathbf{L} : \mathbf{Y}\right)^{1/2} \tag{17}$$

where B_0 is a material constant and represents the initial threshold of the damage evolution and

$\mathbf{L}$ is the fourth order tensor which depend on the state of damage $\mathbf{D}$. However, according to the assumption of (17), $\mathbf{L}$ may be expressed the isotropic tensor as follows:

$$L_{ijkl} = 1/2\,(\,\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk}\,) \tag{18}$$

Therefore, the evolution equations are finally established by the dissipation potential of eq. (16) as follows:

$$\dot{\mathbf{D}} = -\,\dot{\lambda}_d \frac{\partial F}{\partial \mathbf{Y}} \qquad \dot{\beta} = \dot{\lambda}_d \frac{\partial F}{\partial(-B)} = \dot{\lambda}_d \tag{19a, 19b}$$

$$\dot{\lambda}_d = \frac{\left(\dfrac{\partial F}{\partial \mathbf{Y}} : \dot{\mathbf{Y}}\right)}{\left(\dfrac{\partial B}{\partial \beta}\right)} = \alpha \frac{\mathbf{L} : \mathbf{Y}}{2K_d\, Y_{eq}} : \dot{\mathbf{Y}} \tag{20}$$

where $\dot{\lambda}_d$ is a multiplier to be determined by the consistency condition on the damage surface and $\alpha = 1$ if $F = 0$ and $\partial F/\partial \mathbf{Y} : \dot{\mathbf{Y}} > 0$ or $\alpha = 0$ if $F < 0$ and $\partial F/\partial \mathbf{Y} : \dot{\mathbf{Y}} \leq 0$.

4. Discussion

Fundamental irreversible thermodynamics concepts is investigated to develope the constitutive and damage evolution equations for damaged concrete. Concrete is assumed a elastic-brittle material. Helmholtz free energy is postulated using three state variables. Although a unilateral load effect is considered in this paper, different load conditions must be considered for a further development of modeling of damaged concrete. Moreover, material constants must be examined with care to reduce an error between real behavior and model.

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