Shear Strength Prediction by Modified Plasticity Theory for High-Strength Concrete Deep Beams

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ABSTRACT

This paper presents the analysis results predicted by the upper bound approach in the limit analysis of concrete incorporating the original plastic and crack sliding solutions for short high-strength concrete beams that varied the compressive strength of concrete, and the shear span-to-depth and vertical shear reinforcement ratios. The significance of the distance away from the support to define the location where the yield line starts and the properties of cracked concrete, particularly related to high-strength concrete, is identified.

1. INTRODUCTION

As the beam becomes shorter or deeper, the stress distribution becomes nonlinear with the tensile stresses concentrating toward the bottom of the beam, thus the stresses at midspan deviate more and more from those predicted by the plane section theory. Examination of the experimental results revealed that beams with shear span-to-depth ratios less than about 2.5 carry the load principally by strut-and-tie action. In this range, the strength of the beams decreases rapidly as the shear span increases, and further it is strongly influenced by details such as the size of the bearing plates supporting the beam.

Recent work[1] on concrete shear problems however reported that slip along the cracks can delay or prevent the development of direct strut action spanning between the loading and supporting points of short beams. Furthermore, measurements of the relative displacements along a diagonal crack carried out by Muttoni[2] demonstrated that at failure the relative displacements are not perpendicular to the crack direction, rather they take an angle smaller than 90 degrees. These aspects certainly imply that sliding or shear displacements can occur along the crack, and the yield

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line, in view of concrete plasticity, can be introduced along the existing crack which is not necessarily linked between the loading and supporting points. Accordingly, the yield line might traverse several cracks, so its sliding resistance containing many cracks would be substantially reduced compared with the one in a state of uncracked concrete.

Hence, the present paper reinvestigates the previous work[3] that included two series of thirty high-strength concrete short beam tests, and some findings which were not recognized previously can be explored to lead to a better understanding of the behavior involved in high-strength short beams.

2. CRACK SLIDING SOLUTIONS

To treat the influence of cracking in beams without transverse reinforcement more rationally, Zhang[4] considered the critical diagonal crack which appeared near the collapse with a finite distance from the support (a-x) rather than the direct connection to the support, as shown in Fig. 1, and also developed the simple method of determining the critical cracking load according to a plastic stress distribution in analogy to an elastic one. Following the usual procedure of the failure mechanism approach, the upper bound solution corresponding to the yield line formed along the crack and some distance away from the support was derived. Then, the condition that the ultimate load equals the cracking load, only valid for slender beams, was finally imposed to determine the finite crack distance and the corresponding solution with a state of cracked concrete can be derived.

Similarly, for beams with transverse reinforcement, Hoang[5] extended the above solution by considering the plane strain state that resulted an additional constraint on the inclination of the yield line. It was also postulated that the ultimate load is greater than the critical cracking load, and the development of cracks with all possible inclinations are permitted instead of only those with finite distances from the support. Disregarding the contribution of one stirrup to treat the case of largely spaced stirrups, the more succinct and practical form of final results for the longitudinally over-reinforced case can be written.

However, as pointed out previously, it is also possible that the yield line develop following a critical crack path or crossing many cracks, resulting in a reduced sliding resistance at the crack interface compared with that formed in the uncracked concrete. To this, Zhang recommended the value of cohesion being 0.5 in addition to ν_0 in which the effect of the shear span-to-depth ratio is excluded since it is regarded as the effect of cracking.

Like slender beams, if the yield line is introduced along or crossing the crack in short beams, similar expressions corresponding to the original plastic solutions can be derived, simply replacing ν by $\nu_0\nu_s$. In doing so, it is noticed that most beams in the normal configuration belong to the longitudinally over-reinforced case, thereby the failure mechanisms previously suggested involving the longitudinally under-reinforced case are somewhat ambiguous. In the present study, this related solution is regarded as another version of the crack sliding solution given below.

3. PREDICTION OF TEST RESULTS

Two series of thirty high-strength short beams (52 MPa and 73 MPa,) were tested with the simply supported end condition subjected to a concentrated midspan load(see Fig. 2).[3] Each series consisted of three groups of specimens depending on the shear span-to-depth ratio such as 1.5, 2.0, and 2.5, and again each group contained five specimens varying of the traverse reinforcement ratio as 0%, 25%, 50%, 75% and 100% of that required by the ACI Code provisions for slender beams.

From the comparison between observed and predicted ultimate loads, it can be seen that the solutions based on the reduced sliding resistance correlate well with test results having a mean of 0.946 and a coefficient of variation (COV) of 13.5% in the Hoang method and a mean of 0.932 and a COV of 15.7% in the Zhang and related methods. It is interesting to note that the related solution reflecting only the reduced sliding resistance in the web crushing criterion exhibits comparable accuracy to the Hoang method for beams with stirrups. In contrast, the critical diagonal cracking loads predicted exhibit somewhat more scattered than one normally would expect. However, considering the highly variable nature of concrete tensile strength and the measurement procedure taken during tests, the ratio of the experimental to predicted cracking loads with a mean of 1.389 and a COV of 30.3% still seems to be acceptable.

4. CONCLUSIONS

The crack sliding and related solutions predicted test strength with reasonable accuracy, but much better than the original plastic theory. The development of yield line along the critical diagonal crack with some distance away from the support is more likely for deep beams, particularly for those with high-strength concrete due to the larger slip expected at the crack interface. The crack sliding is also of critical importance for beams with sufficient transverse reinforcement as well as beams without transverse reinforcement. When applying the crack sliding or related solution, most beams in the normal configuration are classified into the longitudinally over-reinforced case, thus the failure mechanisms previously suggested for the under-reinforced case in the uncracked concrete seem somewhat ambiguous. The related solution extended from the usual web crushing criterion, simply including the crack sliding, provided equally accurate results as the solution developed by Hoang for beams with stirrups.

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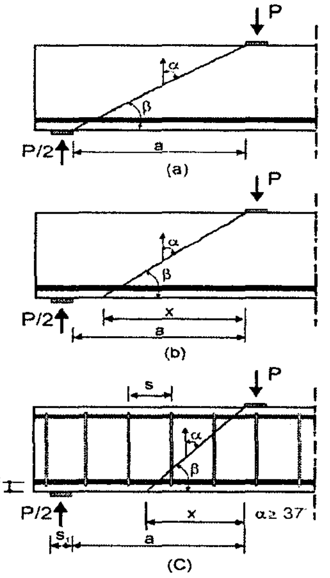
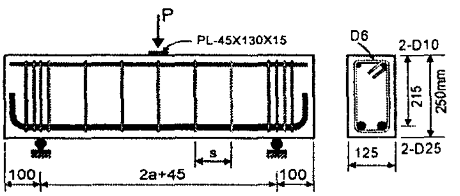


Fig. 1 Shear Failure Mechanisms Considered



Shown here for MHB or HB-1.5-25.
One stirrup at loading point consistently.
D6 (area=32mm²), D10 (area=71mm²) and D25 (area=507mm²).

Specimens	2a+45 (mm)	Stirrup no. and s (mm)
MHB or HB-1.5-25	645	7@100
MHB or HB-1.5-50		13@50
MHB or HB-1.5-75		19@33
MHB or HB-1.5-100		25@24
MHB or HB-2.0-25	860	7@140
MHB or HB-2.0-50		13@70
MHB or HB-2.0-75		19@47
MHB or HB-2.0-100		25@35
MHB or HB-2.5-25	1075	7@172
MHB or HB-2 5-50		13@86
MHB or HB-2.5-75		19@57
MHB or HB-2.5-100		25@43

Fig. 2 Test Specimen Details