샘플치 데이터 퍼지 시스템의 다중레이트 제어기

김도완*, 박진배*, 장권규**, 주영훈**
*연세대학교 전기전자 공학과 **군산대학교 전자정보공학부

Multirate Control of Sampled-Data Fuzzy System

Do Wan Kim*, Jin Bae Park*, Kwon Kyu Jang**, and Young Hoon Joo**
*Yonsei University **Kusan National University

Abstract - In this paper, a new multirate digital control technique for the Takagi-Sugeno (T-S) fuzzy system is suggested. The proposed method takes account of the stabilizablity of the discrete-time T-S fuzzy system at the fast-rate sampling points. Our main idea is to utilize the lifted control input. The proposed approach is to obtain the multirate discrete-time T-S fuzzy system by discretizing the overall dynamics of the T-S fuzzy system with the lifted control, and then to derive the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for this system. An example is provided for showing the feasibility of the proposed discretization method.

1. Introduction

In practice, there are not all systems in which the A/D and the D/A conversions are made uniformly at one single rate. The faster D/A converter is used to take into account of the effects of the intersampling behavior of the system. There are also situations where the converse is true. For example, it is difficult to implement antialiasing filters with long time constants using analog technique. In such cases, it is much easier to apply the faster A/D. Above and beyond these causes, formulating the faster D/A or the faster A/D aries from the hardware restrictions [1]. In both case, A/D and D/A converters are operated at different rates. This is called as *multirate control system*.

There have been fruitful researches in the digital control system focusing on the multirate sampling. Systems with multirate sampling were first analyzed in Kranc [2]. Additional researches are given in Jury [3,4], and Kalman and Bertram [5].

Motivated by the above observations, this paper aims at merging the Takagi-Sugeno (T-S) fuzzy model-based digital control and the multirate control technique for a class of nonlinear systems. The main contribution of this paper is to derive some sufficient conditions, in terms of the linear matrix inequalities (LMIs), such that the digitally controlled system is asymptotically stable at every intersampling points. Specifically, our main idea is to utilize the lifted control input. The proposed approach is to obtain the dual-rate discrete-time T-S fuzzy system by discretizing the overall dynamics of the T-S fuzzy system with the lifted control, and then to derive the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for this system. An example is provided for showing the feasibility of the proposed discretization method.

2. Preliminaries and Problem Description

The T-S fuzzy model is described by fuzzy IF-THEN rules, which represent local linear input-output relations of a nonlinear system. Consider the ith fuzzy rule of a sampled-data T-S fuzzy model with the sampling time T governed by

$$R_i: \text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{ip}$$
$$\text{THEN } \frac{d}{dt}x(t) = A_i x(t) + B_i u(t) \qquad (1)$$

where $R_i, i \in I_q = \{1, 2, \cdots, q\}$, is the ith fuzzy rule, $z_h(t), h \in I_p = \{1, 2, \cdots, p\}$, is the hth premise variable, $\Gamma_{ih}, (i, h) \in I_q \times I_p$, is the fuzzy set, and $u(t) = u(kT)$ is the piecewise-constant control input vector to be determined time interval $[kT, kT+T)$. . Given a pair $(x(t), u(t))$, Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of this samped-data T-S fuzzy model (1) is described by

$$x(t) = \sum_{i=1}^{q} \theta_i(z(t)) (A_i x(t) + B_i u(t)) \qquad (2)$$

where $w_i(z(t)) = \prod_{h=1}^{p} \Gamma_{ih}(z_h(t))$, $\theta_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}$,

and $\Gamma_{ih}(z_h(t))$ is the grade of membership of $z_h(t)$ in Γ_{ih}. Based on the PDC [12,13], we consider the following fuzzy digital control law for the fuzzy model (2):

$$R_i: \text{IF } z_1(kT) \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(kT) \text{ is } \Gamma_{ip}$$
$$\text{THEN } u(t) = K_{di} x(kT) \qquad (3)$$

for $t \in [kT, kT+T)$. The overall state feedback fuzzy-model-based digital control law is represented by

$$u(t) = \sum_{i=1}^{q} \theta_i(z(kT)) K_{di} x(kT) \qquad (4)$$

Problem 1: *Because the fuzzy model (2) is subject to the sampling time T, in general, stabilizing controller at the intersampling points do not exist. The aim of this paper is to design the digital control law (4) such that the closed-loop system is asymptotically stable at every intersampling points.*

3. Main Results

3.1 Fast Discretization for Continuous-Time T-S Fuzzy System

Our proposed method of descritizing the continuous-time T-S fuzzy system is to apply the fast discretization technique [6] to the T-S fuzzy system. The fast discretization leads to a dual-rate discrete-time system which can be lifted to a single-rate discrete-time system. Specifically, the continuous T-S fuzzy system is dicretized with fast-rate sampling $T_f\left(=\frac{T_s}{n}\right)$. For this discretized version, lifting the control input so that the lifted signals correspond to the slow-rate sampling T_s results in a lifted system. To maintain the polytopic structure of the lifted system for designing the digital fuzzy model-based controller, we utilize the following assumption.

Assumption 1 **[7,8]** *Suppose that the firing strength* $\theta_i(t)$ *for* $t\in[kT_s,(k+1)T_s)$ *is* $\theta_i(kT_s)$. *That is*

$$\theta_i(t)\approx\theta_i(kT_s) \qquad (5)$$

Then, the nonlinear matrices $\sum_{i=1}^{q}\theta_i(z(t))A_i$ *and* $\sum_{i=1}^{q}\theta_i(z(t))B_i$ *of (2) can be approximated as the piecewise constant matrices* $\sum_{i=1}^{q}\theta_i(z(kT_s))A_i$ *and* $\sum_{i=1}^{q}\theta_i(z(t))B_i$, *repectively.*

Theorem 1 *The sampled-data T-S fuzzy system (2) can be converted to the following pointwise dynamical behavior with a slow sampled system and a lifted sampled input.*

$$x[k+1]=G(\theta[k])x[k]+\tilde{H}(\theta[k])\tilde{u}[k] \qquad (6)$$

where $x[k]=x(kT_s)$, $u[k]=u(kT_s)$, $\theta[k]=\theta(z(kT_s))$,

$G(\theta[k])=G_f^n(\theta[k])=\left(\sum_{i=1}^{q}\theta_i[k]G_{fi}\right)^n$,

$H(\theta[k])=(G_f^{n-1}(\theta[k])+G_f^{n-2}(\theta[k])+\cdots+I)H_f(\theta[k])$,

$H_f(\theta[k])=\sum_{i=1}^{q}\theta_i(z(kT_s))H_{fi}$,

$G_{fi}=\exp(A_iT_f)$, $H_{fi}=(G_{fi}-I)A_i^{-1}B_i$, *a lifted sampled input* $\tilde{u}[k]$ *is defined as*

$$\tilde{u}[k]=\begin{bmatrix}\tilde{u}_1[k]\\ \tilde{u}_2[k]\\ \vdots\\ \tilde{u}_n[k]\end{bmatrix}=\begin{bmatrix}u(lT_f)\\ u((l+1)T_f)\\ \vdots\\ u((l+n-1)T_f)\end{bmatrix} \qquad (7)$$

and the matrices $G(\theta[k])$ *and* $\tilde{H}(\theta[k])$ *are given by*

$G(\theta[k])=G_f^n(\theta[k])$

$\tilde{H}(\theta[k])=[\,G_f^{n-1}(\theta[k])H_f(\theta[k])\;\; G_f^{n-2}(\theta[k])H_f(\theta[k])\;\cdots\; H_f(\theta[k])\,]$

Proof: The proof is omitted due to lack of space.■

Corollary 1 *The fast-sampled discrete-time system of (2) is obtained as follows:*

$$x[l+1]=G_f(\theta[l])x[l]+H_f(\theta[l])u[l] \qquad (12)$$

where $x[l]=x(lT_f)$ *and* $\theta[l]=\theta(lT_f)$.

Proof: When $n=1$ and $T=T_f$, it can be straightforwardly proved by Theorem 1. ■

3.2 Stability Conditions

In this subsection, we derive the stability conditions for the dual-rate T-S fuzzy system (6). Consider the open-loop system for (6).

$$x[k+1]=G(\theta[k])x[k] \qquad (13)$$

The following theorem gives a set of conditions for ensuring the stability of (13).

Theorem 2 *The equilibrium of (13) is globally asymptotically stable in the sense of Lyapunov stability criterion if there exists a common positive definite matrix* P *such that*

$$G_{fi}^TPG_{fi}^T-P<0 \qquad I\in[1,q] \qquad (14)$$

Proof: The proof is omitted due to lack of space.■

Remark 3 *From Theorem 2, we know that if* $G_f(\theta[l])$ *is globally asymptotically stable, so is* $G(\theta[k])$. *This is very useful property for the design of digital controller.*

3.3 Design of Stabilizable Digital Controller

Our main objective is to construct the stabilizable controller for (2) at fast-rate sampling points. We first design a stabilizable controller for the fast-sampled discrete-time system, and then convert the controlled system into (6).

We consider the following state feedback fuzzy control law for (12):

$$u[l]=\sum_{i=1}^{q}\theta[l]K_{di}x[l] \qquad (15)$$

Then, the closed-loop system can be rewritten as

$$x[l+1]=\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i[l]\theta_j[l](G_{fi}+H_{fi}K_{dj})x[l] \qquad (16)$$

The following theorem provides the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for (6).

Theorem 3 *For the dual-rate T-S fuzzy system (6), the closed-loop system under the state feedback controller law*

$$\tilde{u}[k]=K_d(\theta[k])\begin{bmatrix}I\\ (G_f(\theta[k])+H_f(\theta[k])K_d(\theta[k]))\\ \vdots\\ (G_f(\theta[k])+H_f(\theta[k])K_d(\theta[k]))^{n-1}\end{bmatrix}\times x[k] \qquad (17)$$

is globally asymptotically stabilizable in the sense of Lyapunov stability criterion if there exist symmetric positive definite matrix Q *and constant matrix* F *such that*

$$\begin{bmatrix}\left(\frac{G_{fi}Q+H_{fi}F_i-Q}{T_f}\right)^T+\frac{G_{fi}Q+H_{fi}F_i-Q}{T_f} & \star\\ \frac{G_{fi}Q+H_{fi}F_i-Q}{\sqrt{T_f}} & -Q\end{bmatrix}<0 \quad i\in[1,q]$$

$$\begin{bmatrix}\left(\frac{G_{fi}Q+H_{fi}F_j+G_{fj}Q+H_{fj}F_i-2Q}{2T_f}\right)^T+\frac{G_{fi}Q+H_{fi}F_j+G_{fj}Q+H_{fj}F_i-2Q}{2T_f} & \star\\ \frac{G_{fi}Q+H_{fi}F_j+G_{fj}Q+H_{fj}F_i-2Q}{2\sqrt{T_f}} & -Q\end{bmatrix}<0 \quad i<j\in[1,q] \qquad (18)$$

where ★ *denotes the transposed element in symmetric position.*

Proof: The proof is omitted due to lack of space.■

4. Computer Simulations

In this section, we use the results in Section 3 to digitally control the continuous-time T-S fuzzy system, which is the fuzzy model of the chaotic Lorenz equation. The Lorenz equation is given by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma x_1(t)+\sigma x_2(t) \\ rx_1(t)-x_2(t)-x_1(t)x_3(t) \\ x_1(t)x_2(t)-bx_3(t) \end{bmatrix} \tag{19}$$

where $\sigma, r, b>0$ are parameters σ is the Prandtl number, r is the Rayleigh number, and b is a scaling constant). The corresponding T-S fuzzy model of the system in (19) is expressed as follows:

R_1: IF $x_1(t)$ is about Γ_{11}

$$\text{THEN}\quad \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_1\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

R_2: IF $x_1(t)$ is about Γ_{21}

$$\text{THEN}\quad \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_2\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \tag{20}$$

where

$$A_1=\begin{bmatrix} \sigma & -\sigma & 0 \\ r & -1 & -x_{1\min} \\ 0 & x_{1\min} & -b \end{bmatrix},\quad A_2=\begin{bmatrix} \sigma & -\sigma & 0 \\ r & -1 & -x_{1\max} \\ 0 & x_{1\max} & -b \end{bmatrix} \tag{21}$$

and the membership functions are

$$\Gamma_1^1(x_1(t))=\frac{-x_1(t)+x_{1\max}}{x_{1\max}-x_{1\min}},\quad \Gamma_1^2(x_2(t))=\frac{x_1(t)-x_{1\min}}{x_{1\max}-x_{1\min}}. \tag{22}$$

where Γ_{ij} are positive semi-definite for all $x\in[x_{1\min}, x_{1\max}]=[-20,30]$.

First, we simulate the continuous-time T-S fuzzy system. The input matrices are arbitrary chosen as

$$B_1=B_2=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

where preserve the controllability of the system.

For the T-S fuzzy system (20) with (23), we seek to a stabilizing dual-rate digital controller (17), where n=1, 2, and 4. Applying Theorem 3 yields the digital gain matrices K_{di} for the sampling T_s=0.08 sec., as

wth n=1,

$$K_{d1}=[-17.0985\ -8.2747\ -10.9188]$$
$$K_{d2}=[-14.0584\ -1.2786\ 10.9158]$$

with n=2,

$$K_{d1}=[-31.2155\ -21.7680\ -13.3059]$$
$$K_{d2}=[-30.0575 - 16.8091 18.1496]$$

with n=4,

$$K_{d1}=[-55.8677\ -39.9455\ -13.6820]$$
$$K_{d2}=[-55.6458\ -37.7265\ 18.7637] \tag{24}$$

Figure 1 depicts the time responses of the digitally controlled system. As shown in these figures, the single-rate digitally controlled system is not stable in spite of obtaining the feasible gain matrices. On the other hand, the dual-rate controllers with $T_u=\frac{T_s}{2}$ and $T_u=\frac{T_s}{4}$ stabilize the given system.

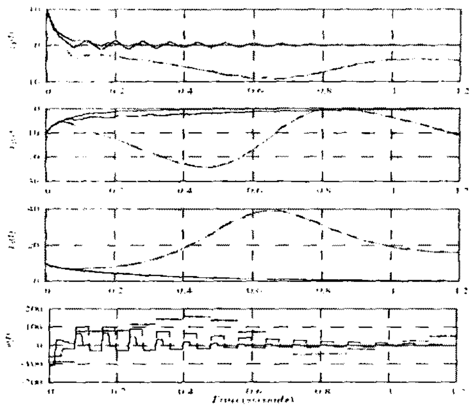

Figure 1. The time resposes of the controlled Lorenz system (T=0.08 sec., dotted line: n=1: dased line: n=2, solid lione n=4).

5. Closing Remarks

In this paper, a new dual-rate digital control method has been proposed for the T-S fuzzy system. We have formulated and solved the intersampling stability problem for the T-S fuzzy system. The proposed fast discretization approach leads to the dual-rate T-S fuzzy system which can be lifted to a single-rate discrete-time system. For this system, the stability conditions at the fast-rate sampling points have been derived. Finally, for the digitally controlled T-S fuzzy system, the sufficient stabilization conditions in the sense of the Lyapunov asymptotic stability have been derived.

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