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Self-Recurrent Wavelet Neural Network Observer Based Sliding Mode Control for Nonlinear Systems

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자기 회귀 웨이블릿 신경 회로망 관측기 기반 비선형 시스템의 슬라이딩 모드 제어

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Abstract - This paper proposes the self-recurrent wavelet neural network (SRWNN) observer based sliding mode control (SMC) method for nonlinear systems. Unlike the classical SMC, we assume that all states of nonlinear systems are not measured and design the SRWNN observer to measure the states of nonlinear systems. The SRWNN in the observer is used for approximating the observer system's gain. To generate the control input for controlling the nonlinear system, the measured states are used. The sliding surface with a boundary layer is defined to remove the chattering of the control input. Simulation result to show the effectiveness of the SRWNN observer is presented.

1. Introduction

The conventional sliding mode control (SMC) theories assume that the states can be available for feedback. However, in real application, only the output of the plant rather than the full state vector can be measured. Therefore, designing a state estimator with a good accuracy is an essential condition for achieving a high performance SMC system.

The nonlinear observers using neural network have suggested during the past years[1-2]. But, since the neural network has some limitation such as convergence, settlement of local minima, the performance of the estimation does not work satisfactorily. Thus, we develop an observer using the SRWNN, which combines the properties of attractor dynamics of recurrent neural network and the fast convergence of wavelet neural network for estimating the states of nonlinear systems. The SRWNN in the observer is applied for approximating the observer system's gain. The estimated states are used for generating the control input of the SMC for nonlinear systems. The sliding surface with boundary layer is defined to remove the chattering of the control input[3]. To verify the performance of the SRWNN observer based SMC, we consider the Duffing system, the representative continuous-time chaotic nonlinear system.

2. Problem formulation and design of the SMC

Consider the nonlinear systems

$$x^n(t) = f(X) + g(X)u \tag{1}$$

where state vector $X = [x, \dot{x}, \ldots, x^{n-1}]^T \in \boldsymbol{R}^n$, $u \in \boldsymbol{R}$ is the control input, $f(X)$ is the general nonlinear function, and $g(X)$ is the control gain. Then, the objective of the SMC is to get the state X to track a reference state $X_d = [x_d, \dot{x}_d, \ldots, x_d^{n-1}]^T$.

Let us define the tracking error vector

$$\begin{aligned} E &= [e_1, e_2, \ldots, e_n]^T \\ &= [x - x_d, \dot{x} - \dot{x}_d, \ldots, x^{n-1} - x_d^{n-1}]^T. \end{aligned} \tag{2}$$

And its derivatives are the state variables as follow:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ &\vdots \\ \dot{e}_n &= f(X) + g(X)u - x_d^n \end{aligned} \tag{3}$$

Now, define a time-varying sliding surface [3]

$$S(e;t) = C^T E = 0 \tag{4}$$

where, $C = [c_1, c_2, \ldots, c_n]^T$, $C \in \boldsymbol{R}^n$, $c_n = 1$, and C is given for $S(e;t) = 0$. It means that the resultant system is stable. Thus the problem of tracking the n-dimensional vector X_d can be replaced by a $1-st$ order stabilization problem in $S(e;t)$. If the states are outside the sliding surface, to drive the states to the sliding surface, we need the sliding condition as follows:

$$S\dot{S} \le -\lambda |S| \tag{5}$$

Then we take the derivative of (4) and obtain

$$\dot{S}(e;t) = \sum_{i=1}^{n-1} c_i e_{i+1} + f(X) + g(X)u - x_d^n = 0 \tag{6}$$

From Eq. (6), obviously, the equivalent control input u_{eq} is defined as

$$u_{eq} = \frac{1}{g(X)}\left(-\sum_{i=1}^{n-1} c_i e_{i+1} - f(X) + x_d^n\right). \tag{7}$$

And to guarantee the convergence of the states trajectory, we define the corrective control input u_c, which is shown as follows:

$$u_c = -\frac{1}{g(X)} K sgn(S) \tag{8}$$

where $K > 0$ and the sign function is a discontinuous functions as follows:

$$sgn(S) = \begin{cases} 1, & S > 0 \\ 0, & S = 0 \\ -1, & S < 0 \end{cases}. \tag{9}$$

Accordingly, the entire control input is

$$u = u_{eq} + u_c \tag{10}$$

Due to the corrective control input (8), chattering is generated. Chattering is undesired because it may excite the high frequency response of the system. Thus we define the corrective control input with the boundary layer as follows[3]:

$$u_c = -\frac{1}{g(X)} K sat(\frac{S}{\Omega}) \tag{11}$$

where Ω denotes the boundary layer thickness and

$$sat(\frac{S}{\Omega}) = \begin{cases} \frac{S}{\Omega}, & \left|\frac{S}{\Omega}\right| \le 1 \\ sgn(\frac{S}{\Omega}), & \left|\frac{S}{\Omega}\right| \ge 1 \end{cases}. \tag{12}$$

Remark 1: The sliding mode control is usually used under assumption that all states can be measured. However, in most realistic systems, only the output of the plant can be measured.

3. SRWNN observer based SMC

In the section, we first design the structure and algorithm of the SRWNN observer, then discuss the training method of the SRWNN. Finally, the SMC method using the states, which are estimated by the SRWNN observer, is discussed.

3.1 SRWNN observer

The SRWNN, a modified model of a wavelet neural network(WNN), has the attractive ability such as dynamic attractor, information storage for later use. Unlike a WNN, since the SRWNN has the mother wavelet layer which is composed of self-feedback neurons, mother wavelet nodes of the SRWNN can store the past information of the network[4]. Thus the SRWNN can be used as a better tool to approximate the nonlinear systems than a WNN. In this paper, we design a nonlinear observer using the SRWNN with these properties.

Consider the general model of nonlinear system again,

$$\begin{aligned} \dot{X}(t) &= q(X, u) \\ y(t) &= h(X) \end{aligned} \tag{13}$$

where $X \in \boldsymbol{R}^n$, q and h are known, only the output $y(t)$ is assumed to be measurable.
An observer that estimates states in (13) is

$$\begin{aligned} \dot{\hat{X}}_p(t) &= q(\hat{X}_p, u) \\ \hat{y}(t) &= h(\hat{X}_n) \end{aligned} \tag{14}$$

where $\hat{X}_p$ denotes the states of the observer as follows:

$$\hat{X}_p = \hat{X}_n + L(\hat{X}_n, e_o) e_o \tag{15}$$

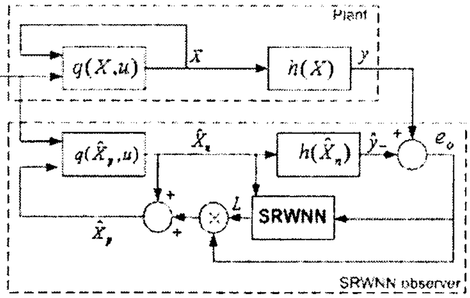

Figure 1 The structure of the proposed SRWNN observer

The SRWNN is used to represent the function $L(\hat{X}_n, e_o)$. In Fig. 1, note that the estimated state $\hat{X}_n$ and the error e_o are the input to the SRWNN, and the output of the SRWNN $L(\hat{X}_n, e_o)$ is multiplied by e_o. This guarantees that, when the error e_o is zero, no update occurs to the estimated states.

3.2 SRWNN Training

Our goal is to minimize the following quadratic cost function:

$$J = \frac{1}{2} [y - \hat{y}] = \frac{1}{2} e_o^2 \tag{16}$$

where, y and $\hat{y}$ are the output of the plant and the observer, respectively.
By using the gradient descent (GD) method, the weight values of SRWNN are adjusted so that the error is minimized after a given number of training cycles. The GD method may be defined as:

$$\begin{aligned} W(n+1) &= W(n) + \Delta W(n) \\ &= W(n) + \eta(-\frac{\partial J}{\partial W}) \end{aligned} \tag{17}$$

where, W is the weighting vector of the SRWNN[4]. The partial derivative of the cost function with respect to W is

$$\frac{\partial J}{\partial W} = -e^2 \frac{\partial \hat{y}}{\partial \hat{X}_n} \frac{\partial \hat{X}_n}{\partial \hat{X}_p} \frac{\partial L}{\partial W} \tag{18}$$

where, $\partial L / \partial W$ is computed by the component of weight vector in [4]. Since Eq. (14) is the nonlinear function, we cannot define $\partial \hat{y} / \partial \hat{X}_n$ and $\partial \hat{X}_n / \partial \hat{X}_p$. Thus, we linearize Eq. (14) as follows:

$$\dot{\hat{X}}_p(t) = A\hat{X}_p(t) + Bu(t)$$
$$\hat{y}(t) = C\hat{X}_n(t) \quad (19)$$

where, $A = \partial q/\partial \hat{X}_p$, $B = \partial q/\partial u$, and $C = \partial h/\partial \hat{X}_n$. Accordingly, Eq. (18) is redefined as follows:

$$\frac{\partial J}{\partial W} = e^2 C^T (A - LC^T)^{-1} \frac{\partial L}{\partial W} \quad (20)$$

3.3 SMC using the SRWNN observer

Figure 2 shows the structure of the SMC using the SRWNN observer. The states estimated by the SRWNN observer are used for generating the control input. From Eqs. (7) and (11), the control input u is defined as follows:

$$u = \frac{1}{g(\hat{X})}\left(-\sum_{i=1}^{n-1} c_i e_{i+1} - f(\hat{X}) + x_d^n - Ksat\left(\frac{\hat{S}}{\Omega}\right)\right) \quad (21)$$

where $\hat{S} = C^T \hat{E}$, $\hat{E} = [\hat{x} - x_d, \dot{\hat{x}} - \dot{x}_d, \ldots, \hat{x}^{n-1} - x_d^{n-1}]^T$.

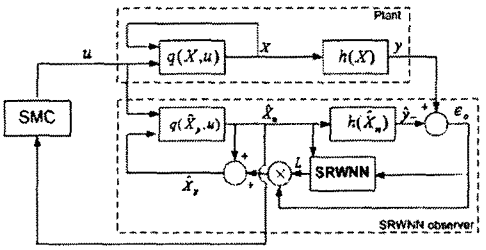

Figure 2 The structure of the SMC using the SRWNN observer

4. Simulation results

Consider the Duffing system, the representative continuous-time chaotic nonlinear system,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ a_1 x_1 - x_1^3 - a_2 x_2 + b\cos(wt) + u \end{bmatrix} \quad (22)$$

where $a_1 = 1.1$, $a_2 = 0.4$, $b = 2.1$, and $w = 1.8$.

The reference signal is defined as one periodic solution in the case of $b = 2.3$. In the simulation, take $\eta = 0.005$, $\lambda = 0.9$, $c_i = 20$, $\Omega = 0.1$, $A = \begin{bmatrix} 0 & 1 \\ a_1 & -a_2 \end{bmatrix}$, and $C = [1\ 0]^T$. And the used SRWNN is a simple structure. That is, the number of the product node of the SRWNN is one. Figure 3 shows the estimation and SMC error. They show that the SRWNN observer is a good estimator of the nonlinear system. And also, note that the SMC using the estimated states has a good tracking ability of the nonlinear system. The estimation and control performance measures are tabulated in Table 1, using the mean-squared error (MSE) as the performance index. Since the sliding surface with the boundary layer has been designed, the control input without chattering is shown in Fig. 4.

3. Conclusion

In this paper, we have developed the SRWNN observer for the nonlinear system. The SRWNN, which has a simple structure, has been utilised in approximating the nonlinear observer gain. And to examine the performance of the SRWNN observer, the SMC using the SRWNN observer has been designed. The performance of the observer and control have been verified through a simulation of the Duffing system.

Table 1. The performance measures

	Estimation error	Control error
State x_1	0.00098	0.00076
State x_2	0.00065	0.00055

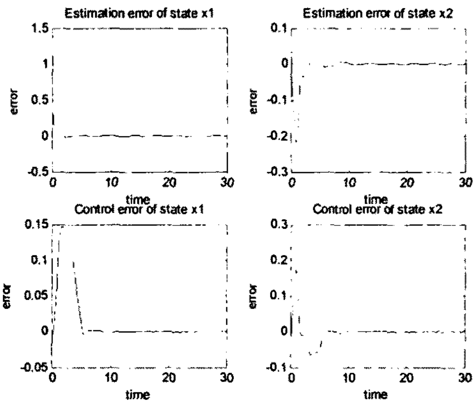

Figure 3. The estimation error and control error

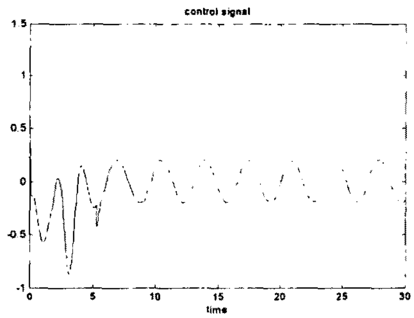

Figure 4. The control input for the Duffing system

[Reference]

[1] F. Abdollahi, H.A. Talebi, and R.V. Patel, "A stable neural network observer with application to flexible-joint manipulators", *Neural Information Processing*, Vol. 4, No. 18-22, pp. 1910-1914, 2002.

[2] M.S. Ahmed and S.H. Riyaz, "Design of dynamic neural observers", *IEE Proc. Control Theory and Appl.*, Vol. 147, No. 3, pp. 257-266, 2000.

[3] J. Slotine and W. Li, *Applied nonlinear control*, Englewwod Cliffs, NJ: Prentice-Hall, 1991.

[4] Sung Jin You, Yoon Ho Choi, and Jin Bae Park, "Generalized predictive control for chaotic systems using a self-recurrent wavelet neural network", *Information and Control Conf.*, pp. 421-424, 2003.

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